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ELEMENTS OF NAVIGATION
CHART, LEAD, AND LOG
DEVIATION AND COMPASS COMPENSATION
PILOTING
DEAD RECKONING
NAUTICAL TABLES

SCRANTON
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CONTENTS

ELEMENTS OF NAVIGATION

SYSTEM OF DETERMINING POSITION OF A SHIP AT SEA

INTRODUCTORY REMARKS

1. Navigation, as a general term, may be defined as the art or science by which a ship is conducted with safety and by the shortest convenient route from one port to another. It is the science by which the position of a ship is determined, or fixed, at the end of certain intervals of time, and, in fact, at any required time. Thus, the navigating officer of a vessel, besides establishing the position of his ship each noon or oftener, should be able, at a few minutes' notice, to give the location of the vessel whenever called on to do so.

2. Navigation, considered as a whole, is divided into two distinct branches, known, respectively, as *navigation by account* and *nautical astronomy*. The former is also called *navigation by dead reckoning*, or simply *navigation*. In deep-sea navigation, or whenever the vessel is out of sight of land or not in range with any object whose position is fixed and known, both branches are carried on together.

3. Navigation by account consists mainly in measuring and recording the courses and distances run by the ship, and in finding, on the basis of these data, the position of the ship in reference to some known point on land or to a previously established position. It also includes the plotting of the ship's position on the chart and the calculation of courses and distances to be run in order to reach the port

of destination or any desired point along the track toward that port.

The principal instruments used in navigation by account are the *chart*, representing the part of the sea in which the ship is navigating; the *lead*, used to ascertain the depth and character of the bottom; the *mariner's compass*, and the *log*, the former to determine the direction in which the ship is proceeding, and the latter to measure the distance run in that direction.

4. Nautical astronomy consists in finding the ship's position from observations made of the sun, the moon, or a star. By measuring at a stated time the position of any of these bodies with reference to some fixed line, usually the sea horizon, the data obtained makes it possible to calculate the exact position of the ship.

The principal instruments used in nautical astronomy are the *sextant*, the *chronometer*, and the *azimuth instrument*. The Nautical Almanac and suitable nautical tables facilitate the solution of the various problems.

THE TERRESTRIAL SPHERE

PRELIMINARY DEFINITIONS

5. The **axis of the earth** is that diameter about which the earth daily revolves with a uniforn motion from west to east, each revolution being completed in 24 hours.

6. The **poles of the earth** are the extremities of its axis, or the points where the axis meets the surface. Thus, if the line PP', Fig. 1, represents the axis of the earth, the points P and P' are the poles; the pole to which the United States is nearest is the *north pole*, while the opposite is the *south pole*.

7. The **equator** is a great circle on the earth's surface, every point of which is at an equal distance from both poles. It divides the earth into two equal parts, or **hemispheres**—

the *northern hemisphere* and the *southern hemisphere.* The poles of the earth are at a distance of 90° from the equator.

8. The **meridians** are great circles that pass through the poles of the earth and are therefore perpendicular to the equator. Thus, if $EbdE'$, Fig. 1, represents the earth's equator, the great circles PaP', PbP', etc., passing through and intersecting at the poles P and P', are meridians. By reason of their passing through the poles, all meridians run in true north-and-south directions.

Of all the innumerable meridians that may be imagined as being drawn on the surface of the globe, one is always, for the purpose of reference, selected as the **prime,** or **first, meridian.** Prior to 1884, most of the maritime nations having a national observatory usually adopted the meridian that passed through that observatory as the first meridian. But at a conference of delegates from the principal maritime countries of the world held at Washington in October, 1884, for the purpose of fixing a universal first or zero meridian, the one passing through the Royal Observatory at Greenwich, England, was selected.

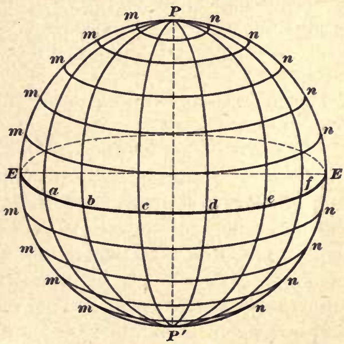


FIG. 1

9. Parallels of latitude or **latitude parallels** are circles (called small circles) whose planes are parallel to the equator. Thus, in Fig. 1, the circles mn are parallels of latitude, each being parallel to the equator and consequently running east and west. Every point on the circumference of a latitude parallel is equidistant from the equator; consequently, all places situated on the same latitude parallel are at an equal distance from the equator. Like meridians,

latitude parallels can be drawn through any place on the surface of the earth.

NOTE.—Great circles are those whose planes pass through the center of the earth. They are thus distinguished from small circles, the planes of which do not pass through the center of the earth.

FORM AND MAGNITUDE OF THE EARTH

10. In geography, as well as for navigational purposes, the earth is regarded as a sphere. From actual measurement at various parts of its surface, it has, however, been found to differ slightly from this, being somewhat flattened at the poles. This departure from the true spherical form is so trifling and inconsiderable that no practical error can result from treating the earth as a perfect sphere when laying down positions and framing directions for sailing over its surface.

The dimensions of the earth, according to Col. A. R. Clarke, are as follows:

Equatorial radius = 3,963.307 miles, or 20,926,202 feet
Polar radius = 3,949.871 miles, or 20,854,895 feet

From this, it is evident that the flattening at the poles is inconsiderable, the difference between the two radii being about 13.5 miles. By using this value of the radius, the equatorial circumference of the earth may be approximately estimated at 25,000 miles, and the surface of the whole earth, land and water, at 49,243,000 square miles.

11. The **rotundity of the earth** is manifested in many ways; of these the most obvious is the curvature of the surface of the sea, which manifests itself in a striking manner. Assume a person to be on board a vessel at sea or at the summit of a hill along the coast. In the latter case, when a steamer appears on the horizon, the first indication of its presence is the smoke from its funnel. After a while, the observer sees the upper parts of its spars and rigging, but the lower parts of the masts, the smokestack, and the hull are invisible. As the steamer approaches, these lower parts gradually come into view, and later on the entire steamer

can be seen, as illustrated in Fig. 2. In the same manner, the successive appearance of the different parts of the coast are manifested to the mariner who observes the land from the steamer. The same appearances are noted everywhere,

FIG. 2

on the Indian Ocean and the Atlantic Ocean as well as on the Great Lakes; hence, the rotundity is uniform and the earth globular. Many other proofs could be submitted, but the one already mentioned will perhaps suffice; they all tend to prove the same fact—the sphericity of the earth.

LATITUDE AND LONGITUDE

12. Latitude.—The **latitude** of any place on the earth's surface *is the distance north or south from the equator measured on the meridian that passes through the place.* Thus, if a place is situated at A, Fig. 3, north of the equator EE', its latitude is the arc, or distance, BA of the meridian PBP' that passes through the place. It is named *north* or *south* latitude according to the situation of the place in relation to the equator; thus, the latitude of A is north

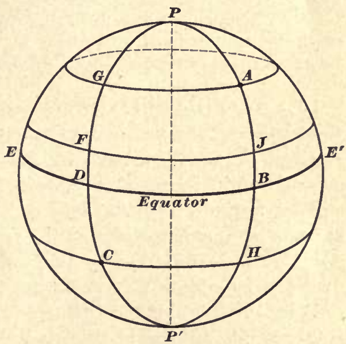


FIG. 3

because A is situated in the northern hemisphere. Again, if a place is situated at C in the southern hemisphere, its

latitude is the arc DC of the meridian PDP', and it is named south.

Latitude is reckoned from the equator toward the poles in degrees, minutes, and seconds, and since the distance from the equator to either pole is 90°, there can be no latitude higher than 90°. When a ship is on the equator, its latitude is 0°; if at the north pole, its latitude would be 90° N; if at the south pole, 90° S.

13. The **difference of latitude** of any two places is the arc of a meridian contained between the two latitude parallels passing through those places. In Fig. 3, the difference of latitude between F and G is the arc FG of the meridian contained between the latitude parallels passing through the two places.

14. All places situated on those latitude parallels have the same difference of latitude. Thus, the difference of latitude between A and F is the same as that between F and G.

The difference of latitude between A and C, Fig. 3, is the arc GC or AH. The difference of latitude between B and F, Fig. 3, is the arc FD or JB.

Difference of latitude is sometimes designated by the term *northing* or *southing*, indicating in which direction, north or south, a vessel has decreased or increased her latitude.

15. To Find the Difference of Latitude Between Two Places.—If the places are both on the same side of the equator, or, in other words, if their latitudes have the same name, the difference of latitude is found by subtracting the smaller latitude from the greater; when the two places are situated on opposite sides of the equator, that is, when their latitudes have different names, the difference of latitude is obtained by adding the two latitudes.

Thus, in Fig. 3, if the latitude of G is 50° N and that of F is 20° N, the difference of latitude between F and G is equal to $50° - 20° = 30°$. Again, if the place H is in latitude 45° S, the difference of latitude between G and H will be $50° + 45° = 95°$.

EXAMPLE 1.—Find the difference of latitude between Boston, Massachusetts, in latitude 42° 22′ N, and Philadelphia, Pennsylvania, in latitude 39° 56′ N.

SOLUTION.—The latitude of both having the same name their difference is taken; thus,

$$\begin{array}{rl} \text{Lat. Boston} = & 42^\circ\ 22'\ \text{N} \\ \text{Lat. Philadelphia} = & \underline{39^\circ\ 56'\ \text{N}}\ (-) \\ \text{Diff. of lat.} = & 2^\circ\ 26'.\ \text{Ans.} \end{array}$$

EXAMPLE 2.—The latitude of Cape Verde is 14° 43′ N; that of Cape St. Roque is 5° 28′ S; find the difference of latitude between the two places.

SOLUTION.—The latitudes being of different names their sum is taken; thus,

$$\begin{array}{rl} \text{Lat. Cape Verde} = & 14^\circ\ 43'\ \text{N} \\ \text{Lat. Cape St. Roque} = & \underline{5^\circ\ 28'\ \text{S}}\ (+) \\ \text{Diff. of lat.} = & 20^\circ\ 11'.\ \text{Ans.} \end{array}$$

EXAMPLE 3.—The latitude of Southampton, England, is 50° 54′ N; the latitude of New York is 40° 42.7′ N; required the difference of latitude between the two places.

SOLUTION.—

$$\begin{array}{rl} \text{Lat. Southampton} = & 50^\circ\ 54'\ \ \text{N} \\ \text{Lat. New York} = & \underline{40^\circ\ 42.7'\ \text{N}}\ (-) \\ \text{Diff. of lat.} = & 10^\circ\ 11.3'.\ \text{Ans.} \end{array}$$

EXAMPLE 4.—The latitude of Rio de Janeiro is 22° 54′ S, and that of Buenos Ayres 34° 36′ S; required the difference of latitude.

SOLUTION.

$$\begin{array}{rl} \text{Lat. Rio de Janeiro} = & 22^\circ\ 54'\ \text{S} \\ \text{Lat. Buenos Ayres} = & \underline{34^\circ\ 36'\ \text{S}} \\ \text{Diff. of lat.} = & 11^\circ\ 42'.\ \text{Ans.} \end{array}$$

16. When a ship sails north in northern latitudes or south in southern latitudes, she increases her latitude; but when sailing south in northern latitudes and north in southern latitudes she decreases her latitude. Therefore, when the difference of latitude and latitude from is given, the latitude in is readily found by addition or subtraction.

NOTE.—By *latitude from* is understood the latitude of the place the ship sailed from; by *latitude in*, the latitude of the place arrived at.

EXAMPLE 1.—A ship sails from a place in latitude 27° 15′ S a distance of 320′ true north; required the latitude of place arrived at.

SOLUTION.—The ship having sailed north in southern latitudes, her latitude in is evidently less than her latitude from.

$$\begin{aligned}\text{Lat. from} &= 27^\circ\ 15'\ \text{S}\\ \text{Diff. of lat. } (320') &= \underline{\ 5^\circ\ 20'\ \text{N}\ (-)}\\ \text{Lat. in} &= 21^\circ\ 55'\ \text{S. Ans.}\end{aligned}$$

EXAMPLE 2.—A ship from the west end of the island of Madeira, in latitude 32° 48′ N, sails a distance of 98′ true north; find her latitude in.

SOLUTION.—The ship sailing to the north in northern latitudes, her latitude in is evidently greater than the latitude from.

$$\begin{aligned}\text{Lat. from} &= 32^\circ\ 48'\ \text{N}\\ \text{Diff. of lat. } (98') &= \underline{\ 1^\circ\ 38'\ \text{N}\ (+)}\\ \text{Lat. in} &= 34^\circ\ 26'\ \text{N. Ans.}\end{aligned}$$

EXAMPLE 3.—A steamer from latitude 3° 8′ S runs a distance of 694′ north; find the latitude arrived at.

SOLUTION.—In this case, the steamer has evidently crossed the equator and is therefore in north latitude.

$$\begin{aligned}\text{Lat. from} &= 3^\circ\ \ 8'\ \text{S}\\ \text{Diff. of lat. } (694') &= \underline{11^\circ\ 34'\ \text{N}}\\ \text{Lat. in} &= 8^\circ\ 26'\ \text{N. Ans.}\end{aligned}$$

EXAMPLE 4.—A vessel from Pensacola, Florida, latitude 30° 20′ N, runs 72′ in a southerly direction; what is her latitude in?

SOLUTION.—

$$\begin{aligned}\text{Lat. from} &= 30^\circ\ 20'\ \text{N}\\ \text{Diff. of lat.} &= \underline{\ 1^\circ\ 12'\ \text{S}\ (-)}\\ \text{Lat. in} &= 29^\circ\ \ 8'\ \text{N. Ans.}\end{aligned}$$

17. When finding the difference of latitude made by a vessel, it should always be named either N or S, according to the direction in which the ship has run. Thus, if a vessel leaves a place in latitude 42° N for another in latitude 46° N, her course is evidently toward the north, and therefore the difference of latitude made by the ship is 46° − 42° = 4° N; if she had left 46° N for 42° N, the difference of latitude would evidently be 4° S, or 4 × 60 = 240′ S.

EXAMPLES FOR PRACTICE

1. A ship from latitude 26° 17′ S sails 190′ south; find her latitude in.
Ans. Lat. in = 29° 27′ S

2. The latitude of one place is 41° 36′ N, of another 33° 42′ N; find the difference of latitude. Ans. Diff. of lat. = 7° 54′, or 474′

3. The latitude from is 3° 2′ S, the difference of latitude sailed is 190′ north; find the latitude in. Ans. Lat. in = 0° 8′ N

4. The latitude from is 2° 48′ S, the difference of latitude sailed is 288′ north; find the latitude in. Ans. Lat. in = 2° 0′ N

5. The latitude from is 3° 42′ S, the latitude in is 1° 40′ N; find the difference of latitude sailed. Ans. Diff. of lat. = 322′ N

6. The latitude from is 0° 10′ N, the difference of latitude sailed is 228′ north; find the latitude in. Ans. Lat. in = 3° 58′ N

7. A ship is in latitude 68° 48′ N and another in 38° 30′ N; what is the difference of latitude between the two?
Ans. Diff. of lat. = 30° 18′, or 1,818′

8. A vessel from latitude 40° 40′ N arrives in latitude 33° 42′ N; what is the difference of latitude made? Ans. Diff. of lat. = 418′ S

---

18. The **colatitude** is the complement of the latitude, or 90° − latitude. Thus, in Fig. 3, the colatitude of the place A is the arc PA; if the latitude of A is 60° N, its colatitude is 90° − 60° = 30°.

19. A **nautical mile** is reckoned as 6,080 feet, or 1,013 fathoms; it is equal to the mean length of a minute of latitude. A **statute,** or **land, mile** is less than a nautical mile and contains only 5,280 feet.

20. Longitude.—The **longitude** of any place *is the distance in arc east or west, measured on the equator, from the first meridian to the meridian passing through the place.*

Thus, in Fig. 4 (a), if PBP' represents the first (Greenwich) meridian, then the arc BD of the equator EE' intercepted between the first meridian and the meridian passing through G is the longitude of that place, and is named *west* because it lies to west of the first meridian. If G had been to the east of PBP', its longitude would have been so many degrees *east.*

Longitude may also be defined as the angle at the pole subtended by the first meridian and the meridian passing through the place. This angle GPF, the arc GF of the parallel, and the arc BD of the equator contains the same number of degrees, minutes, and seconds, although the linear distance of BD is greater than GF.

21. Longitude is reckoned from 0° to 180° *east* or *west*, beginning at Greenwich, but is never considered greater than 180° either way; if it exceeds 180°, it is subtracted from 360°, the result being taken and given the contrary name. Thus, longitude 186° W is equal to 360° − 186° = 174° E, but the former expression is never used. No place, therefore, can exceed 180° in longitude, whether east or west. Fig. 4 (*b*), which represents the globe looked at from above the pole *P*, illustrates the foregoing statements.

Besides being measured in degrees, etc., longitude is also measured in time, that is, in hours, minutes, and seconds, each hour being equal to 15°.

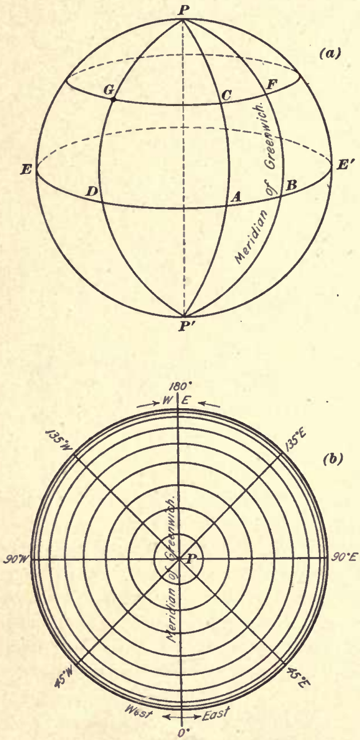


Fig. 4

22. Conversions of Time and Angular Measure.—Since longitude can be expressed either in time or in angular measure, it is necessary to be able to reduce time to angular measure and angular measure to time. This may be done by the following rules:

Rule I.—*To convert time to angular measure, multiply the number of hours by 15; the product is so many degrees. Divide the minutes and seconds by 4 and reckon every unit of remainder as 15′, if minutes be the dividend, and as 15″, if seconds be the dividend; the quotients obtained will be, respectively, degrees and minutes.*

EXAMPLE 1.—Convert $3^h\ 14^m\ 23^s$ to angular measure.

SOLUTION.—

$$
\begin{array}{rl}
3^h \times 15 = & 45^\circ \\
14^m \div 4 = 3 \text{ (Rem. 2)} = & 3^\circ\ 30' \\
23^s \div 4 = 5 \text{ (Rem. 3)} = & \underline{\quad 5'\ 45''} \\
 & 48^\circ\ 35'\ 45''. \text{ Ans.}
\end{array}
$$

Rule II.—*To convert angular measure to time, multiply the degrees and minutes by 4; then the minutes of angular measure become seconds of time, and the degrees become minutes of time, which is reduced to hours by dividing by 60.*

EXAMPLE 2.—Convert 137° 26′ into time.

SOLUTION.—

$$
\begin{array}{r}
137^\circ\ 26' \\
\underline{\times 4} \\
60)549^m\ 44^s = 9^h\ 9^m\ 44^s. \text{ Ans.} \\
\underline{540} \\
9
\end{array}
$$

NOTE.—The term *arc* is sometimes, for sake of convenience, used as a substitute expression for "angular measure."

23. Both of these processes are greatly simplified by the use of the table on the opposite page. In the first column of this table are contained degrees and minutes in angular measure; in the second column, the corresponding hours and minutes, or minutes and seconds of time; the other columns are a continuation of the first and second, respectively. The use of this table will become clear by a few examples.

EXAMPLE 1.—Required the time corresponding to 54° 23′.

SOLUTION.— Opposite 54° in column 1 is $3^h\ 36^m\ 0^s$
and opposite 23′ in column 1 is $+\quad 1^m\ 32^s$
adding, the required time is $3^h\ 37^m\ 32^s$. Ans.

EXAMPLE 2.—Required the degrees and minutes corresponding to $4^h\ 15^m\ 20^s$.

SOLUTION.— Opposite $4^h\ 12^m\ 0^s$ in column 4 is $63°\ 0'$
and opposite $+\ 3^m\ 20^s$ in column 2 is $+\ 0°\ 50'$
Hence, $4^h\ 15^m\ 20^s$ corresponds to $63°\ 50'$. Ans.

EXAMPLE 3.—Convert 137° 29′ 27″ into time.

SOLUTION.— Opposite 137° in column 5 is $9^h\ 8^m\ 0^s$
opposite 29′ in column 1 is $+\ 1^m\ 56^s$
and opposite 27″ in column 1 is $+\ 0^m\ 1.8^s$
adding, the required time is $9^h\ 9^m\ 57.8^s$. Ans.

In order to obtain the number of seconds of time corresponding to 27″ look for the number opposite 27 in column 1; the numbers found there are 1 and 48, which, according to the note at the foot of the table, must be read $1\frac{48}{60}^s$ or 1.8^s.

EXAMPLE 4.—Convert $8^h\ 19^m\ 27^s$ into degrees, minutes, and seconds of arc.

SOLUTION.— Opposite $8^h\ 16^m\ 0^s$ in column 6 is $124°\ 0'\ 0''$
opposite $+\ 3^m\ 24^s$ in column 2 is $+\ 0°\ 51'\ 0''$
and opposite $+\ 3^s$ in column 2 is $+\ 0°\ 0'\ 45''$
Hence, $8^h\ 19^m\ 27^s$ corresponds to $124°\ 51'\ 45''$. Ans.

When finding the number of seconds of arc corresponding to 3^s of time, column 2 is entered as though it were 3^m, and the required number of seconds of arc, 45, is found opposite.

24. The student should memorize the following equivalents in arc and time:

$24^h = 360°$	$1^h = 15°$
$12^h = 180°$	$1^m = 15'$
$6^h = 90°$	$1^s = 15''$
$3^h = 45°$	$4^m = 1°$
$2^h = 30°$	$4^s = 1'$

EXAMPLES FOR PRACTICE

Find the equivalents to the following values:

1. 96° 10′ 45″. Ans. $6^h\ 24^m\ 43^s$
2. 178° 48′ 45″. Ans. $11^h\ 55^m\ 15^s$
3. 2° 59′ 30″. Ans. $0^h\ 11^m\ 58^s$
4. 15° 0′ 15″. Ans. $1^h\ 0^m\ 1^s$
5. 0° 59′ 45″. Ans. $0^h\ 3^m\ 59^s$
6. $8^h\ 17^m\ 15.5^s$. Ans. 124° 18′ 52.5″
7. $17^h\ 8^m\ 22^s$. Ans. 257° 5′ 30″
8. $1^h\ 12^m\ 30^s$. Ans. 18° 7′ 30″
9. $0^h\ 1^m\ 40^s$. Ans. 0° 25′ 0″
10. $10^h\ 0^m\ 12^s$. Ans. 150° 3′ 0″

CONVERSION OF ARC AND TIME

D.	H. M.	D.	H. M.	D.	H. M.	D.	H. M.	D.	H. M.	D.	H. M.
M.	M. S.	M.	M. S.	M.	M. S.	M.	M. S.	M.	M. S.	M.	M. S.
1	0 4	61	4 4	121	8 4	181	12 4	241	16 4	301	20 4
2	0 8	62	4 8	122	8 8	182	12 8	242	16 8	302	20 8
3	0 12	63	4 12	123	8 12	183	12 12	243	16 12	303	20 12
4	0 16	64	4 16	124	8 16	184	12 16	244	16 16	304	20 16
5	0 20	65	4 20	125	8 20	185	12 20	245	16 20	305	20 20
6	0 24	66	4 24	126	8 24	186	12 24	246	16 24	306	20 24
7	0 28	67	4 28	127	8 28	187	12 28	247	16 28	307	20 28
8	0 32	68	4 32	128	8 32	188	12 32	248	16 32	308	20 32
9	0 36	69	4 36	129	8 36	189	12 36	249	16 36	309	20 36
10	0 40	70	4 40	130	8 40	190	12 40	250	16 40	310	20 40
11	0 44	71	4 44	131	8 44	191	12 44	251	16 44	311	20 44
12	0 48	72	4 48	132	8 48	192	12 48	252	16 48	312	20 48
13	0 52	73	4 52	133	8 52	193	12 52	253	16 52	313	20 52
14	0 56	74	4 56	134	8 56	194	12 56	254	16 56	314	20 56
15	1 0	75	5 0	135	9 0	195	13 0	255	17 0	315	21 0
16	1 4	76	5 4	136	9 4	196	13 4	256	17 4	316	21 4
17	1 8	77	5 8	137	9 8	197	13 8	257	17 8	317	21 8
18	1 12	78	5 12	138	9 12	198	13 12	258	17 12	318	21 12
19	1 16	79	5 16	139	9 16	199	13 16	259	17 16	319	21 16
20	1 20	80	5 20	140	9 20	200	13 20	260	17 20	320	21 20
21	1 24	81	5 24	141	9 24	201	13 24	261	17 24	321	21 24
22	1 28	82	5 28	142	9 28	202	13 28	262	17 28	322	21 28
23	1 32	83	5 32	143	9 32	203	13 32	263	17 32	323	21 32
24	1 36	84	5 36	144	9 36	204	13 36	264	17 36	324	21 36
25	1 40	85	5 40	145	9 40	205	13 40	265	17 40	325	21 40
26	1 44	86	5 44	146	9 44	206	13 44	266	17 44	326	21 44
27	1 48	87	5 48	147	9 48	207	13 48	267	17 48	327	21 48
28	1 52	88	5 52	148	9 52	208	13 52	268	17 52	328	21 52
29	1 56	89	5 56	149	9 56	209	13 56	269	17 56	329	21 56
30	2 0	90	6 0	150	10 0	210	14 0	270	18 0	330	22 0
31	2 4	91	6 4	151	10 4	211	14 4	271	18 4	331	22 4
32	2 8	92	6 8	152	10 8	212	14 8	272	18 8	332	22 8
33	2 12	93	6 12	153	10 12	213	14 12	273	18 12	333	22 12
34	2 16	94	6 16	154	10 16	214	14 16	274	18 16	334	22 16
35	2 20	95	6 20	155	10 20	215	14 20	275	18 20	335	22 20
36	2 24	96	6 24	156	10 24	216	14 24	276	18 24	336	22 24
37	2 28	97	6 28	157	10 28	217	14 28	277	18 28	337	22 28
38	2 32	98	6 32	158	10 32	218	14 32	278	18 32	338	22 32
39	2 36	99	6 36	159	10 36	219	14 36	279	18 36	339	22 36
40	2 40	100	6 40	160	10 40	220	14 40	280	18 40	340	22 40
41	2 44	101	6 44	161	10 44	221	14 44	281	18 44	341	22 44
42	2 48	102	6 48	162	10 48	222	14 48	282	18 48	342	22 48
43	2 52	103	6 52	163	10 52	223	14 52	283	18 52	343	22 52
44	2 56	104	6 56	164	10 56	224	14 56	284	18 56	344	22 56
45	3 0	105	7 0	165	11 0	225	15 0	285	19 0	345	23 0
46	3 4	106	7 4	166	11 4	226	15 4	286	19 4	346	23 4
47	3 8	107	7 8	167	11 8	227	15 8	287	19 8	347	23 8
48	3 12	108	7 12	168	11 12	228	15 12	288	19 12	348	23 12
49	3 16	109	7 16	169	11 16	229	15 16	289	19 16	349	23 16
50	3 20	110	7 20	170	11 20	230	15 20	290	19 20	350	23 20
51	3 24	111	7 24	171	11 24	231	15 24	291	19 24	351	23 24
52	3 28	112	7 28	172	11 28	232	15 28	292	19 28	352	23 28
53	3 32	113	7 32	173	11 32	233	15 32	293	19 32	353	23 32
54	3 36	114	7 36	174	11 36	234	15 36	294	19 36	354	23 36
55	3 40	115	7 40	175	11 40	235	15 40	295	19 40	355	23 40
56	3 44	116	7 44	176	11 44	236	15 44	296	19 44	356	23 44
57	3 48	117	7 48	177	11 48	237	15 48	297	19 48	357	23 48
58	3 52	118	7 52	178	11 52	238	15 52	298	19 52	358	23 52
59	3 56	119	7 56	179	11 56	239	15 56	299	19 56	359	23 56
60	4 0	120	8 0	180	12 0	240	16 0	300	20 0	360	24 0

NOTE.—When turning seconds of arc into time, and vice versa, it should be remembered that the fractions are sixtieths. Thus, the value in time of 42″ is not $2^s.48$, but $2^s.\frac{48}{60} = 2^s.8$.

25. The **difference of longitude** of any two places is the arc of the equator contained between the meridians passing through those places. Thus, the difference of longitude between the two places A and B, Fig. 5, is the arc CD of the equator EE' contained between the meridians PDP' and PCP' passing through B and A, respectively. When both places are in east, or both in west, longitude, the difference of longitude is equal to the difference between their longitudes expressed in time, or in degrees, minutes, and seconds; but if one place is in west longitude and the other in east longitude, the difference of longitude between the two is equal to the sum of their longitudes, or that sum subtracted from 360°.

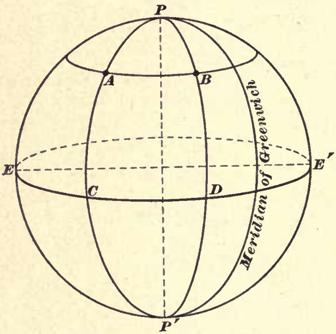


Fig. 5

Example 1.—Find the difference of longitude expressed in minutes of arc between a place in longitude 5° 3′ E and another in longitude 16° 39′ E.

Solution.—Since both places are in east longitude, the difference of longitude is equal to the difference of their longitudes; thus,

1st long. = 16° 39′ E
2d long. = 5° 3′ E
Diff. of long. = 11° 36′
Expressed in min. of arc = 696′. Ans.

Example 2.—Find the difference of longitude between New York and Charleston, S. C.; the former being in longitude 74° 0′ W, the latter in 79° 54′ W.

Solution.—Both places being in west longitude, take their difference; thus,

Long. New York = 74° 0′ W
Long. Charleston = 79° 54′ W
Diff. of long. = 5° 54′
Expressed in min. of arc = 354′. Ans.

Example 3.—One place is in longitude 1° 40′ W, another in 3° 20′ E; find the difference of longitude between the two.

Solution.—Since one place is in east, and the other in west, longitude, the difference of longitude is equal to the sum of their longitudes; thus,

$$\begin{aligned} \text{1st long.} &= 1^\circ\ 40' \text{ W} \\ \text{2d long.} &= 3^\circ\ 20' \text{ E} \\ \hline \text{Diff. of long.} &= 5^\circ\ \ 0' \\ \text{Expressed in min. of arc} &= 300'. \quad \text{Ans.} \end{aligned}$$

Example 4.—A ship is in longitude 177° 45′ W; another is in longitude 175° 27′ E; find the difference of longitude between the two positions.

Solution.—The sum of their longitudes subtracted from 360° will give the required difference of longitude; thus,

$$\begin{aligned} \text{1st long.} &= 177^\circ\ 45' \text{ W} \\ \text{2d long.} &= 175^\circ\ 27' \text{ E} \\ \hline \text{Sum} &= 353^\circ\ 12' \\ \text{Subtracted from} &\quad\ 360^\circ \\ \hline \text{Diff. of long.} &= \quad 6^\circ\ 48' \\ \text{Expressed in min. of arc} &= 408'. \quad \text{Ans.} \end{aligned}$$

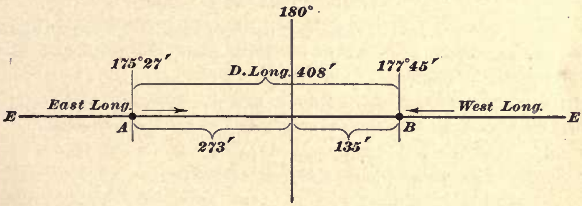


Fig. 6

Or, the difference of longitude may be found also by subtracting each longitude from 180° and adding the remainders; thus,

$$\begin{aligned} 180^\circ - 177^\circ\ 45' &= 2^\circ\ 15' = 135' \\ 180^\circ - 175^\circ\ 27' &= 4^\circ\ 33' = 273' \\ \hline \text{Diff. of long.} &= 408'. \quad \text{Ans.} \end{aligned}$$

This operation is graphically shown in Fig. 6, where the heavy line $E\,E'$ represents the equator and A and B the two places considered.

26. When a ship in west longitude sails west, and in east longitude sails east, she evidently increases her longitude. But when sailing toward the east in west longitude, and

toward the west in east longitude, her longitude decreases. Therefore, when the longitude from and the difference of longitude is known, the longitude arrived at is readily found, as shown in the following examples:

NOTE.—By *longitude from* is understood the longitude of the place the ship sailed from; by *longitude in*, the longitude of the place arrived at.

EXAMPLE 1.—A ship leaves a place in longitude 97° 45′ W, the difference of longitude sailed is 71′ east; find the longitude in.

SOLUTION.—The longitude from being west and the difference of longitude toward the east, the longitude in is found by subtracting the latter from the former; thus,

$$\begin{array}{rl} \text{Long. from} = & 97° \ 45' \text{ W} \\ \text{Diff. of long. } 71' = & \underline{\ 1° \ 11' \text{ E}} \\ \text{Long. in} = & 96° \ 34' \text{ W. Ans.} \end{array}$$

EXAMPLE 2.—A ship in longitude 1° 20′ W changes her longitude 236′ to the eastward; required her longitude in.

SOLUTION.—

$$\begin{array}{rl} \text{Long. from} = & 1° \ 20' \text{ W} \\ \text{Diff. of long. } 236' = & \underline{3° \ 56' \text{ E}} \\ \text{Long. in} = & 2° \ 36' \text{ E. Ans.} \end{array}$$

In this case, the ship changes her position from west to east longitude by an amount equal to the longitude from subtracted from the difference of longitude.

EXAMPLE 3.—Find the longitude in, the longitude from being 160° 20′ W and the difference of longitude 2,480′ to the westward.

SOLUTION.—

$$\begin{array}{rl} \text{Long. from} = & 160° \ 20' \text{ W} \\ \text{Diff. of long. } 2{,}480' = & \underline{\ 41° \ 20' \text{ W}} \\ \text{Sum} = & 201° \ 40' \text{ W} \\ \text{Subtracted from} & \underline{360°} \\ \text{Long. in} = & 158° \ 20' \text{ E. Ans.} \end{array}$$

It should be noted in the preceding examples that the difference of longitude is named E or W according to the direction in which the ship has run. Thus, if a vessel in longitude 35° W sails 3° east, the difference of longitude made is 180′ E; if she sails 3° west, the difference of longitude is 180′ W.

EXAMPLES FOR PRACTICE

1. The longitude from is 110° 42′ W, longitude in is 101° 42′ W; find the difference of longitude. Ans. Diff. of long. = 540′ E

2. Longitude from is 2° 30′ E, the difference of longitude is 126′ E; find the longitude in. Ans. Long. in = 4° 36′ E

3. Longitude in is 1° 40.4′ W, the difference of longitude sailed is 100.4′ W; what is the longitude from? Ans. Long. from = 0° 0′

4. Longitude from is 3° 10′ W, the difference of longitude is 380′ E; find the longitude in. Ans. Long. in = 3° 10′ E

5. Longitude from is 62° 32′ E, the longitude in is 45° 51.5′ E; find the difference of longitude. Ans. Diff. of long. = 1,000.5′ W

6. Longitude from is 178° 15′ E, the longitude in is 178° 45′ W; find the difference of longitude. Ans. Diff. of long. = 180′ E

---

27. Relation Between Time and Longitude.—Since the circumference of the earth is 360°, the sun, in making its apparent daily circuit in 24 hours, moves through 360°; hence, in 1 hour it moves through $\frac{360}{24}$, or 15°. When the sun has attained its greatest altitude, or is on the meridian of any place, it is noon there; hence, the time at any place 15° east of that meridian will be 1 hour past noon, and at any place 15° west of that meridian, 1 hour before noon.

Thus, at the instant of high noon at Greenwich, the time in longitude 30° W is 10 A. M. and in longitude 30° E it is 2 P. M. In other words, the time increases or decreases at the rate of 1 hour for every 15° of longitude, depending on whether the direction is east or west of Greenwich or any other meridian. This is shown in the chart, Fig. 7, where the heavy black vertical line represents the meridian of Greenwich and the vertical light lines the meridians east and west, 15° or 1 hour apart. Referring to this chart, find the time at Philadelphia, Pennsylvania, when it is noon at Greenwich. Philadelphia being in longitude 75° W, about, the time at that place would be 7 o'clock in the morning, and therefore the difference in time between Philadelphia and Greenwich is 5 hours, or 75°. Again, when it is 7 o'clock A. M. in Philadelphia it is 10 A. M. in longitude 30° W and

2 P. M. in longitude 30° E; and so on. From this, it is evident that if the navigator knows the difference between his

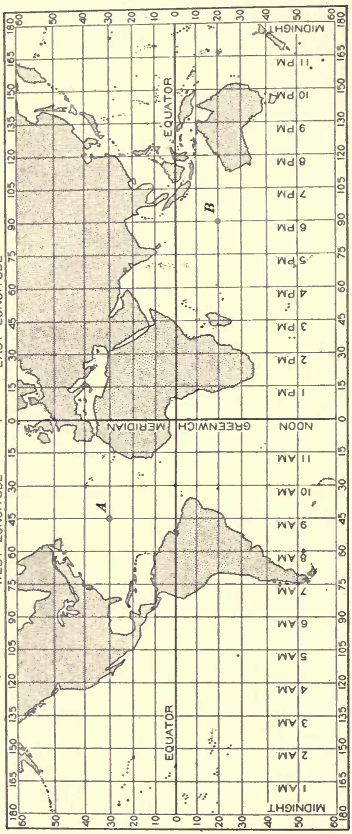


FIG. 7

local time and that of any standard meridian—Greenwich, for instance—he has a means of determining the longitude

of his ship. The greater the difference of longitude, the greater is the difference of time between any two places; from this, it will be seen that the relation between time and longitude are so intimate that they may almost be said to be identical.

28. Determination of a Position on the Earth's Surface.—From the foregoing explanations, the student will readily understand that any point, or place, on the surface of the earth is determined when its latitude and longitude are known. Hence, the principal object of navigation is the determination of these quantities. When the navigator, by one method or another (hereafter to be described), finds his latitude, he knows how many degrees, minutes, etc. he is north or south of the equator, and by obtaining his longitude he knows how many degrees, minutes, etc. he is east or west of the first, or Greenwich, meridian. Then by consulting his chart he will at once be able to mark on it the exact position of his vessel.

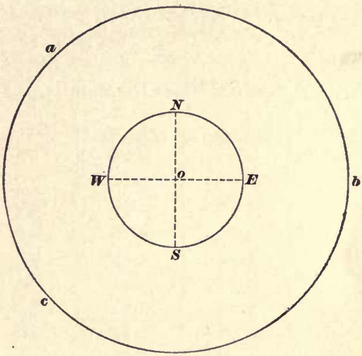


FIG. 8

For example, if his latitude and longitude are, respectively, 30° N and 45° W, the position of his vessel is at the intersection of the given parallel and meridian, or at *A*, Fig. 7; if the latitude is 20° S and the longitude 90° E, its position will be at *B*, where the 90° meridian crosses the 20° parallel of south latitude.

COURSE, DISTANCE, AND DEPARTURE

29. Sea Horizon.—The apparent boundary between the sky and sea, which to an observer, when not in sight of land, appears to be encircling him, is called the **sea, or visible, horizon.** It may also be defined as a small circle drawn on the surface of the earth, the center of which is the observer's eye. Thus, in Fig. 8, if the large circle *a b c* represents the earth and an observer is stationed at *o*, the small circle *N E S W* is the visible, or sea, horizon. Its principal points, called the *horizon*, or *cardinal*, *points*, are four, viz., north, east, south, and west; north and south, east and west being diametrically opposite each other, as indicated in the figure. The whole circumference is divided into thirty-two equal parts called *points*, thus making the number of points in each quadrant equal to eight. The length of the radius of this circle or, in other words, the sea horizon's distance from the observer is varied, depending on the height of the observer's eye above the surface of the sea.

TABLE I

Height of Eye Feet	Distance of Sea Horizon		Height of Eye Feet	Distance of Sea Horizon	
	Nautical Miles	Statute Miles		Nautical Miles	Statute Miles
5	2.56	2.95	45	7.69	8.87
10	3.62	4.18	50	8.11	9.35
15	4.44	5.12	55	8.50	9.81
20	5.13	5.91	60	8.88	10.24
25	5.73	6.61	70	9.59	11.06
30	6.28	7.24	80	10.26	11.83
35	6.78	7.82	90	10.88	12.54
40	7.25	8.36	100	11.47	13.22

30. Distances of Visibility.—Table I will give an idea of the sea horizon's distance at different heights of the eye, expressed in both nautical and statute miles.

Thus, an object seen on the horizon on a clear day, the height of the eye being, for instance, 20 feet, is 5.13 nautical, or 5.91 statute, miles away.

31. Course.—By the course run or steered by a vessel is understood the angle that the ship's fore-and-aft line makes with a meridian. It is reckoned from north or south toward east or west, and is measured either in degrees or in *points* of 11° 15′ each. *True course*, or the course made good, is the angle contained between a true, or geographical, meridian and the ship's track or path over ground.

32. The Rhumb Line.—When a ship proceeds from one place to another in one uniform direction, her track will intersect all meridians at the same angle. Thus, in Fig. 9 (*a*), if a ship sails from *a* to *d*, steering a straight

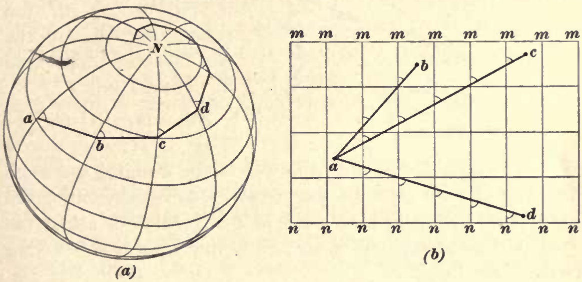


FIG. 9

continuous course, the line *a b c d*, representing her track, will intersect all meridians at the same angles; in other words, *N a b*, *N b c*, *N c d*, etc. are all equal. Since the meridians all converge toward the pole, it is evident that this line, called the **rhumb line,** in its continuity is spiral, as shown in the figure, always approaching the pole but never actually reaching it.

This is perhaps better illustrated in Fig. 9 (*b*), which represents a Mercator's chart, the vertical lines *m n* being the meridians running north and south. There, if a ship runs from *a* to *b*, *c*, or *d*, following a straight track, the angles

at which this track intersects the meridians are all equal, as is evident by inspecting the figure.

33. Bearing.—The **bearing** of an object or place is the angle that the direction of the object or place makes with the meridian, and is the same as the course toward it. Thus, in Fig. 9 (*b*), if an observer is standing at *a*, the bearing of the object *b* is the line *a b* running directly toward it, and is measured by the angle contained between the line of bearing and the meridian. The bearing between two places is identical to the rhumb course from one place to the other.

34. Distance.—The **distance** between two places, or the distance run by a vessel on any course, is the length of the rhumb line joining the two places, expressed in nautical miles.

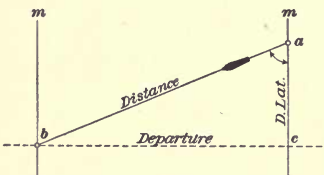


FIG. 10

35. Departure. The distance made good by a ship due east or due west when running between any two places is called **departure.** It is usually expressed as so many miles east or west, and is measured along the parallel connecting one place with the meridian running through the other.

In order to more fully explain the foregoing terms the student is referred to Fig. 10. There, if a ship sails from *a* to *b*, the line *a b* is the *distance;* and if the lines *m n* represent meridians running north and south, *a c* is the *difference of latitude*, the angle at *a* the *course*, and the distance *c b* the *departure*.

NAUTICAL TERMS

36. To those students not sufficiently versed in the meanings of nautical terms, or terms relating to the rig and fittings of a ship, appearing throughout this Course, the following brief explanations will be useful.

37. Bow and Stern.—The fore end of a ship or boat is designated as **bow**, the after part as **stern**, and the middle part, with regard to either length or breadth, is known as the **midship** section of the ship.

38. The **fore-and-aft line** of a ship runs from bow to stern or vice versa; it coincides with the direction of the keel and divides the ship in two equal parts, known, respectively, as the **port** and **starboard sides**, port being to the left and starboard to the right when looking from stern toward the bow.

39. Athwartship means across the length of a vessel, or a direction perpendicular to any point along the fore-and-aft line. **Abeam** and **abreast** designates a direction at right angles to the ship's fore-and-aft line; also, opposite the center of a ship. Thus, an object is said to be abeam or abreast when it is exactly opposite the center portion of a ship.

40. Ahead is a term that refers to any object, moving or otherwise, forwards of a line that passes through the center of the ship at right angles to the fore-and-aft line. A ship is going ahead when it moves forwards. A *head wind* is a wind blowing against the direction in which the ship is progressing.

41. Dead ahead is a direction exactly perpendicular to the ship's athwartship line. When an object is sighted dead ahead, its bearing from the ship coincides with the direction of the fore-and-aft line. **Astern** is the opposite of ahead;

it also means the after part of the ship, or in the direction of the stern.

42. Relative Bearings.—**Relative bearings** are bearings that refer exclusively to the direction in which a ship is heading. Thus, in Fig. 11, if a ship is headed north and an object is sighted dead ahead, the same object will be abeam when the ship is headed east or west, as shown.

43. How Relative Bearings Are Named.—Relative bearings are named in points, as follows: Beginning from dead ahead on the starboard side, they are:

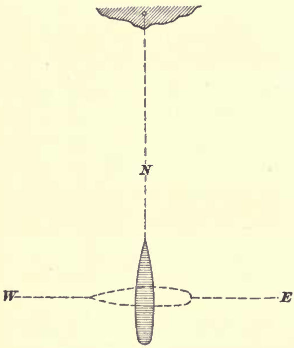


FIG. 11

1 point on starboard bow
2 points on starboard bow
3 points on starboard bow
Broad on (or off) starboard bow
3 points forward of starboard beam
2 points forward of starboard beam
1 point forward of starboard beam
Starboard beam (abeam or abreast)
1 point abaft starboard beam
2 points abaft starboard beam
3 points abaft starboard beam
Broad on (or off) starboard quarter
3 points on (or off) starboard quarter
2 points on (or off) starboard quarter
1 point on (or off) starboard quarter
Astern

Thus, in Fig. 12, from dead ahead to abeam is eight points; from dead ahead to astern is sixteen points; from broad off bow to quarter is eight points. The bearings on the port side are designated in a similar manner, the word starboard in each case being replaced by port.

44. Windward and Leeward.—Wind is named according to the direction from which it comes. Thus, a wind blowing from the north is a northerly wind, and from the east an easterly wind. **Windward** is the direction from which the wind blows and **leeward** is the direction toward which the wind blows.

45. Working to Windward.—When a sailing vessel, by reason of an adverse wind, cannot steer directly for the port of destination, her head is laid as close to the wind as her sails will permit. She is then said to be **close hauled** on either port or starboard **tack,** depending on which side of the ship is facing the wind. When the wind is on the starboard side, the vessel is on the *starboard tack;* when on the port side, it is on the *port tack.* Thus, in Fig. 13, the ships *a* and *c* are both on the port tack, while *b* and *d* are on the starboard tack, the arrows indicating the direction of the wind. A ship heading alternately on port and starboard tacks, trying to reach her destination by a zigzag course in the face of a head-wind, as shown in Fig. 13, is

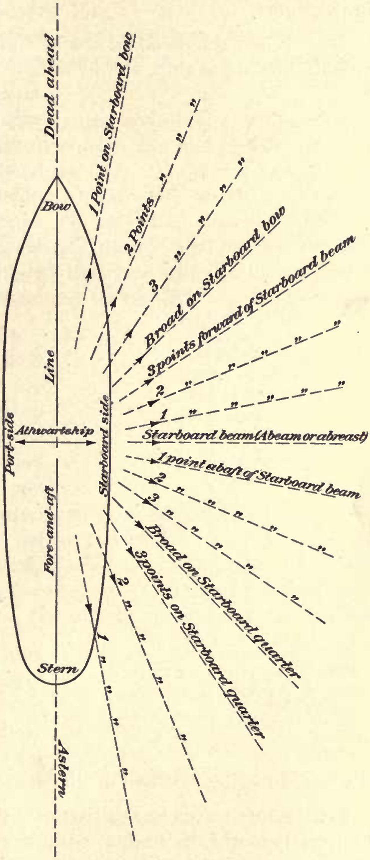


FIG. 12

said to be *beating* or *working to windward.* Few ships can lie closer than five points to the wind. A square-rigged

vessel usually cannot get closer than six points, as indicated in the figure.

46. When the wind blows at right angles to the fore-and-aft line of a ship, she is said to have the *wind on the beam*. The head of the ship is then eight points from the wind. When the head is more than eight points from the wind, the wind is said to be *abaft the beam;* when less than eight points, it is said to be *before the beam*.

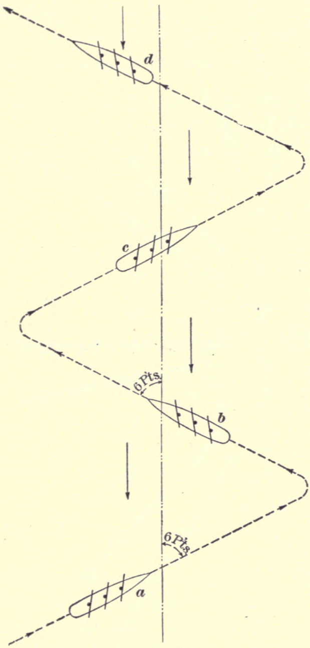


FIG. 13

47. Adrift means floating at random, without control. Thus, a boat or vessel is said to be adrift when broken from her moorings, or when her machinery or sails cannot be used to control her progress in any desired direction.

48. Aground indicates the situation of a ship or any floating vessel, log, or raft that is in direct contact with, or resting on, the ground. It also signifies stranded, or the state of being disabled or hindered by having run ashore.

49. Aloft refers to any part of a ship situated above the hull, particularly the higher, or loftiest, part of the rigging. *Going aloft* means going up in the rigging of a ship.

50. Ballast is a certain amount of weighty substances, such as stone, pig iron, water, etc. that is placed in the ship's

hold to give her proper stability when she has either no cargo or an insufficient amount of cargo. In steam vessels, water tanks are usually provided for this purpose, the water being let in from the outside and pumped out when the voyage is completed.

51. Division of Time on Shipboard.—The day on shipboard is divided into **watches**, the *port* and the *starboard watch*, consisting of 4 hours each, except the period from 4 to 8 P. M., which is divided into two watches of 2 hours each, called *dog watches*. The object of this is to make an odd number of watches during the 24 hours, so that the starboard and port watches of the crew will not be on duty at the same time every day. The watches are designated as follows:

12 noon to 4 P. M.	Afternoon watch
4 to 6 P. M.	First dog watch
6 to 8 P. M.	Second dog watch
8 P. M. to midnight	First watch
Midnight to 4 A. M.	Mid watch
4 to 8 A. M.	Morning watch
8 A. M. to 12, noon	Forenoon watch

The time on shipboard is marked by strokes on the ship's bell, and is expressed by the number of bells (strokes) that have been struck; thus, 1 bell is one stroke of the bell, 6 bells is six strokes of the bell, and so on. Counting from 12 o'clock, noon, which is 8 bells, the half hours through the day and night run as follows:

12:30 P. M.	1 bell	7:00 P. M.	6 bells
1:00 P. M.	2 bells	7:30 P. M.	7 bells
1:30 P. M.	3 bells	8:00 P. M.	8 bells
2:00 P. M.	4 bells	8:30 P. M.	1 bell
2:30 P. M.	5 bells	9:00 P. M.	2 bells
3:00 P. M.	6 bells	9:30 P. M.	3 bells
3:30 P. M.	7 bells	10:00 P. M.	4 bells
4:00 P. M.	8 bells	10:30 P. M.	5 bells
4:30 P. M.	1 bell	11:00 P. M.	6 bells
5:00 P. M.	2 bells	11:30 P. M.	7 bells
5:30 P. M.	3 bells	12:00 midnight	8 bells
6:00 P. M.	4 bells	12:30 A. M.	1 bell
6:30 P. M.	5 bells	And so on as before	

52. Currents.—A current may be defined as a large volume of water, or a portion of the sea, which is kept in motion by the wind or by some other cause or causes. The direction of a current or the point of the compass toward which it flows is called the *set* of the current; and its velocity or rate is called the *drift*. The mode adopted in speaking of the direction of a wind, which is named according to the point from which it blows, is thus reversed in speaking of the direction of a current. Hence, a current setting toward the north is called a northerly current, and a current setting toward the east is called an easterly current.

53. Tides.—Briefly stated, the phenomenon of **tide** is the periodical and alternate rise and fall of the water level, as seen from sea beaches, cliffs, etc. When the water rises to the highest point it is capable of reaching on any particular day, it is called *high water* or *high tide;* when it sinks to the lowest corresponding point, it is said to be *low water* or *low tide*. High tides follow each other at intervals of 12 hours and 25 minutes. Low tides succeed each other at the same intervals.

Tides do not always rise to the same height, but every fortnight after new and full moons they rise much higher than in the alternate weeks, or after the first and last quarters of the moon. These high tides are called *spring tides*, and the lower ones *neap tides*.

During the first and third quarters of the lunar month, the sun's influence tends to draw the tide to the westward of its position under the influence of the moon, and so tends to hasten the time of high water; this is called *priming*. During the second and fourth quarters the sun has an opposite effect, and tends to delay the time of high water, thus causing what is known as *lagging*.

Slack water is the interval of time when the tide has reached its lowest or its highest point and has not yet commenced to rise or recede.

The term *set* applies to tides as well as to currents, and indicates the compass direction toward which the tide or

current is flowing. The *drift* of a tide or current is its rate of movement, usually expressed in knots.

The *range* of a tide is the vertical distance between the high and low water of any tide.

The *age* of a tide is the interval between the time of new or full moon and the succeeding spring tide.

54. Tide Tables.—For practical purposes, the time of high and low water for any port along the seaboard of the United States may be conveniently found from Tide Tables or Tide Books published annually by the United States Coast and Geodetic Survey and also by private concerns. In such books, the time and range of tides at all principal ports are predicted for every day of the year. Full instructions for using the tables are usually given in the books.

55. Sternboard is a term indicating the backward motion of a vessel. A sailing vessel is said to be *gathering sternway* when, by misadventure, or purposely, her head is brought up against the wind with her sails aback, causing her to drift backwards.

56. Lie-To.—When a vessel encounters a strong gale and her head is laid as close to the wind as possible, in order to prevent seas from boarding her, she is said to **lie-to** or **heave-to.**

57. Under Way.—A vessel is said to be **under way** when she is moving through water in any desired direction; or, in general, when she is not at anchor, or made fast to the shore, or aground.

58. Under Weigh.—This term **under weigh** applies, in particular, to the operation of preparing a vessel to get under way, such as loosing the sails or getting up steam and weighing the anchor.

59. Wake.—By the **wake** of a ship is understood the glossy and sometimes smooth track left on the surface of the water behind an advancing ship.

60. Taffrail.—The rail that is affixed to the stern and quarters or that runs around the stern of a ship is called the **taffrail.**

61. Type of Rigging.—Vessels using wind as their motive power may be divided in two general classes; namely, *fore-and-aft rigged* and *square rigged vessels*. **Fore-and-aft rigged vessels** are those that carry only fore-and-aft sails, such as jibs, staysails, spankers, and trysails. **Square-**

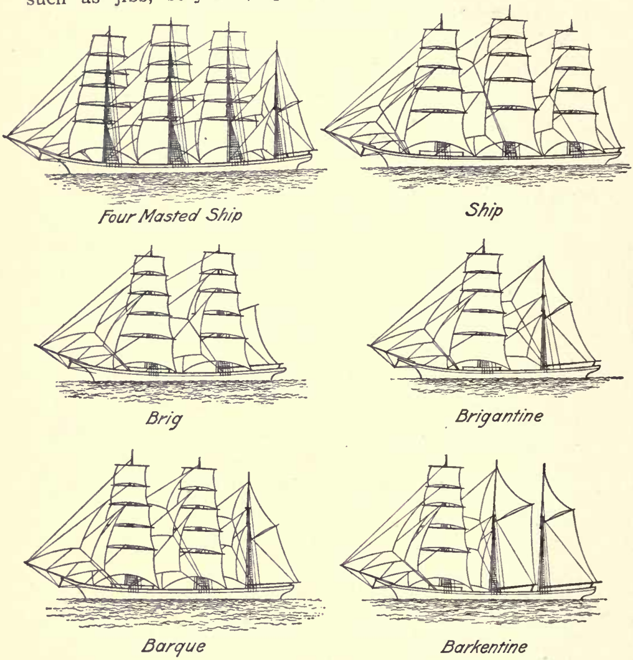


FIG. 14 (*a*)

rigged vessels carry square or rectangular sails on one or more masts, in addition to fore-and-aft sails. The former can sail closer to the wind than the latter. Steamers may be rigged the same as sailing ships. At the present time, however, the masts of a steamer are used principally as a support

for booms and derricks, instead of sails. In Fig. 14 (*a*) and (*b*) are shown the principal types of rigging carried by sailing vessels.

62. A **ship** is a vessel having square sails on at least three masts. The first mast is called the *foremast*, the second the *mainmast*, and the third the *mizzenmast;* if

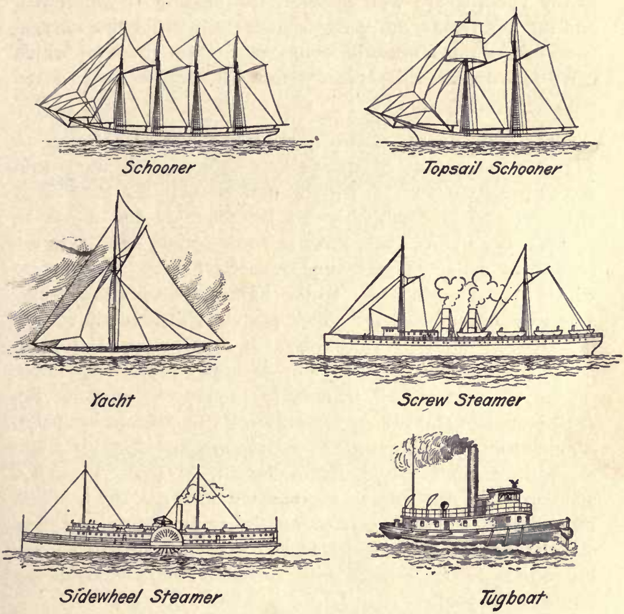


FIG. 14 (*b*)

equipped with four masts, the fourth one is known as the *jigger-mast*.

A **brig** has two masts, both of which carry square sails.

A **bark,** or **barque,** has three masts, two of which carry square sails, and the third a spanker and a gaff-topsail.

The **barkentine** has three masts, but carries square sails only on the foremast, the second and third mast being equipped with fore-and-aft sails, as shown.

The **brigantine** has two masts, the foremast being equipped with square sails.

The **schooner** may have from two to five or even seven masts, all carrying fore-and-aft sails. They are extremely handy vessels, are well adapted for beating to windward, and for this reason are used exclusively in coastwise navigation. A **topsail schooner** has two masts, both of which carry fore-and-aft sails, the foremast in addition being fitted with square topsails.

63. Names of Sails and Fittings.—Referring to Fig. 15, the names of the various sails carried by a ship are as given below. It will be noticed that the identity of any one sail is established by reference to the masts to which it belongs. Thus, there are fore-, main-, and mizzen-topgallant sails, distinguished respectively by the mast to which they are attached. In the figure, the sails are designated by letters, and the other principal fittings shown are indicated by numbers, as follows: *A*, royal; *B*, upper topgallantsail; *C*, lower topgallantsail; *D*, upper topsail; *E*, lower topsail; *F*, foresail; *F'*, mainsail; *G*, outer jib; *H*, inner jib; *I*, foretopmast-staysail or forestaysail; *J*, mainroyal-staysail; *K*, maintopgallant-staysail; *L*, maintopmast-staysail or mainstaysail; *M*, mizzentopgallant-staysail; *N*, maintrysail; *O*, spanker. In the vessel represented in Fig. 15, a single topgallant sail only is carried on the mizzenmast; nor does she carry a flying jib, which is a staysail attached to the jibboom outside *G*. Square sails carried above the royals are known as skysails.

64. The fittings, spars, etc. shown in the figure are as follows: *1*, foremast; *2*, mainmast; *3*, mizzenmast; *4*, mizzentopmast; *5*, mizzentopgallantmast; *6*, mizzenroyalmast (corresponding numbers on the other masts have the same names, mizzen being replaced by fore and main, respectively); 7 is a platform known as foretop, maintop, and

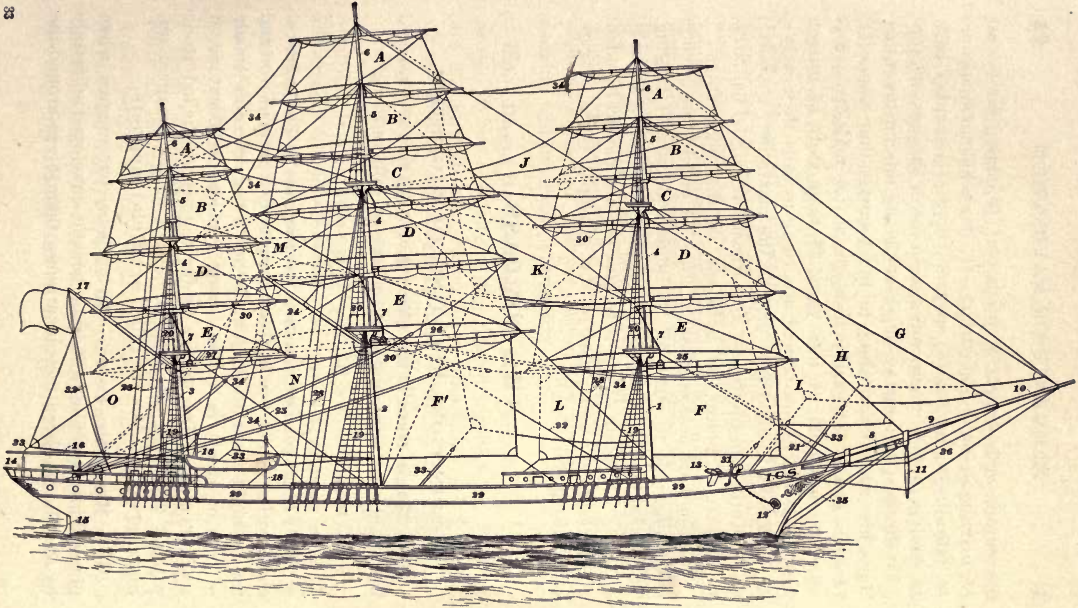


FIG. 15

mizzentop; *8*, bowsprit; *9*, jib boom; *10*, flying-jib boom; *11*, martingale or dolphinstriker; *12*, hawsehole; *13*, anchor; *14*, taffrail; *15*, rudder; *16*, spanker boom; *17*, spanker gaff; *18*, davits; *19*, fore, main, and mizzen lower shrouds; *20*, topmast shrouds; *21*, forestay; *22*, mainstay; *23*, mizzenstay (other stays are named according to the topmast they support); *24*, main gaff (if a trysail is used on the foremast its gaff is called the foregaff); *25*, fore yard; *26*, main yard; *27*, mizzen yard (other yards are named according to sails they carry); *28*, backstays (named according to the masts they support); *29*, bulwarks; *30*, footropes; *31*, cathead; *32*, signal halliards; *33*, sheets; *34*, braces; *35*, bobstays; *36*, martingalestays.

It should be noticed that, in Figs. 14 and 15, no attempt has been made to show all the minute details of rigging. This could hardly be expected from illustrations on so small a scale. The main object is to give the non-nautical student a comparatively clear idea of the principal types of rigs, sails, and spars.

THE COMPASS

DEFINITIONS RELATING TO MAGNETISM

65. Magnetism is the name given the power displayed by magnets of attracting small pieces of iron and steel.

66. Magnets are of two kinds—*natural* and *artificial*. The ore commonly known as lodestone, which possesses the property of magnetism, is a **natural magnet.** The chemical composition of this ore is about seventy-two parts of iron and twenty-eight parts of oxygen. When a bar or needle (made of iron or steel) is rubbed with a piece of lodestone, it acquires magnetic properties similar to those of the lodestone without depriving the latter of any of its own magnetism. Such bars or needles are called **artificial magnets.**

67. Magnetic Poles.—An ordinary bar magnet when plunged into iron filings does not become enveloped uniformly by the filings; these instead arrange themselves around the

ends of the bar in feathery tufts that gradually grow smaller as the middle of the bar is approached, leaving that portion bare, as shown in Fig. 16. The points p, p' around which the filings concentrate are called the **poles** of the magnet; the central portion of the bar where there is no apparent magnetic force and to which the filings refuse to adhere is called the **neutral zone**, and the line $p p'$ connecting the two poles is known as the **magnetic axis.**

68. Magnetic Polarity.—A magnetized needle when suspended on its center of gravity will lay itself in a definite direction pointing north and south. This tendency, called **polarity**, applies to all magnets. The end pointing northward is called the *north-seeking*, or *red*, *pole*, and the opposite the *south-seeking*, or *blue*, *pole*. In other words, the north-seeking end of the needle is said to have red polarity, while the south-seeking end has blue polarity.

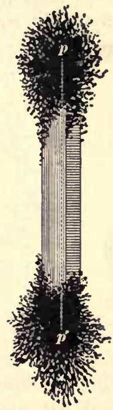


FIG. 16

69. Magnetic Attraction and Repulsion.—When two magnetized bars or needles are brought close together, the north-seeking, or red, pole of one magnetic needle will repel the north-seeking end of the other needle, while it will attract the south-seeking end. From this, the following law for magnetic attraction and repulsion may be enunciated:

Law.—*Poles of contrary names attract each other, while poles of the same name repel each other; or, the red pole of one magnet will repel the red pole of another, but will attract the blue, and vice versa.*

Thus, in Fig. 17, if the blue pole of a bar magnet is held close to the red pole of a magnetic needle, the latter is drawn toward the bar; and, likewise, if the bar is reversed, its red pole will attract the blue, or south-seeking, end of the needle, as shown. This law has an important bearing on the handling and adjustment of ships' compasses and should

therefore be thoroughly understood by the student. Every magnet, it should be remembered, has two poles, distinguished as red and blue poles, and the ends of every artificial magnet, usually, are either painted in these colors or marked, respectively, with the letters *N* and *S*, the former indicating red and the latter blue polarity.

70. Magnetic Property of the Earth.—The fact of a suspended needle taking up a fixed position has led to the theory that the earth itself is a huge magnet, having its red and blue magnetic poles in the neighborhood of the geographical poles, and that the magnetic needle turns to these poles as to the poles of an ordinary magnet, according to the law

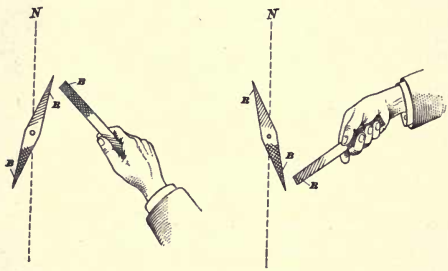


FIG. 17

explained in the preceding article. Furthermore, since the north-seeking end of a needle has red polarity, it follows that the magnetic pole of the earth that is situated in the northern hemisphere must be a blue pole, and that in the southern hemisphere a red pole.

71. The magnetic force of the earth may be resolved into two components, or forces, one horizontal and one vertical. The former represents the directive element of the compass needle; the latter acts only in a vertical direction. A magnetic needle mounted at its center of gravity would be acted on by both forces. The effect of the vertical component is called *magnetic dip*, which will be described in *Variation and Deviation.*

Having explained the matters of polarity as applied to magnets, the compass itself will now be considered.

72. General Description of the Compass.—The **mariner's compass** is an instrument in which the remarkable property of the magnetic needle is utilized. In its simple form, it consists of a needle attached to the under side of a very light circular card, the circumference of which is graduated into a certain number of equal divisions. This card *a b*, Fig. 18, being fitted in its center with a conical brass socket *c*, called the *cap*, is delicately balanced on a central pivot *p* around which it is free to move in a horizontal plane. By this arrangement, the card will partake of the motion of the magnetic needle and maintain a position in accordance with its directive power. The pivot on which the card rests is affixed to the bottom of a metallic vessel, the *bowl*, which is hung in gimbals so as to preserve the horizontal position of the card notwithstanding the rolling and pitching motions of the ship when at sea. This bowl is placed in the top of a strong cylindrical case of wood or bronze called the *binnacle*, which is firmly secured to the deck. As stated before, this is the simplest form of the compass. At the present time, instead of using a single needle, several are used in order to increase the directive power of the compass; and, for the purpose of reducing the friction on the pivot due to the weight of the card and needles, the card in a certain class of compasses almost floats in liquid.

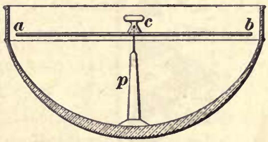


Fig. 18

73. Division of the Compass Card.—The **compass card**, a representation of which is shown in Fig. 19, is divided at its circumference into 360°, and also into thirty-two divisions of 11° 15′ each, called **points.** The four principal points are named after the principal horizon points —north, east, south, and west—and are usually termed the

cardinal points, while the points midway between the cardinal—that is, northeast, southeast, southwest, and northwest—are called the **intercardinal, or quadrantal, points.** Every point has a name, and these the student should learn, so as to be able to repeat them in regular order from north around by way of east or west and back to north. The process of naming the points and fractional parts of points of the

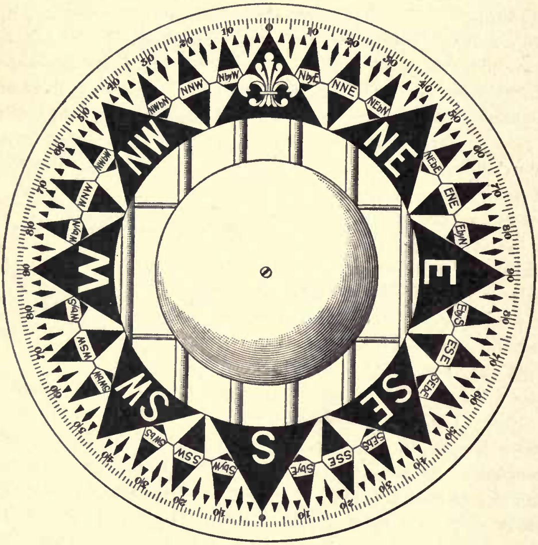


Fig. 19

compass in regular sequence and of naming the points diametrically opposite any given point is commonly known as *boxing the compass.*

74. Names of Compass Points.—Commencing from the north at the top of the card, Fig. 19, and going to the right, or in the direction in which the hands of a watch move, the names of the points are as follows: *NORTH,*

north-by-east, north-northeast, northeast-by-north, northeast, northeast-by-east, east-northeast, east-by-north. EAST, east-by-south, east-southeast, southeast-by-east, southeast, southeast-by-south, south-southeast, south-by-east. SOUTH, south-by-west, south-southwest, southwest-by-south, southwest, southwest-by-west, west-southwest, west-by-south. WEST, west-by-north, west-northwest, northwest-by-west, northwest, northwest-by-north, north-northwest, north-by-west, NORTH.

From this, it can be seen that the cardinal points (given in capital letters) are eight points apart and four points away from the nearest quadrantal points. The name of the opposite point to any given point is known at once by simply reversing the names or the letters that indicate the name of the given point. Thus, the opposite point of N E by E is S W by W, that of N N W is S S E, that of E by S is W by N, and so on.

75. The space between each point is divided into four equal parts, called **half** and **quarter points.** Thus, the expression N $\frac{1}{4}$ W means "north a quarter point to the west," and S E by E $\frac{3}{4}$ E means "southeast-by-east three-quarters of a point to the east." There is no particular system in expressing half and quarter points, but experience has taught that the simplest is the best. Hence, the expression N N E $\frac{1}{4}$ E is preferable to N E by N $\frac{3}{4}$ N, although they both indicate the same point; and likewise N by E $\frac{1}{2}$ E is preferable to N N E $\frac{1}{2}$ N. Absurdities like W by N $\frac{1}{4}$ W and N by E $\frac{3}{4}$ N should never be used. The system given in Table II is the one that should be followed.

This table showing the relation between points and degrees will be very convenient to refer to when it is required to convert a course expressed in degrees into points, and vice versa.

76. Quite frequently a ship's course is given in degrees, especially on steamships, as they can be steered more closely than a sailing vessel, and a course of this kind is expressed as so many degrees east or west from either north or south; for example, N 23° E, S 48° W, S 60° E, etc.

TABLE II

NAMES OF POINTS AND NUMBER OF DEGREES, MINUTES, AND SECONDS CORRESPONDING TO ANY NUMBER OF POINTS AND FRACTIONS THEREOF

North to East	North to West	South to East	South to West	Points	Points in Degrees, Etc.
North	North	South	South		
N ¼ E	N ¼ W	S ¼ E	S ¼ W	¼	2° 48′ 45″
N ½ E	N ½ W	S ½ E	S ½ W	½	5° 37′ 30″
N ¾ E	N ¾ W	S ¾ E	S ¾ W	¾	8° 26′ 15″
N by E	N by W	S by E	S by W	1	11° 15′ 0″
N by E ¼ E	N by W ¼ W	S by E ¼ E	S by W ¼ W	1¼	14° 3′ 45″
N by E ½ E	N by W ½ W	S by E ½ E	S by W ½ W	1½	16° 52′ 30″
N by E ¾ E	N by W ¾ W	S by E ¾ E	S by W ¾ W	1¾	19° 41′ 15″
N N E	N N W	S S E	S S W	2	22° 30′ 0″
N N E ¼ E	N N W ¼ W	S S E ¼ E	S S W ¼ W	2¼	25° 18′ 45″
N N E ½ E	N N W ½ W	S S E ½ E	S S W ½ W	2½	28° 7′ 30″
N N E ¾ E	N N W ¾ W	S S E ¾ E	S S W ¾ W	2¾	30° 56′ 15″
N E by N	N W by N	S E by S	S W by S	3	33° 45′ 0″
N E ¾ N	N W ¾ N	S E ¾ S	S W ¾ S	3¼	36° 33′ 45″
N E ½ N	N W ½ N	S E ½ S	S W ½ S	3½	39° 22′ 30″
N E ¼ N	N W ¼ N	S E ¼ S	S W ¼ S	3¾	42° 11′ 15″
N E	N W	S E	S W	4	45° 0′ 0″
N E ¼ E	N W ¼ W	S E ¼ E	S W ¼ W	4¼	47° 48′ 45″
N E ½ E	N W ½ W	S E ½ E	S W ½ W	4½	50° 37′ 30″
N E ¾ E	N W ¾ W	S E ¾ E	S W ¾ W	4¾	53° 26′ 15″
N E by E	N W by W	S E by E	S W by W	5	56° 15′ 0″
N E by E ¼ E	N W by W ¼ W	S E by E ¼ E	S W by W ¼ W	5¼	59° 3′ 45″
N E by E ½ E	N W by W ½ W	S E by E ½ E	S W by W ½ W	5½	61° 52′ 30″
N E by E ¾ E	N W by W ¾ W	S E by E ¾ E	S W by W ¾ W	5¾	64° 41′ 15″
E N E	W N W	E S E	W S W	6	67° 30′ 0″
E N E ¼ E	W N W ¼ W	E S E ¼ E	W S W ¼ W	6¼	70° 18′ 45″
E N E ½ E	W N W ½ W	E S E ½ E	W S W ½ W	6½	73° 7′ 30″
E N E ¾ E	W N W ¾ W	E S E ¾ E	W S W ¾ W	6¾	75° 56′ 15″
E by N	W by N	E by S	W by S	7	78° 45′ 0″
E ¾ N	W ¾ N	E ¾ S	W ¾ S	7¼	81° 33′ 45″
E ½ N	W ½ N	E ½ S	W ½ S	7½	84° 22′ 30″
E ¼ N	W ¼ N	E ¼ S	W ¼ S	7¾	87° 11′ 15″
East	West	East	West	8	90° 0′ 0″

77. The Lubber Line.—On the inside of the compass bowl is a narrow vertical line called the **lubber line,** or **lubber's point** (see *a*, Fig. 20). This line, when the compass is properly placed, will show the heading of the ship. Usually there are two pair of lubber lines drawn on the inner surface of the bowl diametrically opposite each other. When the compass is placed in position, two of these lines should coincide or be exactly parallel with the ship's fore-and-aft line. The line connecting the other pair should be perpendicular to the fore-and-aft line.

The pivot, which is attached to the bottom of the bowl, is of a conical form (see Fig. 18) and should be screwed into the exact center of the bowl, so that the rim of the card will be at an equal distance everywhere from the sides of the bowl. The card being evenly balanced and remaining stationary by virtue of the attached magnetic needles, the least movement of the ship to either side will be instantly shown by the lubber lines.

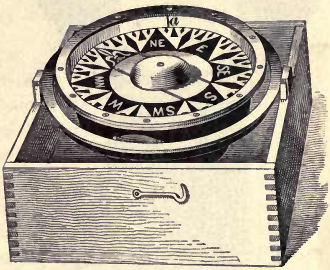


FIG. 20

78. Binnacle.—The **binnacle,** a type of which is shown in Fig. 21, is a stand firmly secured to the deck in front of the helmsman's position (in case the compass it contains is used for steering) or in any other suitable place, where disturbing forces will have the least effect on the compass. It is made either of wood or brass and may be of any shape—round, square, or octagonal. In its upper part is located what is known as the compass chamber, within which are supports on which the gimbals of the compass bowl are placed. The top of the binnacle, which is movable, and which can be removed if necessary, is furnished with glass so that the card may be seen, and also with lamps, at the sides or at the top, to light up the card at night.

79. The requirements of a good compass may be summed up as follows:

1. The pivot and the cap working on it should be accurately in the center of the bowl as well as in the center of the card.

2. The divisions on the rim of the card, particularly the points, should be perfect.

3. The magnetic axis of the needle or needles should be parallel to the north and south line of the card.

4. The compass should be comparatively steady when subjected to the rolling and pitching motions of the ship, and sensitive when at rest or when the ship is in smooth water.

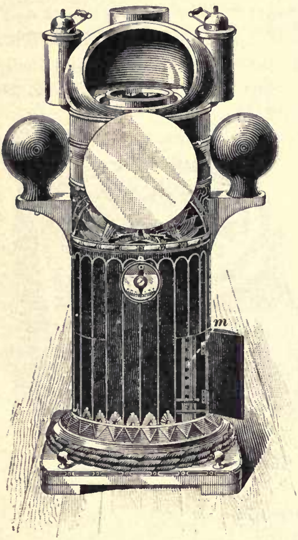


Fig. 21

80. To keep a compass in the best possible condition, so far as its mechanism is concerned, the pivot, cap, and other parts should be examined from time to time to see that they are in good order and working freely; when the card works sluggishly, or an injury from any cause occurs, a new cap or pivot should be put in, taking good care when screwing the pivot into the bowl to preserve its point from injury and to place the card lightly on it. When the bowl does not work freely in the gimbals, the axis of the latter and their bushings should be examined and, if necessary, slightly rubbed with plumbago or graphite. No oil, grease, or other fatty substances should be used for this purpose.

81. Types of Compasses.—The types of compasses now in general use are the *liquid* and *dry compasses*. The **liquid**

compass is simply an ordinary compass the bowl of which is filled with liquid instead of air. It was introduced to overcome the disturbances of the card caused by the sometimes violent motion of small vessels and boats. It is now in general use, and by mariners the world over is considered far superior to the ordinary **dry compass,** the bowl of which is filled with air. When well made, it is a very efficient instrument, especially adapted to withstand the continuous vibrations due to a screw propeller, as well as the shocks of heavy gun firing.

82. According to the use to which it is applied on board ships, the mariner's compass is called the *steering compass* or the *standard compass.*

83. The **steering compass** is the compass by which the ship is steered in a certain direction. It is placed in front of the wheel in a position that leaves the compass card in plain view of the wheelsman. The requirements of this compass, as well as of any other compass whereby the ship's course is set, is that the center of the compass card and the lubber's point a, Fig. 22, shall coincide with the line of the keel or the fore-and-aft line $x\,y$ of the ship or a line parallel thereto. Then the point on the compass card that is in line with the lubber's point a is said to be the direction of the ship's head, or, if the ship is moving, its compass course. It is evident that if the compass is sensitive, the slightest movement of the ship's bow, either to right or left, may be noticed at once by the helmsman by the motion of the lubber's point in relation to the compass points.

FIG. 22

84. The **standard compass** is very correctly called the *navigating compass.* By it, the ship's course should be set, and all bearings taken for ascertaining the ship's position as well as the bearings of celestial bodies. To this end, it should be placed where an unobstructed view of the horizon

can be obtained, and where it is as free as possible from masses of iron and steel; it should be, also, of the best possible workmanship.

Since this compass is superior, and by its position least affected by the disturbing elements of the ship, the steering compass should be frequently compared with it, in order to insure steering the correct course.

85. Lord Kelvin's Compass.—Among the compasses that have been adopted as standard is one invented by Lord Kelvin, which was first patented in 1876. Its principal features are as follows: The card of the ordinary compass is replaced by a thin aluminum ring, about 10 inches in diameter, connected by radial silk threads to a cap of the same metal having a sapphire centerpiece poised on an iridium point. Instead of one single needle there are eight strips of magnetized steel secured to the silk threads radiating from the center. On top of the aluminum ring is placed a paper rim on which are printed the degrees and points of the compass. This rim is broken at intervals by slits in order that the contractions and expansions due to change of temperature may not produce warping of the aluminum. The entire weight of the whole arrangement is about $170\frac{1}{2}$ grains, which is little more than $\frac{1}{3}$ ounce and but one-seventeenth of the weight of the ordinary 10-inch compass previously in common use on merchant steamers and large sailing ships.

The bottom of the bowl of this compass is provided with a closed cavity containing castor oil, which is intended to modify the motion of the bowl in rough sea. It is also provided with a simple device that prevents the card from jumping off the pivot when heavy guns are fired. The bowl and gimbal ring are mounted on knife edges, the whole being hung by two chains from an elastic grummet suspension ring, which in turn is attached to the rim of the binnacle by a ball-and-socket fitting. The binnacle has complete provision for the reception of magnets, soft-iron bars, and spheres used to counteract the magnetism of the iron surrounding the compass.

The latest pattern of this compass has a number of valuable improvements, chief among which are an antivibration suspension of entirely new design, and an arrangement by which the card can be lighted from below, either by oil lamps or by electricity.

86. The Ritchie Compass.—The compass adopted as standard for vessels of the United States navy is made by E. S. Ritchie & Sons, of Boston, Massachusetts. The distinctive feature of this compass is the attachment to the compass card of an air chamber floating in liquid by which almost the entire weight of the card is supported, thus reducing the friction and the wear on the pivot to a very small amount and increasing the sensitiveness of the compass. The liquid used in the bowl is composed of alcohol and distilled water in such proportions that it will not freeze at 20° F. below zero.

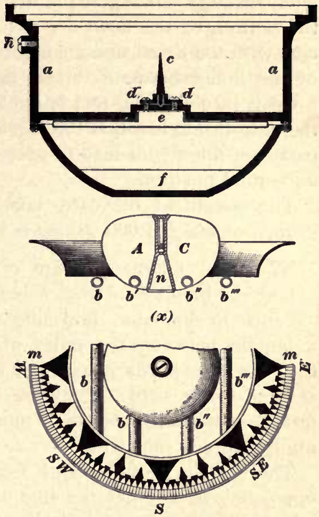


FIG. 23

The card, a representation of which is given in the central and lower part of Fig. 23, is of the curved type, the outer ring being convex on the upper and inner side as shown in (x); it is graduated to quarter points and is provided also with an outer rim $m\,m$ that is divided into half degrees, with legible figures at each 3° for use in reading bearings or in laying the course to degrees.

As stated, the card is provided with a concentric spheroidal air chamber $A\,C$ that buoys up the card and magnets, allowing

a pressure of between 60 and 90 grains on the pivot at 60° F. or to within 18 grains at 13° F. The air chamber contains a hollow cone *n* that is open at its lower end, and is provided with the pivot bearing, or cap, containing a sapphire that rests on the pivot and so supports the card. The cap is provided with two small screws for accurate adjustment in centering the card.

The pivot is rigidly secured by screws to the center of the bowl through a flanged plate *d d*. Through this plate and the bottom of the bowl are two small holes that communicate with the expansion chamber *e* and admit of a circulation of the liquid between it and the bowl. The point of the pivot is of bell metal and is moderately sharp; *h* is a hole in the side through which the liquid is poured, and *f* is a compartment filled with lead to assist in keeping the whole in a horizontal position.

The weight of the card, complete, in air is about 3,060 grains, of which 1,600 grains is in the steel of the magnets.

87. The magnet system of the card consists of four cylindrical bundles of steel wires b, b', b'', b''', each wire being .06 inch in diameter, laid side by side and magnetized as a bundle between the poles of a powerful electromagnet; they are afterwards placed in a cylindrical case, sealed, and secured to the card. They are known as the *Ritchie patent facicular magnets*, and their magnetism is retained indefinitely without impairment.

The bowl is provided with four lubber lines diametrically opposite one another, so that the compass may be placed with the gimbal bearings in the fore-and-aft line, or athwartship, as preferred.

88. Pole Compass.—A compass affixed to a wooden pillar or post at some distance above the deck is called a **pole compass**. If a ship is equipped with a pole, mast, or tripod compass, it has been recommended to consider it as a standard compass, the one on the deck, otherwise known by that name, to be called the navigating compass. Pole compasses, when properly constructed by experienced makers

and well placed, are as a rule very reliable. On account of their altitude above deck, their deviation is very small in amount and much more constant than those nearer the hull. On the other hand, by reason of their elevated position, they are subjected to more violent disturbances when the ship is laboring in heavy seas, and are consequently at such times less reliable. The pivot and cap of a pole compass should be examined frequently, as they are, through the more constant motion of the card, likely to wear out sooner than those of a compass placed on deck.

The pole bearing the compass must be prevented from turning one way or another through changes of temperature or other causes, and should be made of teak and painted white, so as to be least affected by the heat of the sun.

CHART, LEAD, AND LOG

CHARTS AND CHART WORK

CONSTRUCTION OF CHARTS

1. Charts are maps of the sea representing the whole or parts of certain regions of navigable waters with their adjoining coast lines. Each chart contains various particulars peculiar to the tract of water it represents that may aid the mariner to successfully navigate his ship; it points out dangers and obstacles to be avoided, and the shortest route, consistent with safety, by which a desired point may be reached.

In order to understand and intelligently use charts, it is necessary to know something about the construction and peculiarities of those in most common use; we will, therefore, present a few facts relative to this subject.

2. Principles of Construction.—Since the form of the earth is globular, it is evident that any representation of its surface on a flat sheet of paper such as a chart is printed on must necessarily be an artificial construction, or, in other words, a projection of the globe on the paper. In order to insure a true representation of the original, there are certain conditions that must be fulfilled—conformity, equivalence, and equidistance.

Conformity requires that each figure of the image shall possess a geometrical similarity to the original; that is, all corresponding angles must be equal.

Equivalence requires that the areas of certain spaces on the chart shall have the same ratio to one another as the corresponding spaces on the surface of the earth.

Equidistance requires that the distances from any two points to the center of the chart shall have the same ratio to each other as the corresponding distances on the earth.

3. The spherical representation of the earth (the globe) is, however, the only image that can satisfy all of these conditions. But a globe is a very inconvenient thing on which to find distances, bearings, and areas. Therefore, when representing some part of the earth's surface on a sheet of paper, the constructor must content himself with strictly fulfilling one of these conditions, while the others are satisfied as nearly as they possibly can be. This may be accomplished in several ways by applying different methods of projection. Among the methods used for chart construction the *polyconic* and *Mercator's* systems of projection are the most extensively used.

4. The Polyconic Projection.—Without touching on the mathematical principles underlying this projection, the characteristic features of the **polyconic chart,** briefly told, are as follows: The space to be represented on the chart is considered as being divided by the latitude parallels into narrow zones projected, according to certain rules, on a cone tangent to the zones' lower parallel (in north latitude the southernmost, in south latitude the northernmost). These zones $a\,b\,b'\,a'$, $c\,d\,d'\,c'$, etc., Fig. 1, are then placed so that the middle

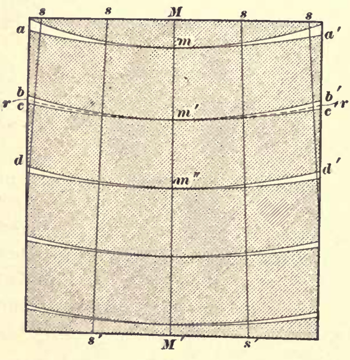


FIG. 1

meridian $m\,m'$, $m'\,m''$, etc. of each constitutes a common meridian MM' for the whole chart; the zones therefore touch or are tangent to one another only at points on or near this line, and leave an open space between their ends, as shown in the figure.

In order to fill these open spaces and make the chart complete, the ends $a\,b$, $c\,d$, etc. of each zone are stretched, so to speak, so that the lower edge $b\,m'\,b'$ of one zone will coincide with the upper edge $c\,m'\,c'$ of the adjoining zone along the center line $r\,m'\,r'$. As a consequence of this stretching, the parts of the chart near the vertical edges are somewhat distorted. The latitude parallels are not parallel with one another and the meridians $s\,s'$, $s\,s'$, etc. consist of curved lines converging toward the poles, with the exception of the middle meridian $M\,M'$, which is a straight line. On charts of large scale, the curvature of the meridians is hardly noticeable; but on charts embracing several degrees of latitude, it is plainly visible.

5. This projection is advantageous for the representation of a coast line that runs north and south, or in the direction of the meridian, and for this reason it is extensively used by the United States Coast and Geodetic Survey in the preparation of charts of the Atlantic and Pacific seaboards.

The central part of a polyconic chart, in fact the greater portion of the chart, the exceptions being those parts near the vertical edges, satisfies the conditions of conformity, equivalence, and equidistance.

6. The course, or line of bearing, between any two places on a polyconic chart will necessarily be represented by a curved line if properly laid out. However, if the scale of the chart is large, the bearing may, without any practical error, be represented by a straight line, especially when the two places are situated in the central part of the chart.

7. If the places are widely separated in longitude, it should be borne in mind that, when using a straightedge for finding the course between them, the ship following this line will, at the end, be nearer the elevated pole by a fraction

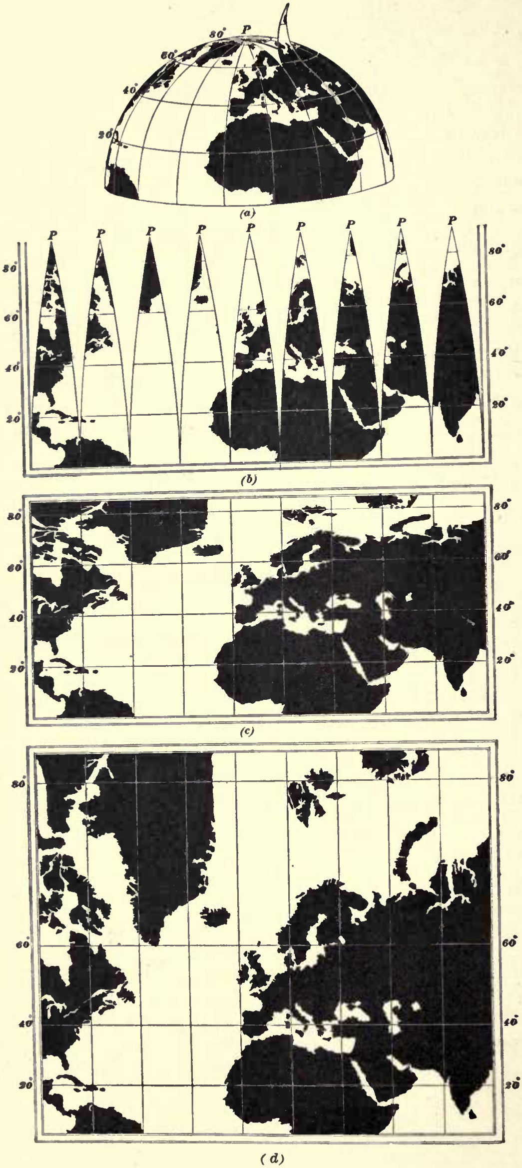


FIG. 25

that amounts to about $1\frac{1}{2}$ miles for every hundred miles on charts including the region between latitudes 45° and 50° N. This error is somewhat augmented on charts constructed on this projection whose scale is small.

8. The Mercatorial, or Mercator's, Projection.—The system on which the **Mercatorial chart** is constructed was invented in 1512 by Gerard Mercator, of Rupelmunde, Flanders. His system was not a projection in the true sense of that word, but may be said to have resulted from operations illustrated in Fig. 2. By stripping off the globe—(*a*) representing the northern hemisphere—the gores formed by the meridians, and placing them in regular order beside one another on a flat surface, a chart is formed similar to the one shown in (*b*). Owing to the openings, or vacant spaces, between the meridians, this chart is very defective; and, in order to remedy this defect, the upper parts of the gores are stretched so as to form the chart represented in (*c*). A glance at this chart, however, will reveal the fact that everything on it, except the equatorial parts, is distorted, and that this distortion increases in the higher latitudes. Now, in order to restore a balance of orientation, or the relative position and direction of spaces that are distributed *horizontally* (or in longitude), it is necessary to distort the chart in an equal proportion *vertically* (or in latitude). When this has been done, the result is as shown in (*d*), which represents a complete map, or chart, of half the northern hemisphere according to Mercator's system.

On account of this distortion of the Mercatorial chart, areas and spaces in the higher latitudes appear on such a chart much larger in comparison with those of the equatorial parts than they really are.

9. Wright's Graphic Explanation.—Mercator did not demonstrate mathematically the principles of his system. Toward the close of the 16th century, however, Edward Wright, an English scientist, gave a mathematical demonstration of the law on which Mercator constructed his chart. He assumed that a cylinder contained a spherical globe

representing the earth, Fig. 3 (*a*), which was to swell, like a bladder, equally in latitude and longitude, until it coincided with the concave surface of the cylinder. At the same time, the meridians widen out until they are everywhere the same distance from one another, as on the equator. In this way, the spherical surface is made to coincide with the cylindrical concave surface. Cutting the cylinder along a meridian and opening it out into a flat surface, Fig. 3 (*b*), we find on it a representation of a Mercator's chart.

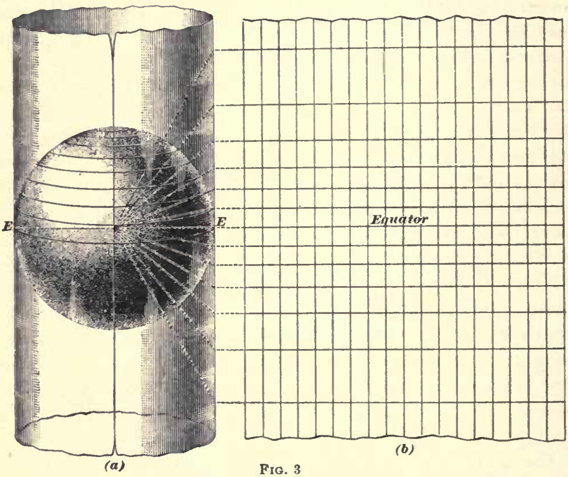


Fig. 3

10. For this reason, all the meridians on a Mercatorial chart are parallel straight lines and the degrees of longitude are all equidistant; the latitude parallels are everywhere at right angles to the meridians, and hence the *rhumb line*, or the track of a ship that steers a straight continuous course, or the relative direction, or bearing, between two places, can be represented as a straight line. For navigational purposes, therefore, the Mercatorial chart has a decided advantage over the polyconic chart.

11. Meridional Parts.—Further examining a Mercator's chart, we find that the degrees of latitude are all unequal, being increased in length from the equator toward the pole. These augmented latitudes, expressed in minutes of the equator, are called **meridional parts,** and the meridional part for any latitude is the line expressed in minutes of the equator into which the latitude is thus expanded.

For example, the true distance from the equator to the twentieth parallel of latitude is $20 \times 60 = 1{,}200'$ (or nautical miles) but on a Mercator's chart this distance is increased to $1{,}217.3'$, and is known as the meridional part of latitude 20°.

12. Tables of meridional parts have been constructed, which show the length in minutes of the equator into which the true latitudes must be expanded when a chart is being constructed on Mercator's projection. These tables are given in the collection of Nautical Tables accompanying this Course. They contain the meridional parts for every minute of latitude. It will be shown later how, by aid of these tables, it is possible to construct a Mercatorial chart of any part of a sea or coast line in general.

13. The **meridional difference of latitude** is the expanded distance between any two latitudes on a Mercatorial chart, or the difference between the meridional parts for any two latitudes. Thus, the meridional parts for latitudes 20° N and 30° N are, respectively, $1{,}217.3'$ and $1{,}876.9'$, and therefore the meridional difference of latitude (M. D. L.) between the parallels is $1{,}876.9' - 1{,}217.3' = 659.6'$.

14. Classification of Charts.—With regard to the objects for which they are constructed, charts may be divided into two classes—*general charts* and *plans.*

15. General charts are those that embrace a comparatively large part of an ocean or the entire ocean or a considerable extent of coast line with its contiguous waters. They are usually subdivided into *sailing charts* and *general charts of the coast.* The former are constructed both on the Mercatorial and the polyconic projections; the latter chiefly on the

polyconic projection. Sailing charts are intended for the use of navigators to fix the position of the ship when approaching the coast from the open ocean, or when navigating between distant ports. On these charts are recorded the offshore soundings, generally giving the 1,000-, 100-, and 50-fathom curves, the principal lights, variation curves, outer buoys, and such landmarks as may be visible at a considerable distance.

16. General charts of the coast are intended for use in coastwise navigation, when the vessel will be most of the time within sight of land, and her position fixed by bearings taken of landmarks, lights, outer buoys, and by characteristic soundings.

It is well to remember that soundings on plain white are in fathoms; those on shaded or dotted parts of the charts are expressed in feet. On general charts of the coast, sounding curves indicating the range of sounding of 10, 20, 30, and 100 fathoms are shown. These curves are of great assistance when approaching or navigating along the coast. On large general charts, for instance, the chart of the North Atlantic Ocean, only the 100-fathom curve is shown. All soundings show the depth at *mean low water*.

17. Plans or **special charts** are charts that comprise a detached portion of a general chart on a large scale, such as a chart of a harbor, a small bay, the entrance to or a part of a river, the channels leading to a port, or a small part of the sea where navigation is difficult and dangerous. They generally contain the lights and buoys, the soundings and character of bottom, the leading marks, the courses through channels, dangers to be avoided, the variation of the compass, and other information that will tend to facilitate the navigation of that locality. Such plans are sometimes inserted for convenience in a corner of the general chart.

Plans are usually designated as *coast charts* or *harbor charts*, according to the objects they are designed to subserve, the latter being on large scales and intended to meet the needs of local navigation.

18. Signs and Symbols Used on Charts.—Charts used in coast navigation along the Atlantic and Pacific seaboard of the United States are prepared and published principally by the United States Coast and Geodetic Survey. Charts of adjoining coast lines, such as those of Newfound-

CONVENTIONAL SIGNS AND SYMBOLS USED ON COAST AND GEODETIC SURVEY CHARTS

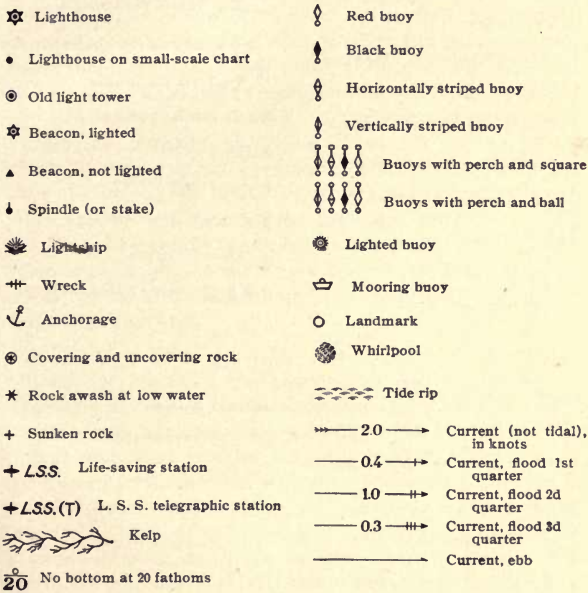


FIG. 4

land, the Gulf of St. Lawrence, Nova Scotia, the Bermudas, the West Indies, the Gulf of Mexico, Central America, etc., are published by the United States Hydrographic Office, as are also the charts of the Great Lakes. The conventional signs and symbols used on the Coast and Geodetic Survey charts are shown in Fig. 4, while those used on

CONVENTIONAL SIGNS AND SYMBOLS USED ON HYDROGRAPHIC OFFICE CHARTS

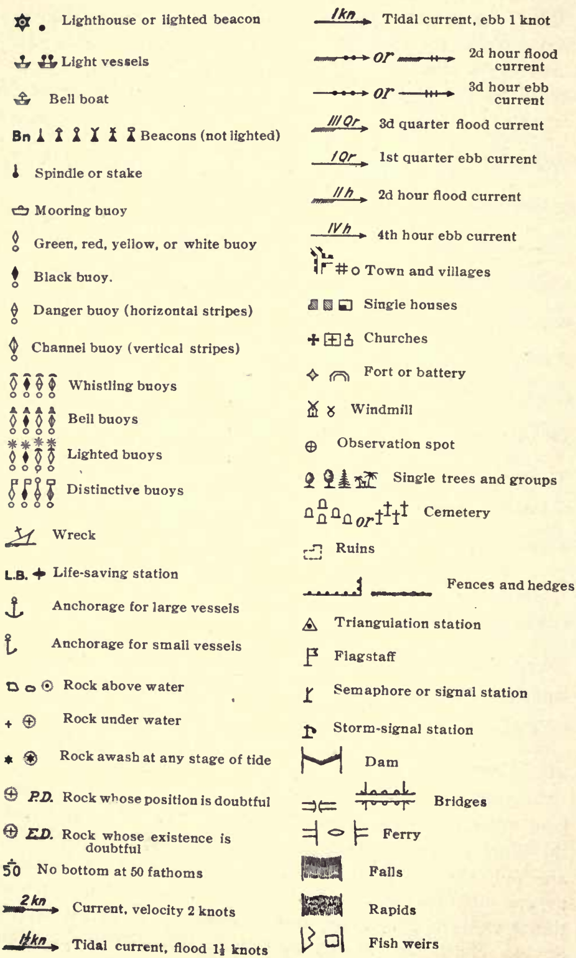


FIG. 5

Hydrographic Office charts are shown in Fig. 5. They should be studied carefully and committed to memory.

19. Correction of Charts.—Great care should be taken to keep charts up to date. Before a chart is used, the navigator should see that all changes in the position or character of lights, the establishment of new or the discontinuation of existing lights, buoys, artificial landmarks, etc. that have taken place since the date stamped on the chart, are properly recorded. Such changes are published in Notices to Mariners issued monthly by the Hydrographic Office. Copies of these notices can be obtained by mariners, free of charge, by applying, personally or in writing, to the Hydrographic Office, Washington, District of Columbia, to one of the branch offices, or to any of the agencies in seaboard or lake ports. They are also on file in all United States Consulates, where every facility will be afforded for their inspection. Shipmasters are especially requested to inform the Hydrographic Office immediately of any newly discovered danger to navigation, or of the establishment or change of any aid to navigation.

20. Pilot Charts.—The pilot charts of the North Atlantic and North Pacific Oceans published monthly by the Hydrographic Office are not intended to be used as sailing charts. They are essentially monthly weather maps, giving a great amount of valuable information about wind, weather, and currents, and should therefore be carefully studied by the navigator. To mariners who assist the Hydrographic Office in its work of collecting and distributing hydrographic information, pilot charts are furnished free of charge as an equivalent for service rendered. To others, a charge of 10 cents per copy is made. They may be obtained per post from the Hydrographic Office, Washington, District of Columbia, or by calling in person at one of the branch offices.

THE USE OF CHARTS

21. General Remarks.—When a chart is properly spread out, the top of it is toward the north, the bottom toward the south, the right side to the east, and the left side to the west. If for any reason it is otherwise, north will be indicated by the direction of the meridians or by the compass diagrams situated somewhere on the chart. On a Mercator chart the latitude scales are found on the right and left sides of the chart, and the longitude scales at the upper and lower margins of the chart. Sometimes graduations are found on a meridian and on part of a parallel situated in the central part of the chart. On every polyconic chart, such as the one shown in Fig. 6, the scale used for measuring distances is inserted at a suitable place, either in a corner or near the borders of the chart. It is usually given in nautical miles.

22. Instruments Used in Chart Work.—The principal instruments used in connection with charts are the *parallel ruler*, the *dividers*, and the *course protractor*.

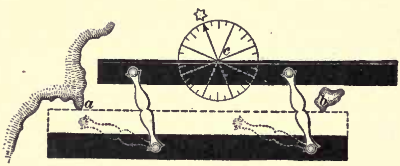
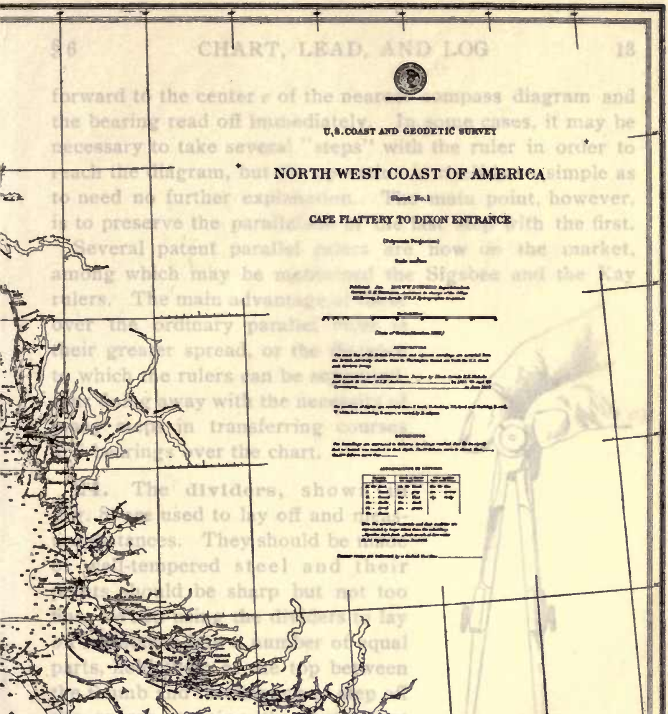


FIG. 7

23. The **parallel ruler,** Fig. 7, is usually made of ebony or gutta percha. The two parts are connected by crosspieces of brass working on pivots in such a manner that they may be spread apart or pushed together and still remain parallel to each other. They are used for the purpose of transferring the direction of a bearing, or course, to the nearest compass diagram, or vice versa. For instance, in Fig. 7, if it is required to find the bearing between *a* and *b*, the edge of a closed parallel ruler is laid between the two places and then the upper part of the ruler is pushed


AND GEODETIC SURVEY
WEST COAST OF AMERICA
TO DIXON ENTRANCE



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forward to the center *c* of the nearest compass diagram and the bearing read off immediately. In some cases, it may be necessary to take several "steps" with the ruler in order to reach the diagram, but the operation in itself is so simple as to need no further explanation. The main point, however, is to preserve the parallelism of the last step with the first.

Several patent parallel rulers are now on the market, among which may be mentioned the Sigsbee and the Kay rulers. The main advantage of these over the ordinary parallel ruler is their greater spread, or the distance to which the rulers can be separated, thus doing away with the necessity of many steps in transferring courses and bearings over the chart.

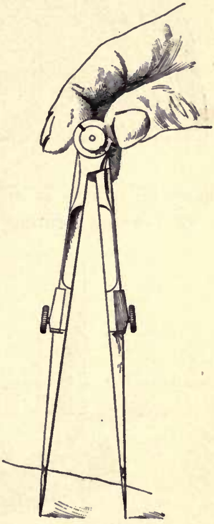


FIG. 8

24. The **dividers**, shown in Fig. 8, are used to lay off and measure distances. They should be made of well-tempered steel and their points should be sharp but not too fine. When using the dividers to lay off a distance into a number of equal parts, hold them at the top between the thumb and forefinger and step off the spaces, turning the instrument alternately to right and left. When laying off a distance in this manner, great care must be exercised not to press the divider points into the chart; they should be turned lightly and the final point indicated by a pencil mark, lightly applied.

25. The **course protractor** is shown in Fig. 9. The outer edge is a semicircle, with a center at *o*, and is divided into degrees and half degrees and graduated so that angles can be read both ways. Thus graduations may be read from *a* to *b* or from *b* to *a*, as indicated by the arrows. The protractor is used for laying off or measuring courses or

bearings, and should be made of celluloid or metal, and with a radius of at least 3 inches. When using the protractor, it

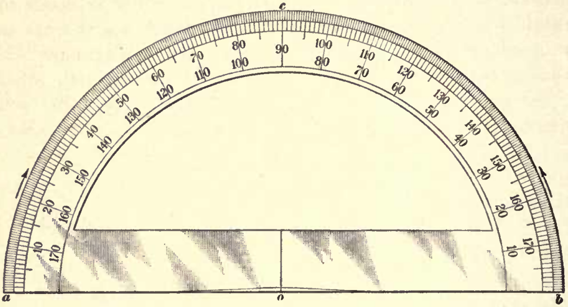


FIG. 9

must be placed so that the line *oa*, *ob*, or *oc* will coincide with the line forming one side of the angle to be laid off or

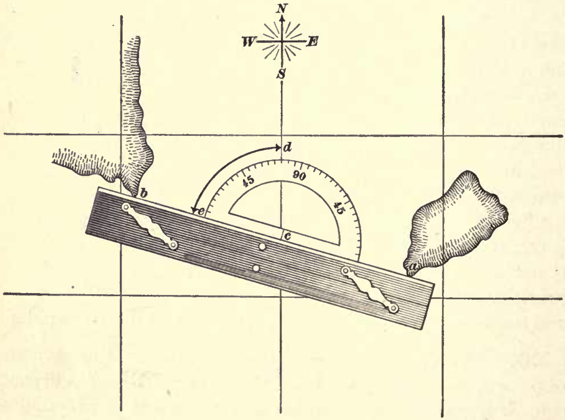


FIG. 10

measured, and the center *o* must be at the vertex of the angle. For example, if it is required to find the bearing

between the two places *a* and *b* in Fig. 10, place any straightedge, such as a parallel ruler, between the two places, as shown; then place the protractor along the ruler with its center *c* on one of the meridians, and read off the bearing as indicated by the number of degrees contained in the arc *d e* reckoned from the meridian—in this case, about N 75° W. Again, the protractor may be used in the manner shown in Fig. 11. Connect the two places *a* and *b* with a

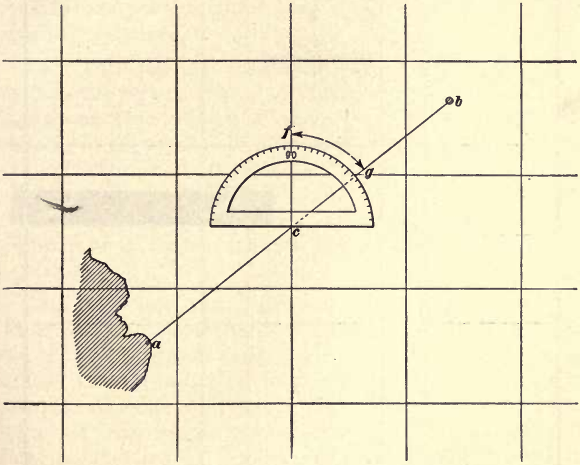


FIG. 11

light pencil line; place the center *c* of protractor on the point of intersection between *a b* and one of the meridians, and the 90° mark on the meridian, as shown; the angle *f c g*, reckoned from the meridian, will then give the required bearing from *a* to *b*, or vice versa, as the case may be. The protractor may be placed in any other position, provided that its center is exactly on the intersection of the line of bearing and one of the meridians. It should be borne in mind that the course or bearing thus found is true.

26. To Plot a Position on the Chart.—When the latitude and longitude of a ship or a place are given, its position on the chart is laid down as follows: Place the edge of a parallel ruler along the latitude parallel nearest the given latitude; then move it until one of its edges passes accurately through the given latitude on the graduated meridian, and

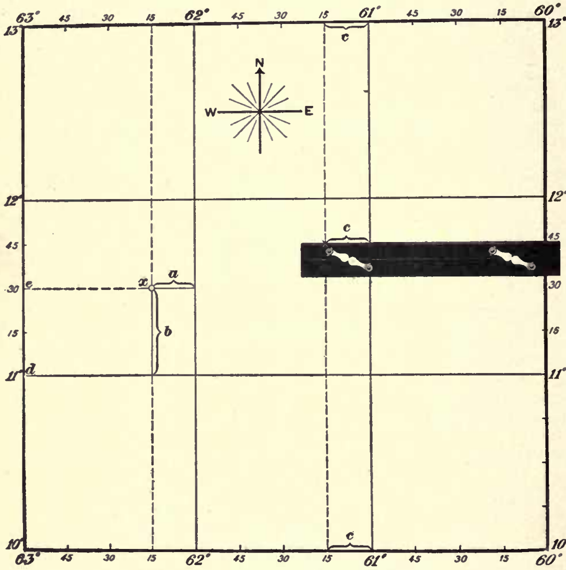


FIG. 12

keep it there. This done, take, with the dividers, from the upper or lower graduated margin, the distance of the given longitude from the nearest meridian, and lay off this distance along the edge of the ruler from the same meridian. The position thus found is the position of the ship or place required, and should be marked on the chart by a light pencil cross or circle.

For example, the latitude and longitude of a place is 11° 45′ N and 61° 15′ W, respectively; plot its position on the chart, Fig. 12. The parallel ruler is first placed along the twelfth parallel and then moved down until its edge is exactly at 11° 45′ on the graduated meridian, as shown in figure, whence the distance c (= 15′ of longitude) is taken by the dividers from either the upper or lower longitude scale and laid off along the edge of the ruler from the sixty-first meridian. The position thus found (indicated by a cross) is the position of the given place according to its latitude and longitude.

27. This position could have been just as easily found by first placing the ruler in the position it occupies in the figure and then placing it along the sixty-first meridian, moving its edge to 61° 15′; the point of intersection between the lines formed by the edge of the ruler in each position would be identical with the point indicated by the cross. On a polyconic chart of small scale the dividers are used exclusively in finding and plotting positions, as explained hereafter.

28. To Find the Latitude and Longitude of a Place or Position From the Chart.—*First Method.*—Place the edge of a ruler through the given point, parallel to the nearest latitude parallel, and read off the degree and minute at which the edge of the ruler cuts the graduated meridian. This will give the required latitude. Similarly, place the ruler parallel to the nearest meridian through the given point and note the degree and minute at which the edge of the ruler cuts the graduated parallel, or longitude scale. This will give the required longitude.

29. *Second Method.*—The latitude and longitude of the same position may also be conveniently found by means of the dividers as follows: Measure the perpendicular distance of the given point from the nearest parallel; refer this distance to one of the graduated meridians or latitude scales, and place one leg of the divider on the same parallel; the other leg should then be at the required latitude. For the

longitude, proceed in precisely the same manner, but use a meridian and the longitude scale at the top or bottom of the chart.

For example, if it be required to find the latitude and longitude of the point x, Fig. 12, measure with the dividers the distance b of the point from the eleventh parallel and place one leg at d; the other leg will then be at e and the latitude read off at once—in this case 11° 30′ N. For the longitude, measure the distance a from the point to the nearest meridian, and lay off this on the longitude scale either at the top or bottom from the same meridian. This will give the longitude of the point x—in this case 62° 15′ W.

30. To Find Distance Between Two Points on the Chart.—If the two points lie on the same meridian, or very nearly so, find on the chart their respective latitudes according to the preceding article; the difference or sum of these, according as the given points are situated on the same or on different sides of the equator, will give the required distance in minutes of arc, or in nautical miles, since 1 minute measured on a meridian is equal to a nautical mile.

31. On a Mercator chart, if the two points are situated on the same parallel, half the distance should be measured on the graduated meridian, or latitude scale, on each side of the parallel; the total length thus measured, expressed in minutes, is very nearly the required distance. For example, if it be required to find the distance between two places a and b, Fig. 13, situated on the same parallel, take half the distance, or $a\,c$, and lay it off on the latitude scale from the parallel to a'; then lay off the other half $c\,b$ downwards from the same parallel to b'. The sum of these lengths, or the distance d, will give very nearly the required distance $a\,b$.

On a polyconic chart, measure the distance between the two points by means of the scale of nautical miles, using a pair of dividers to step off the distance.

32. If the two points differ both in latitude and longitude, and the chart is constructed on Mercator's projection, place the parallel ruler between the two points, and if the

distance is short use the dividers and measure it on the graduated meridian, as nearly as possible opposite or between the two points. A better and more accurate method, however, is to take off from the graduated meridian, opposite and near the middle latitude, with the dividers, a convenient unit, such as 2, 3, or 5 minutes, and find how many times this unit is contained in the distance between the two points.

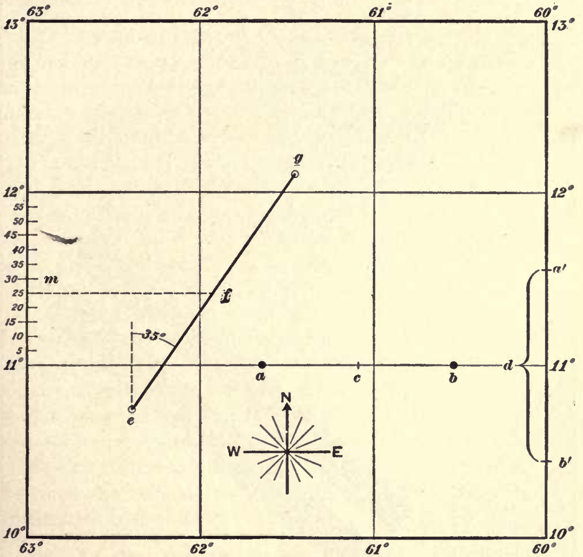


FIG. 13

For example, let it be required to measure the distance between *e* and *g*, Fig. 13. Find the center *f*, and from this point draw a line *m f* parallel to the latitude parallels. This is the *middle latitude* between *e* and *g*. On the graduated meridian, and on the opposite sides of the middle latitude parallel, take a distance, say 5 miles or minutes, and step this off from *e* to *g* or from *g* to *e*. Suppose *eg* contained

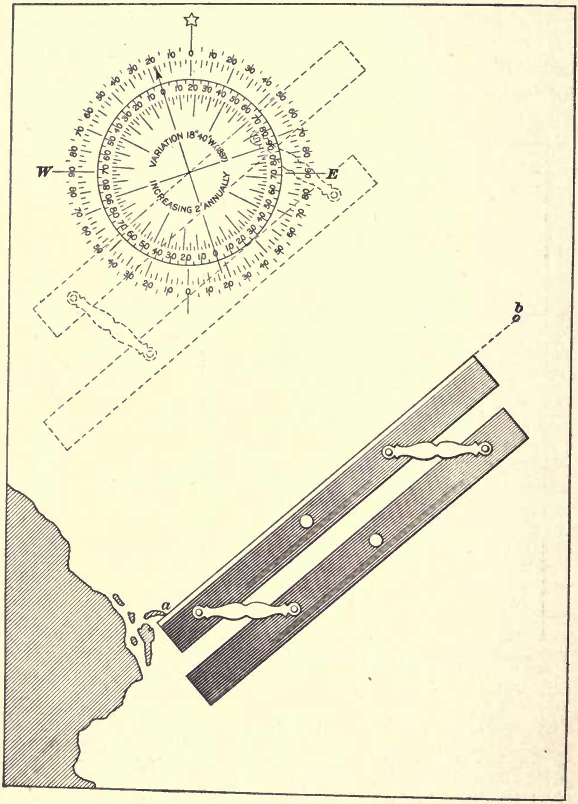


FIG. 14

twenty of these steps; the distance between the two points would then be $5 \times 20 = 100$ miles. In practice, it is not necessary to draw the middle parallel, but simply to be guided by the eye in taking the unit from that part of the scale midway between the two places.

If the chart is constructed on the polyconic projection, a suitable unit should be taken from the scale of nautical miles and stepped off along the line *eg*, as usual.

33. To Find Course or Bearing Between Any Two Points on Chart.—For this purpose use either the parallel ruler or the course protractor. When using the former, place an edge over the two given points; then move the two parts of the ruler in succession until the edge of one passes through the center of a compass diagram. This edge will then point out, by its intersection with the graduated circumference of the compass rose, the required course or bearing. Thus, in Fig. 14, if it is required to find the course from *a* to *b*, place the parallel ruler along the straight line connecting the two places, as shown. Then move the ruler until its edge passes through the center of the compass rose. In this case, the edge of the ruler coincides with the E N E point of the inner diagram and with the 49° mark of the outer diagram. The course from *a* to *b* is accordingly E N E magnetic, or N 49° E true, since the outer diagram represents true, and the inner diagram magnetic, directions.

When a course protractor is used, the course, or bearing, between the two places is found by methods described and illustrated in Art. **25.**

34. If the compass course is to be shaped from the course, or bearing, thus obtained, the various corrections must be applied according to the compass diagram showing true or magnetic directions. Thus, if the course is taken from the true diagram, variation and deviation must be applied to get the compass course; if taken from the inner, or magnetic, card, deviation only should be applied. The operation of correcting courses will be fully treated in *Piloting*.

35. To Find Ship's Position on Chart When Course and Distance From Any Given Place is Known.—Place the edge of a parallel ruler over the given place in the direction of the given course according to the nearest compass diagram, and take the given distance with the dividers from that part of the graduated meridian that lies opposite the given place and the supposed position of the ship. Thus, if *e*, Fig. 13, is the given point, N 35° E the ship's true course, and 100 miles the distance run, the position arrived at *g* is found by drawing a line from *e* in a N 35° E direction, and on this line laying off a distance of 100 miles according to a selected unit taken from the latitude scale, as near as possible to the middle parallel between the given and supposed place of the ship, or from the distance scale if the chart is on the polyconic projection.

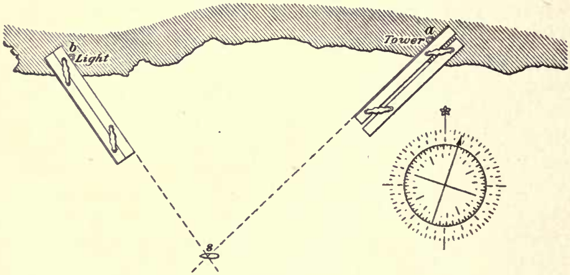


FIG. 15

36. To Lay Off Ship's Position on Chart by Cross-Bearings.—When the bearings of two selected objects are corrected for deviation, due to the direction of the ship's head at the time of observing them, place the parallel ruler on the nearest compass diagram so that the edge passes through the center and the requisite degree or point on the circumference. Then move the ruler, step by step, until the edge passes through the object, when the line drawn along the edge will represent one of the bearings. The ship will then be somewhere on that line. Proceed similarly with the

other bearing. Now, the ship will be somewhere on this line also, and since the only common point of two lines intersecting each other is at their point of intersection, the position of the ship on the chart must necessarily be at the point where the two bearings intersect. This is illustrated in Fig. 15, where *s a* is the bearing of the object *a* from the ship *s*, and *s b* is the bearing of *b* from *s*. The point of intersection of the two lines is the position of the ship.

37. It is evident that the objects selected for cross-bearing should be so situated that the lines of bearing do not intersect at a very acute angle, since the point of intersection in such a case is somewhat doubtful. To obtain good results, the angle between the bearings should be as near as possible to 90°, or 8 points.

METHOD OF CONSTRUCTING A MERCATOR'S CHART

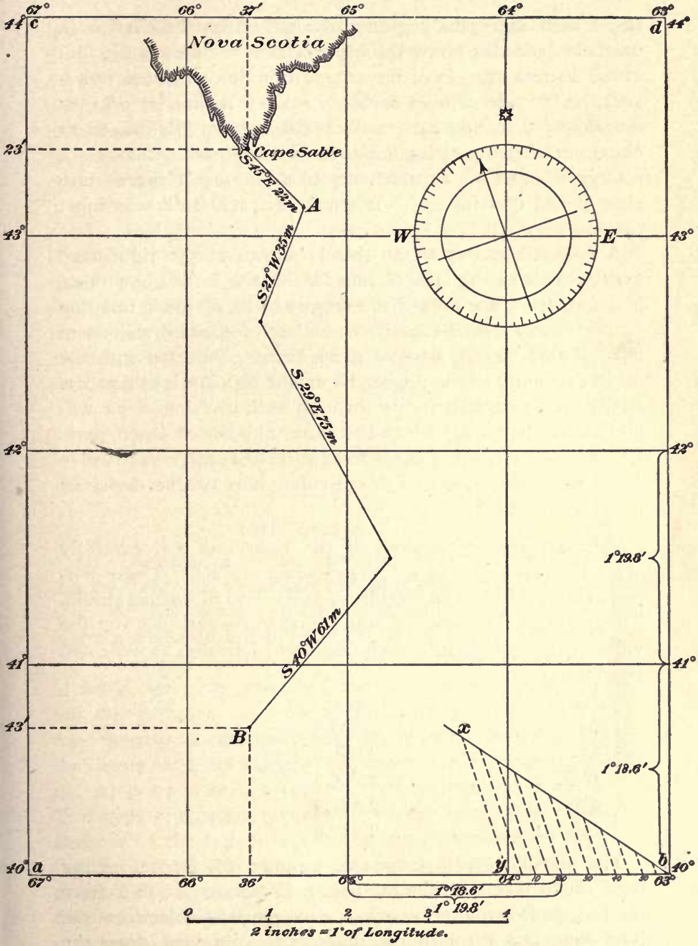
38. In connection with the use of charts and the solution of chart problems in general, it may prove useful to know how to construct a Mercator's chart. Among seamen, this is considered a complicated problem and one that cannot be mastered by the average man. That the problem is comparatively simple, however, is evident from the following exposition. All that is needed is a Table of Meridional Parts and the usual drawing materials.

The order of procedure is then as follows: First determine the limits of the proposed chart—in other words, the number of degrees and minutes it is to contain, both of latitude and of longitude. Spread out a sufficiently large sheet of paper and fasten it securely to the drawing table by means of thumbtacks, one on each corner, taking care that the paper is evenly stretched so as to present a smooth surface. Then draw a straight line near the lower margin of the paper if the chart is to represent north latitude; near the upper margin if it is to represent south latitude; or at a suitable position toward the center of the sheet if both north and south latitudes are to be represented. Divide this line into as many equal parts as the number of degrees of longitude

required; for instance, if the chart is to contain 15° of longitude, divide the line into fifteen equal parts. At each extremity of this base line erect lines perpendicular to it. Take from the tables referred to in Art. **12** the meridional parts for each convenient degree of latitude, for the limits between which the chart is to be drawn, and take the difference between each successive pair, thus obtaining the meridional differences of latitude. Reduce these meridional differences to degrees by dividing them by 60; the result will be the lengths on the longitude scale between the chosen degrees of latitude. Lay off these lengths successively on the perpendicular lines, and through the points thus obtained draw straight lines parallel to the base line to represent latitude parallels. At convenient intervals, or through each division on the base line, draw lines parallel to the perpendiculars to represent meridians.

The accuracy of the frame of the chart thus completed should be tested by measuring the two diagonals of the rectangle formed; if they are of the same length, the frame is perfect. The principal points in the chart are now laid down according to their respective latitudes and longitudes, and whatever formations and contours of water or land are required, together with other useful items, are drawn in freehand. A compass diagram may also be inserted, remembering that the direction of the meridians indicates true north and south.

39. Illustration of Method.—In order to illustrate the foregoing, let it be required to construct a chart on the Mercator projection, extending from latitude 40° N to 44° N, and from longitude 63° W to 67° W, on a scale of 2 inches to 1° of longitude. On this chart lay down the position of Cape Sable lighthouse (latitude 43° 23′ N, longitude 65° 37′ W) with outlines of the adjoining coast. Then, from a point situated S 45° E, 22 miles, from Cape Sable light, plot the following true courses and distances and find the latitude and longitude of the position arrived at: S 21° W, 35 miles; S 29° E, 75 miles; S 40° W, 61 miles. To



67°
66°
37′
65°
64°
63°
44° c
d 44°
Nova Scotia
23′
Cape Sable
S 45° E 22 m
A
43°
W
E
43°
S 21° W 35 m
S 29° E 75 m
42°
42°
1° 19.8′
S 40° W 61 m
41°
41°
43′
B
x
1° 18.6′
40° a
y
b 40°
67°
66°
36′
65°
64°
10
20
30
40
50
63°
1° 18.6′
1° 19.8′
0
1
2
3
4
2 inches = 1° of Longitude.

FIG. 16

begin with, since the region to be shown on the chart is in northern latitudes, draw the horizontal base line *a b*, Fig. 16, at the bottom margin of the paper; this line will then represent the latitude parallel of 40° N. On this line, lay off four lengths of 2 inches each and divide each length into sixty equal parts, representing minutes of longitude. It may not always be possible or necessary to divide each degree into sixty parts; divisions of 5 minutes each will suffice in most cases.

A convenient way to do this is shown at the right-hand lower corner of Fig. 16. A line *b x* forming an angle with *a b* is drawn from one end of the degree to be divided; this line is then divided into a desired number of equal divisions of any suitable length, the last mark being connected with the other extremity of the degree by means of a straight line *x y*. Lines drawn parallel to *x y* through each division of *b x* will divide the distance *y b* into the same number of equal parts into which *b x* is divided. Thus, in this case, *b x* has twelve equal divisions; hence, *b y* is divided into twelve divisions of 5 minutes each.

40. At the extremities of the base line *a b*, carefully erect the lines *b d* and *a c* perpendicular to *a b*. These perpendiculars will be the meridians of 63° and 67° of longitude. Then consult the Table of Meridional Parts and pick out the values corresponding to each degree of latitude, as indicated below:

Lat.	M. P.		M. D. L.
40°	2,607.9		
		78.6 ÷ 60 =	1° 18.6′
41°	2,686.5		
		79.8 ÷ 60 =	1° 19.8′
42°	2,766.3		
		81.1 ÷ 60 =	1° 21.1′
43°	2,847.4		
		82.5 ÷ 60 =	1° 22.5′
44°	2,929.9		

The meridional differences of latitude (M. D. L.) having been found take with the dividers a distance of 1° 18.6′ from the longitude scale, lay it off on each perpendicular from the base line, and through the points thus obtained draw the parallel of 41°. In like manner, from the parallel of 41°,

lay off 1° 19.8′, taken from the scale of longitude, and draw the parallel of 42°. Proceed similarly and get the parallels of 43° and 44°. Divide this last parallel into degrees and minutes, the same as the parallel of 40°, and draw the meridians of 64°, 65°, and 66° west longitude. The result, in reduced form, will be as shown in the figure, representing a chart on Mercator's projection according to the limits required.

41. On this chart may now be plotted the position of Cape Sable lighthouse according to its latitude and longitude. Point *A*, S 45° E, 22 miles from the light, gives the position from which the given courses and distances are plotted. The point *B* or the position arrived at will be found to be in latitude 40° 43′ N and longitude 65° 36′ W, very nearly.

LEAD AND LOG

THE LEAD

42. Sounding.—The operation of measuring the depth of the water and investigating the character of the bottom is called **sounding.** The instruments used for this purpose are the **lead** and the **sounding machine.** There are three kinds of leads, viz., the *hand lead*, the *coasting lead*, and the *deep-sea lead.* All these leads are similar in form to the one shown in Fig. 17, being widest at the lower end *a*, which is hollowed out for the reception of a lump of tallow (see bottom view). This cavity is called the *arming* of the lead. The purpose of the tallow is to bring up a specimen of the bottom that it touches, so that the quality of the ground struck by the lead may be compared with the description of the bottom given on the chart or in sailing directions, and the ship's position therefrom approximately determined.

FIG. 17

43. The **hand lead** weighs from 7 to 14 pounds, and is therefore readily thrown by hand. It is used in shallow water, when in the vicinity of land, and for soundings in channels, rivers, and harbors where the depth is inconsiderable.

44. Manner of Marking the Lead Line.—The hand-lead line, which usually has a length of 20 fathoms, is marked thus:

2 fathoms	2 strips of leather
3 fathoms	3 strips of leather
5 fathoms	a piece of *white* bunting
7 fathoms	a piece or *red* bunting
10 fathoms	a piece of leather with a hole in it
13 fathoms	a piece of *blue* bunting
15 fathoms	a piece of *white* bunting
17 fathoms	a piece of *red* bunting
20 fathoms	a strand with two knots in it

In river and lake navigation, the first four fathoms are usually subdivided into feet.

The lines used for large hand leads and coasting leads, and which are longer than 20 fathoms, are marked above the 20-fathom mark with an additional knot at every 10-fathom point (at 30, 40, 50, etc.) and with a single knot at each intervening 5-fathom point (at 25, 35, 45, etc.).

It should be noticed that by this marking of the lead line, the intervening fathoms of 4, 6, 8, 9, 11, etc. are without any markings and consequently the leadsman has to depend on his own judgment concerning the depth between the marked fathoms. In order to obviate the uncertainty of guesswork, however, a lead line should be marked at every fathom and half fathom up to 12 or 15 fathoms, and the first 4 or 5 fathoms should always be subdivided into feet by suitable markings.

45. The **deep-sea lead** is of much larger size, weighing from 80 to 150 pounds, and is attached to a much longer line, in order to take soundings in depths of 100 or more fathoms. The line used for this lead is marked in the same

manner as the coasting-lead line; at 100 fathoms is a piece of bunting, and then the knots recommence.

46. Methods of Sounding.—The operation of sounding, or measuring the depth of water, is always performed on the windward side of the vessel. Usually the leadsman selects a place that will insure a free use of his arms without the danger of falling, and which is, at the same time, within hearing distance of the officer in charge of the deck. Before sounding, the lead line should be coiled up near by in such a

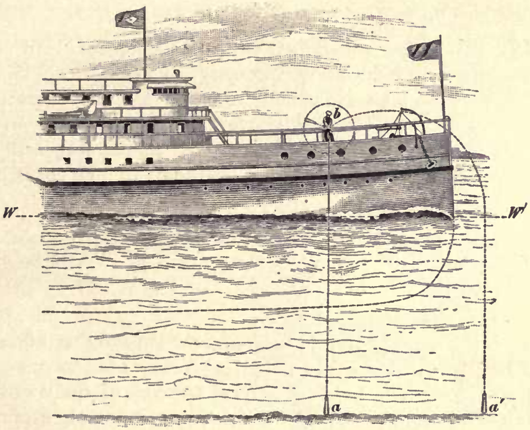


FIG. 18

manner as to insure its running out freely and without a hitch when the lead is thrown. Holding the line a few feet from the lead the operator swings it back and forth, or above his head so as to impart to the lead a certain velocity and then throws it as far forwards as is deemed necessary, according to the speed of the vessel. The point *a'*, Fig. 18, where the lead touches the bottom, will then, by the ship's forward motion, be directly underneath the leadsman and the sounding an up-and-down one, as indicated by the line *a b*. Before reading off the cast, all slack line must be pulled in

and the line be absolutely taut. To make sure that the lead has reached the bottom, the leadsman pulls his line up and drops it, in rapid succession, a foot or two each way, in order to get a true "touch" of bottom, and then reads off the depth by the mark on the line near the water-line WW'. For sounding during the night, when marks cannot be seen, the distance from the operator's hand to the water-line should be known, and at each cast this distance should be deducted from the amount of line out. At night, expert leadsmen usually read off the depth by using their sense of touch in distinguishing one mark from the other.

47. To take soundings with the deep-sea lead, the ship's speed is reduced, if necessary, to about 1 or 2 knots. Three or more men are usually required for this operation, in case the depth is considerable. Generally, it is carried out as follows: One man is stationed at the forecastle *a*, Fig. 19, with the lead and several fathoms of line coiled up;

FIG. 19

at *d* on the quarter-deck is the operator (usually an officer), and between *a* and *d* are placed as many men as may be required, each man holding several fathoms of the lead line in his hand ready to let go when he feels the weight of the lead. Everything being in readiness, the operator at *d* gives the signal to heave, and the man at *a* throws the lead, whereupon the line is paid out freely by the men along the rail, so that the lead is surely at the bottom before the operator is directly above it.

It should be remembered that the up-and-down sounding is the only true one, and that errors in sounding are generally in *excess*, that is, the actual depth is less than that indicated by the lead line. When the line is run in, the tallow in the arming of the lead is examined in order to determine the nature of the bottom.

THE SOUNDING MACHINE

48. An excellent substitute for the lead is the sounding machine, of which there are at the present time several types in use; among them, Massey's, Walker's, and Lord Kelvin's are the best known. The last named is especially favored by navigators on account of its many excellent qualities. This instrument, represented in Fig. 20, is described by Captain Lecky in his "Wrinkles of Navigation" as follows: "Lord Kelvin's sounding machine consists of a drum *D* about 1 foot in diameter and 4 inches wide, on which about 300 fathoms of steel pianoforte wire are tightly wound. To the wire is attached 9 feet of log line, and to this is fastened a heavy sinker about twice the length of the ordinary lead, but not so thick. On the log line between the wire and the sinker, a small copper tube is securely fastened, the lower end of which is perforated; the upper end being opened and shut at pleasure by means of a close-fitting cap. When ready for sounding, the copper tube contains a smaller-sized glass one. This latter is open at the bottom end and hermetically sealed at the other. The interior surface is coated with a chemical preparation of a light salmon color (chromate of silver). The drum is fitted with a brake, which, on a cast being taken, controls its speed and ultimately arrests it when the

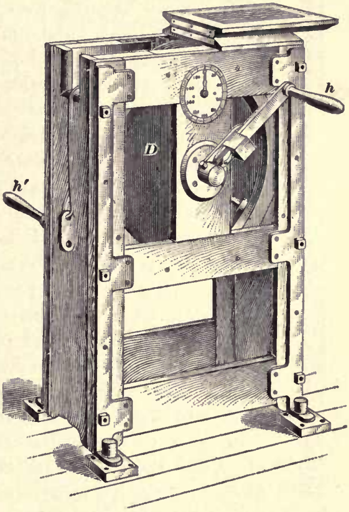


FIG. 20

lead touches the bottom. A pair of small winch handles h, h' wind up the wire again, and the depth is indicated by the height of the discoloration on the inside of the glass tube. The water is forced up the tube in its descent, in obedience to certain well-known laws, and the chemical action of the salt turns the salmon color to a milky white (chloride of silver). The junction of these two colors, when applied to a graduated boxwood scale, tells the depth to which the lead descended."

Fig. 21

49. The Depth Recorder.—Instead of the arrangement with tube and chemicals, the **depth recorder** may be used. This instrument is shown in Fig. 21. Its construction is based on the assumption that the water pressure increases in direct ratio to the depth. As the sinker descends, the increased pressure of the water forces a piston d up into the tube, while the spiral spring attached to the piston pulls it back. The amount that the piston is forced up against the action of the spring depends on the depth. To record the depth, a *marker* is used. As the recorder goes down, the marker c is pushed along the piston; when the recorder is brought up to the surface of the water, the piston comes back to the original position, but the marker remains at the place on the piston to which it was pushed and shows the depth reached by the instrument.

Between each cast, the nut a should be unscrewed to slacken the valve b and the recorder turned upside down in order to empty out any water that may have leaked in. Before each cast, the nut a should be firmly tightened and the marker set to zero.

50. Mechanism of the Sounding Machine.—In Fig. 22, representing a section of Lord Kelvin's sounding machine, a is the drum carrying the wire and b, the center line of the axis around which the drum revolves; c and c' are circular plates to which bevel-shaped blocks of wood are attached. When these plates are pressed together the wooden

blocks form a friction brake acting directly on the drum. The plate *c* is rigidly secured to the axis while *c'* is adjustable by means of the handle *h*. The nut *e*, engaging a thread on the axis, has an arm *f* extending upwards to a clamp *g*; when this arm is secured to the clamp, which is fixed to the frame of the machine, the nut *e* cannot turn, and, therefore, when the handle *h* is turned the plate *c'* moves in or out, relaxing or applying the friction brake, as desired.

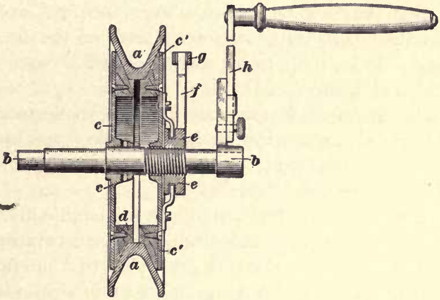


FIG. 22

51. Operating the Sounding Machine.—The following directions for operating Lord Kelvin's sounding machine are furnished by John Bliss & Co., of New York, the American agents for the makers. It is assumed in this description that a depth recorder is used.

(*a*) The work of taking a cast is done by two men under the superintendence of an officer. For brevity, the men will be referred to as brakesman and leadsman. The regular post of the brakesman is at the starboard side of the sounding machine and that of the leadsman at the taffrail fair lead.

(*b*) The men go to their posts, and without further orders the brakesman puts on the handles and fixes them securely by means of the screws. At the same time the leadsman sees that the lead is properly armed and carries it to the fair lead. The officer examines the recorder, sets the marker to zero, and replaces it in the sinker.

(*c*) The brakesman standing on the starboard side of the machine sees that the arm *f* is prevented from turning by means of the clamp *g*, provided for that purpose. He then grasps the handle and applies the brake by turning the handle in the direction for winding in the wire. When the brake is sufficiently tightened, the brakesman calls out "brake on." The leadsman then lets down the sinker without a jerk until it hangs on the rope. The brakesman, holding the handle in one hand, releases the arm *f* and pays out by turning the handle until the link has passed over the fair lead. The leadsman then calls out "on brake," and, on the brakesman answering "brake on," takes away his hand from the link and calls out "ready."

(*d*) The brakesman now, having seen that the index of the dial is at zero, takes the brass finger pin,* and holding it lightly by its handle, presses it against the wire and waits for the officer to give the order "let go."

(*e*) The brakesman instantly turns his handle in the direction for paying out until the drum with wire rotates freely. While the wire is running out, he holds the handle in one hand and the finger pin pressing against the wire in the other hand. The brakesman watches the dial, and if the bottom has not been reached before coming to 250, he commences to apply the brake, so as to stop before 300 is reached. As soon as the brakesman feels the wire slacken he begins turning the handle in the direction for hauling in until the brake is tightened up and the egress of the wire stopped. He then releases the arm *f* and commences to wind in.

(*f*) The leadsman winds with his left hand and guides the wire to the drum with a piece of waste canvas in his right hand. The brakesman, winding with both hands, watches the dial from time to time during the winding in, and when

NOTE.—When the speed exceeds 10 knots, it is desirable to have another man to help in the winding. He is to stand looking aft, and to work with both his hands on the port handle, the leadsman working on the same handle with his left hand.

*The brass finger pin is supplied with the machine, but if it should be missing or lost a piece of hardwood will do to press on the wire while the sinker is going to the bottom.

the link is five fathoms from the fair lead, he calls out "hand the lead."

(*g*) The leadsman instantly leaves the machine, goes to the taffrail, and steadies the link and cord by his hand as they come up, and guides the link over the fair lead, while the brakesman continues slowly winding in until the link reaches the wire drum; then, placing it properly on the wire drum he winds in one turn more; taking care that the link is a little above the middle of the after side of the drum, so that its weight may help to keep the wire stretched, he puts on the brake. In the meantime, the leadsman hauls by hand on the sinker. The leadsman then takes the lead on board, shows the depth recorder to the officer, examines the arming for a specimen of the bottom, shows it to the officer, prepares the arming for a fresh cast, and then goes forward to the machine and stands by on its port side.

(*h*) It takes from a few seconds to a minute for the sinker to reach the bottom from the time it is let go, and from $\frac{1}{4}$ minute to 4 minutes for two men to wind home the lead, if the depth does not exceed 100 fathoms. Thus it is easy to take soundings every 10 minutes if there is an extra hand or two to relieve.

(*i*) Two men can with ease take a sounding every quarter of an hour, and this should be the rule whenever, in keeping the machine thus going, useful information as to the ship's position can be had. The reading on the dial shows approximately the number of fathoms of wire run out. This may be something more than twice the depth for speeds up to 11 knots; or it may be almost as much as three and a half times the depth if the speed is 15 or 16 knots. The proportion of wire to depth differs not only with the speed of the ship, but also with the roughness of the sea and with the depth itself.

(*j*) When the chemical tubes are used instead of the depth recorder, one of the copper guard tubes is lashed to the rope between the wire and the sinker, about 3 feet from the end of the wire. Before taking a cast, the officer places a glass tube with the open end down in the guard tube, puts

on the cap, and then proceeds as directed in sections (*c*) to (*g*). When the guard tube is brought up close to the fair-lead pulley, it should be lifted on board, care being taken to keep it right side up. If it is turned on its side or upside down, the water will run up the glass tube and make a misleading mark.

(*k*) *Precautions.*—The wire will break at a kink under a very moderate pull: 30 pounds steady force, or a very slight jerk, will break it at a kink. Without a kink, and with proper care, the wire can scarcely be broken in practice with the machine. No wire should ever be lost in service, unless by some extremely rare accident, not foreseen, and therefore not provided against.

(*l*) Absolute security against kinks would be had if the wire could be absolutely prevented from ever slacking. It does slacken somewhat the moment the lead touches the bottom, but not to a dangerous degree if the ship is going at more than 5 knots, and if the brake is instantly applied when, by the wire's yielding to the brass pin, the commencement of slacking is shown. The brake, according to instructions in paragraph (*e*), should be instantly applied, so as to slow the motion of the wheel, but not with force enough to stop the wheel suddenly. There is much more danger of losing the wire through a kink in taking an up-and-down cast than in a flying cast with the ship running at 12 or 14 knots. Whenever a cast is taken at any speed less than 5 knots, it is advisable to manage the brake so as to moderate the rate at which the wire pays out, according to judgment, letting the wheel revolve at something like three turns per second. If the ship's speed is more than 5 knots, observe all the rules just given.

52. Of late, several improvements have added to the efficiency of this sounding machine. The latest pattern, shown in Fig. 23, differs in many respects from older types. It embodies a number of alterations that greatly enhance its usefulness, among its many advantages being the fact that the different parts of its mechanism are easily accessible for

cleaning and repairs. The line used in this machine is a seven-strand, galvanized steel wire, which is not liable to kink or rust.

The advantages of the sounding machine over the lead are many, the main advantage being that the speed of the ship need not be reduced, and the time required to sound great depths is minimized to a fraction of the time required when using the common deep-sea lead.

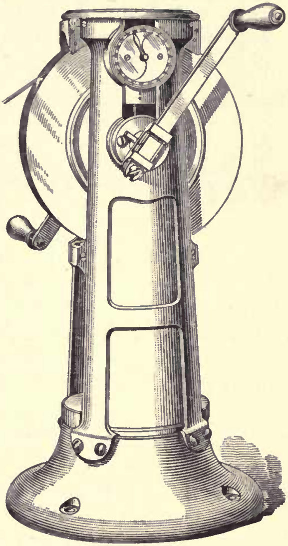


Fig. 23

53. Importance of Using the Lead.—A navigator should bear in mind that he fails to do his duty when he neglects to take soundings when approaching or sailing near land. While off the coast, the lead should be kept going frequently, no matter how fine and clear the weather may happen to be, or how confident he may feel as to the exact position of his vessel.

Too much attention cannot be given to the lead, log, and lookout. Frequent and careful soundings, correct logging, a good lookout, and a correct compass course should be maintained by all good seamen and careful navigators.

54. Reduction of Soundings.—As previously stated, the soundings recorded on charts correspond to the ordinary mean low-water level; hence, at any other condition of the tide a correction should be applied to the sounding obtained in order to compare it with the sounding shown on the chart. This is important, for instance, when trying to locate the

position of a ship by means of a chain of soundings. For general purposes, the following table for reducing soundings to the mean low-water level will be found sufficiently correct:

At high water deduct the whole range of the tide.
At the 1st hour after *high water* deduct seven-eighths of the range.
At the 2d hour after *high water* deduct three-fourths of the range.
At the 3d hour after *high water* deduct one-half of the range.
At the 4th hour after *high water* deduct one-fourth of the range.
At the 5th hour after *high water* deduct one-eighth of the range.
At the 6th hour after high water, or at low water, the soundings should agree with those given in the charts.
At the 1st hour after *low water* deduct one-eighth of the range.
At the 2d hour after *low water* deduct one-fourth of the range.
At the 3d hour after *low water* deduct one-half of the range.
At the 4th hour after *low water* deduct three-fourths of the range.
At the 5th hour after *low water* deduct seven-eighths of the range.
At the 6th hour after low water, or at high water, deduct the whole range of the tide.

Thus, if the range of the tide, or the vertical distance between high and low water, is 12 feet and a cast is taken, for instance, 2 hours after high water, three-fourths the range, or 9 feet, should be deducted from the number of feet in the sounding.

THE LOG

55. Logging.—The operation of measuring the progress or speed of a ship through water is called **logging;** the two principal instruments used for logging are the *common log* and the *patent log*, of which there are several types in use. In steamers, the revolutions of the propeller afford a valuable means of ascertaining the speed, provided that a number of such measurements have been previously compared and found to agree with those recorded by the log.

56. Principle of the Common Log.—The principle on which the **common log** is founded is as follows: Some light floating object is thrown overboard; as soon as it strikes the water, it ceases to partake of the ship's onward motion and becomes stationary. The distance of this stationary object from the ship is then measured after a certain interval

of time has passed, and from this measurement the approximate speed of the ship is ascertained.

57. The log consists of three parts—the *chip*, the *line*, and the *glass*. The chip is the floating object thrown overboard, the line measures the distance, and the glass defines the interval of time.

58. The **log chip** is a triangular piece of light wood *c*, the lower edge of which is rounded and weighted with a strip of lead sufficiently heavy to make it float in an upright position, as shown in Fig. 24. In each corner is a hole, the

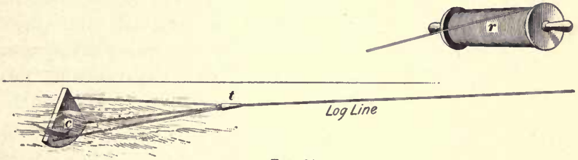


Fig. 24

log line being knotted to the one pointing upwards; in the lower holes is fastened a sling, or bridle, at the bight of which is a peg that fits snugly into a wooden socket *t*, commonly called the **toggle.** This peg can be released from the toggle by a jerk on the log line, thus allowing the chip to be pulled in with little resistance. The inboard end of the line is attached to a **reel** *r*, around which it is wound.

59. The Log Line.—The first 15 to 20 fathoms of the log line from the chip is called the **stray line,** and is usually marked by a small piece of white or red bunting; the purpose of the stray line is to allow the chip to get clear of the vessel's eddy, or wake, before the measuring commences. The rest of the line, or the log line proper, is divided into parts of equal length, called **knots,** by pieces of cord fastened between the strands of the line. Each piece of cord carries the requisite number of knots according to its order from the stray-line mark. The length of each knot, when the nautical mile is taken as the unit, is 47 feet 3 inches.

60. The **log glass,** Fig. 25, is a sand glass of the same shape and construction as the old hour glass. A vessel usually carries two log glasses, one of which runs out in 28 seconds and the other in 14 seconds; the latter is used when the vessel is going at a high rate of speed, when the number of knots of the line run out is doubled.

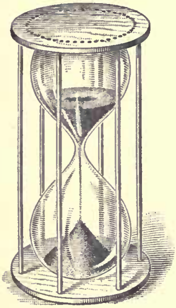


FIG. 25

61. Relation Between the Knot and the Nautical Mile.—In order to determine the speed of a ship per hour, the length of each knot must bear the same ratio to the nautical mile (6,080 feet) as the time of the log glass does to the hour. Hence, the following proportion:

As the number of seconds in an hour is to the number of feet in a mile, so is the number of seconds of the log glass to the number of feet in the knot; or,

$$3{,}600 \text{ seconds} : 6{,}080 \text{ feet} = 28 \text{ seconds} : x$$

whence, the length of the knot represented by x is

$$\frac{6{,}080 \times 28}{3{,}600} = 47.29 \text{ feet, or } 47 \text{ feet } 3 \text{ inches}$$

EXAMPLE.—What should be the length of the knot, using the statute mile (5,280 feet) as a unit, and a glass of 30 and 28 seconds, respectively?

SOLUTION.—The length of the knot in the former case would be $\frac{5{,}280 \times 30}{3{,}600} = 44$ feet, and in the latter case $\frac{5{,}280 \times 28}{3{,}600} = 41.07$ ft. Ans.

62. Method of Heaving the Log.—Generally an officer and two men constitute the logging party, one man holding the reel and the other attending to the glass. The officer throws the log chip into the water on the lee side of the ship. By its vertical position, the chip remains practically stationary in the water while the line is reeled off as the vessel moves forwards. When the stray-line mark crosses the rail, the officer says "turn," whereupon the assistant holding the

glass turns it over instantly, watches the sand running down, and says "stop" when the last grain passes through the opening. The officer then checks the line and reads off the number of knots run out. The chip should be thrown out well to the leeward of the stern, and the reel should be held in such a position as to allow the line to run out freely.

63. Errors of the Common Log.—In order to guard against error caused by an incorrect length of the knot, the log line should be examined quite frequently by comparing each knot with its proper length, as determined by permanent marks fastened to the deck. There may also be errors attached to the log glass; the condition of the sand or the size of the hole through which it runs may be affected by changes in temperature. Hence, it is important to know how to allow for any of these errors.

It is evident that the longer the time taken by the sand to run out, the more line will pass over the rail; and the longer the knot, the smaller will be the number of knots passed out in a given interval of time. Therefore, the speed shown is directly proportional to the length, or time, of the glass, and inversely proportional to the length of the knot. In other words, when an erroneous glass is used, the distance shown by the log is to the correct distance as the number of seconds of the erroneous glass is to 28 seconds; when the line is wrong, the distance shown by the log is to the correct distance as 47.3 feet is to the length of the knot used.

64. Correction of Errors.—Let

x = correct distance;
d = distance shown by log;
g = erroneous time of glass;
l = incorrect length of knot.

We then have the following proportions:

(Glass wrong) $d : x = g : 28$; whence, $x = \frac{28\,d}{g}$ (1)

(Knot wrong) $d : x = 47.3 : l$; whence, $x = \frac{d\,l}{47.3}$ (2)

If both glass and knot are wrong, by substituting in formula **1** the value of x from formula **2**, we obtain the general formula,

$$x = \frac{28\,d\,l}{47.3g} \qquad \textbf{(3)}$$

EXAMPLE 1.—The speed of a vessel, as indicated by the chip log is 8 knots; the glass after being examined and compared with a watch is found to indicate 34 seconds instead of 28. Find the true rate of speed.

SOLUTION.—Insert the value of d and g in formula **1**, whence the true rate of speed, or

$$x = \frac{8 \times 28}{34} = 6.6 \text{ knots, nearly. Ans.}$$

EXAMPLE 2.—The distance run according to the log is 195 miles; but when measured, the length of the knot is found to be 50 feet; what is the true distance?

SOLUTION.—In this case, the length of the knot being incorrect, in formula **2**, $d = 195$ and $l = 50$; hence true distance, or

$$x = \frac{195 \times 50}{47.3} = 206.1 \text{ mi. Ans.}$$

EXAMPLE 3.—The length of the knot is only 45 feet and the glass runs out in $26\frac{1}{2}$ seconds; the distance run, according to the log, is 265 miles. Find the true distance.

SOLUTION.—In this instance both the glass and the knot are wrong; $d = 265$, $l = 45$, and $g = 26\frac{1}{2}$. Inserting these values in formula **3**, the true distance, or

$$x = \frac{265 \times 45 \times 28}{26.5 \times 47.3} = 266.4 \text{ mi. Ans.}$$

EXAMPLE 4.—The distance sailed according to the log was 304 miles, and the true distance was 295 miles; the glass ran out in 27 seconds. What was the error in the length of the knots on the log line?

SOLUTION.—We have $d = 304$, $x = 295$, and $g = 27$. Inserting these values in formula **3**, and solving for l, $l = \frac{47.3 \times 295 \times 27}{28 \times 304}$ $= 44.3$ ft. Hence, the knot was $47.3 - 44.3 = 3$ ft. too short. Ans.

EXAMPLES FOR PRACTICE

Solve the following problems:

1. According to the log, the distance sailed was 217 miles; both the glass and the length of knot were incorrect, the former being 3 seconds too long and the latter 48 feet. Find the true distance.
Ans. 198.9 mi.

2. The true distance was 160 miles, and the distance according to the log 150 miles, the length of the knot being 47.3 feet. Find the error of the glass. Ans. 1.8 sec. too short

3. According to the chip log, a steamer covers a distance of 195 miles; it is found afterwards, by measuring the line, that the knot is 1 foot too long. What is the true distance covered by the steamer?
Ans. 199 mi.

NOTE.—Before a long line is marked off, it should be stretched and boiled in very salty water; by this process the line will be less influenced by the constant wetting to which it is subjected.

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65. Patent Logs.—Besides the chip log, there are at the present time several **patent logs** in use, standard types of which are shown in Figs. 26, 27, 28, and 29. Most of them are based on the indications given by the revolution of a small fan or screw that is towed by the ship.

A general description of the patent log is as follows: A *rotator r* (Figs. 26 and 27) attached to a line is thrown over-

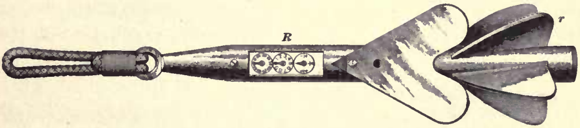


FIG. 26

board and towed by the ship, thus causing it to revolve more or less rapidly, according to the rate at which it is drawn through the water; this rotary motion is transmitted to a *register R*, which by means of a series of gears and a dial records the number of miles run. In the **harpoon log**, Fig. 26, the rotator is connected directly with the register and the whole is towed behind the ship; in the different types of **taffrail log**, in use at present, the register is attached

FIG. 27

to the rail or other suitable place and the revolutions of the rotator are transmitted to the register by means of a plaited line, as shown in Figs. 27, 28, and 29. In the harpoon type of log, it is necessary to pull in the log in order to read off the distance run; whereas in the taffrail type, this inconvenience is done away with and the distance covered can be read off at any moment.

66. Of the two logs, those of the harpoon type, when in good order, are considered the more reliable, so far as accuracy in recording distance is concerned. Its disadvantages, however, are that its recording mechanism may get out of order *without the fact being known* until the log is pulled in, and that if the log is lost through damage to the line the cost of replacing it is considerable. With the taffrail log, if the rotator is lost, it can be replaced at small cost; and its dial can be examined at any time without taking the rotator from the water.

67. Patent logs are not always trustworthy; a log that runs accurately at a

certain speed, may not do so when the speed is changed considerably. If possible, the error for different speeds should be determined and tabulated, and a correction applied accordingly when the log is used in practice. At a high speed, the rotator sometimes has a tendency to jump

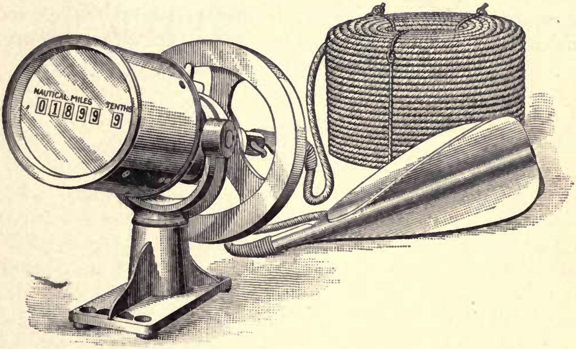


Fig. 28

above the surface of the water, causing the register to record a smaller distance than that actually covered by the ship. This may to a certain extent be overcome by *service*, that is, by wrapping the towing line with strips of sheet lead at suitable distances from the rotator; but this does not

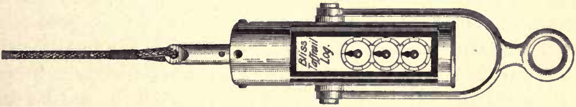


Fig. 29

always prove effective. When in open sea, fast ocean liners usually dispense with the patent log and use the chip log every 2 hours. Both the chip and the patent log should be used when nearing land.

68. Before setting out a patent log to work, the number of miles indicated by the register must be recorded; this

should also be done when any change of the ship's course occurs. Under any circumstances, the distance covered should be noted at least once every other hour, or oftener, if necessary, either by a patent or by a common chip log.

Like any other mechanical device from which a certain degree of accuracy is expected, the patent log should be given a reasonable amount of care. Its mechanism should be oiled daily, and when not in use it should be kept clean and dry.

DEVIATION AND COMPASS COMPENSATION

ERRORS OF THE COMPASS

1. The compass may be affected by three kinds of errors, known respectively as *variation*, *deviation*, and *local attraction*. Of these, **variation** is caused by the magnetism of the earth; **deviation**, by the magnetism of the iron and steel in the ship itself; and **local attraction**, by some temporary magnetic influences in close proximity to the compass. In wooden ships, the error caused by deviation is very small, and in some cases entirely absent.

VARIATION

2. Magnetic Meridian.—When a magnetic needle is suspended at its center by means of a thread, so as to hang freely in a horizontal position, it will, after swinging from one side to the other, come to rest in a certain direction. This direction, assumed by the needle when influenced only by the attraction of the earth, is called the **magnetic meridian** of a place. When extended to the whole earth, the magnetic meridian may be defined as the track made by a magnetic needle conveyed over the earth in a direction always corresponding to that indicated by its magnetic axis.

3. Variation, or Magnetic Declination.—The angle that the magnetic meridian makes with the geographical meridian, or, what is the same, the angle that the direction of the suspended needle makes with the true meridian, is

called the **variation**, or **declination**, of the magnetic needle. Thus, in Fig. 1, if NS represents the true, or geographical, meridian and n the north-seeking end of the magnetic needle, the angle formed by the needle and the meridian is the variation of the compass; it is named *easterly* or *westerly*, according to the direction in which the needle is deflected. Thus, if the north point of the compass is deflected to the east of the true meridian, as in Fig. 1 (a), the variation is easterly; if deflected to the west, as in Fig. 1 (b), the variation is westerly.

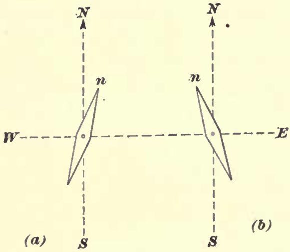


FIG. 1

In Fig. 2 are shown the magnetic meridians for the hemisphere that embraces the two magnetic poles of the earth. These irregular curves show approximately the direction in which a magnetic needle will point for a given locality, and the angle between a magnetic meridian and a true meridian will give the amount of variation for the place considered.

4. Agonic and Isogonic Lines.—The variation of the compass is different at different places on the earth's surface, and at a given place it undergoes a gradual, though small, change from year to year. At places where the magnetic meridian coincides with the true meridian, the variation is said to be zero, and lines connecting these places and plotted on charts are called **lines of no variation**, or **agonic lines.**

In Fig. 3, which represents a map of the world, the lines of no variation are indicated by heavy lines, westerly variation by continuous fine lines, and easterly variation by dotted lines. A glance at this map shows that the variation is westerly nearly all over the North Atlantic Ocean, increasing in amount toward the north, and that the variation is easterly in the greater part of the Pacific Ocean. The lines

connecting places on the earth's surface where the amount of variation is the same are called **isogonic lines,** and charts on which such lines are shown are known as **isogonic charts.**

The isogonic lines covering the Atlantic and Pacific Oceans depend almost entirely on results obtained in wooden ships

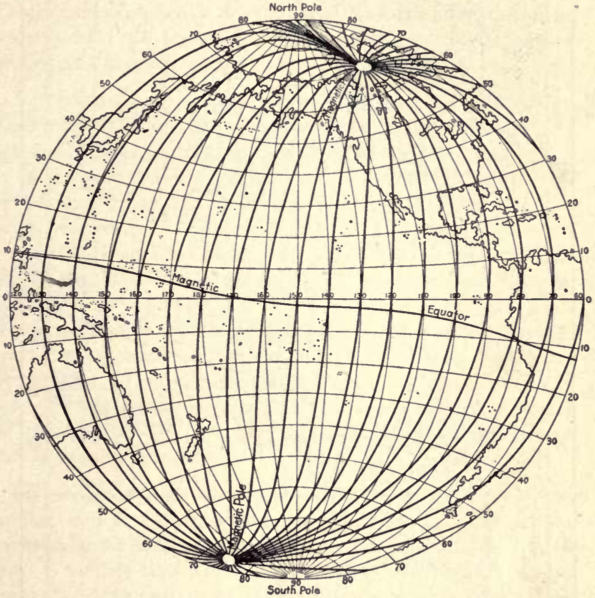


FIG. 2

from 50 to 100 years or more ago. It is therefore impossible to state just how accurate they are. Fortunately, all evidence goes to show that over the deep waters of the ocean most frequently traversed—the Atlantic—the present lines of equal magnetic declination are doubtless correct to within 1°. The study and investigation of the earth's magnetic properties and the determination of the magnetic declination on both

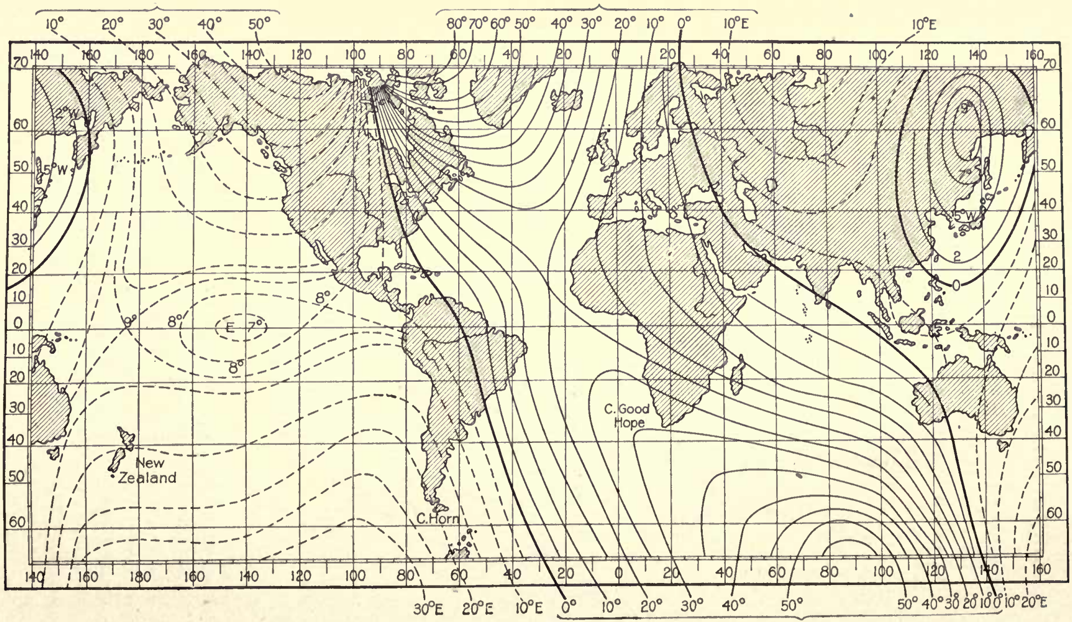


FIG. 8

land and water have lately been taken up with great energy. The United States Coast and Geodetic Survey now has four magnetic observatories that cooperate with a number of foreign observatories and also with the various arctic and antarctic expeditions in the collection of reliable data relating to magnetic phenomena of the earth. The Coast and Geodetic Survey has also fitted out several vessels with the necessary instruments for determining the magnetic elements at sea.

5. **Changes in Variation.**—Magnetic variation undergoes a slow, progressive change in amount, and after long periods, changes of direction, or secular changes, take place; in other words, the variation does not remain the same for a given locality for any considerable length of time. The line of no variation that now crosses the American continent, passed through London in 1657, and through Paris in 1669. At the present time, it is moving toward the west at the rate of about 1° in 14 years. Places to the east of this line have westerly variation, which is gradually increasing, while to the west the variation is easterly, slowly decreasing in amount as the line of no variation approaches. These changes in the variation are important to the navigator, and care should be taken that the charts used by him contain the latest corrections of this kind.

6. **Magnetic Dip.**—Another deflection of the compass needle, called **inclination,** or **magnetic dip,** was discovered by an English instrument maker in 1576. He found that a needle, however well balanced, would, after being magnetized, depart from the horizontal position and point downwards with its north end. Further investigations in different parts of the world disclosed the fact that in the southern hemisphere the south end of the needle has a similar tendency to deflect downwards; but at places situated within the equatorial region this inclination is zero, or the needle assumes a horizontal position. By connecting all places where the magnetic dip is zero, a line, or zone, called the **magnetic equator** is established.

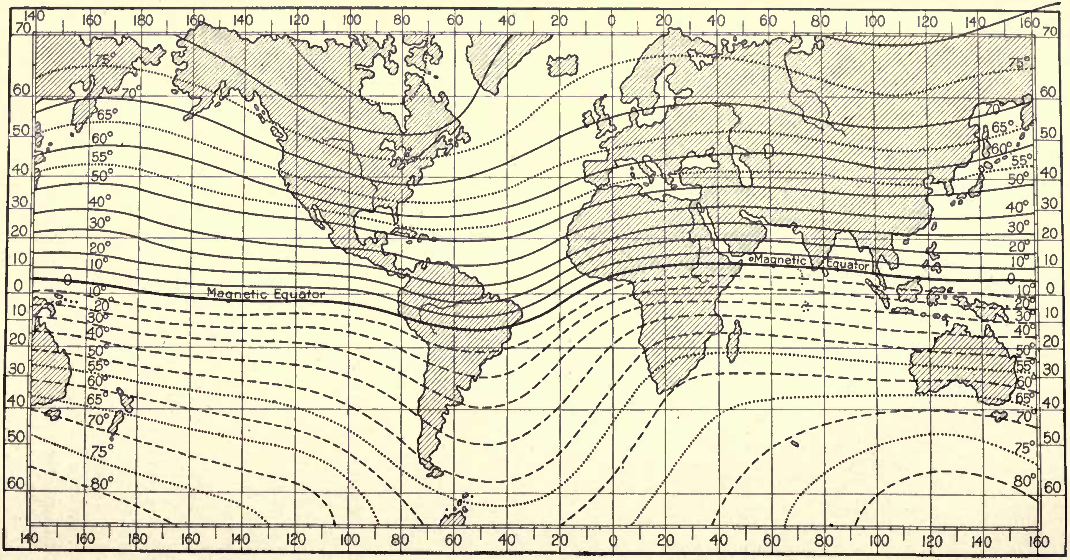


Fig. 4

7. The magnetic equator does not coincide with the geographical equator, but crosses it at two places, one near longitude 12° W and the other near 168° W longitude, never receding more than 16° on either side, as shown in the chart, Fig. 4. From the magnetic equator, the dip of the needle increases gradually until the magnetic poles of the earth are reached, where the needle points vertically downwards. Thus, when going northwards from the equator, the north-seeking end of the needle will commence to incline until the north magnetic pole is reached, where it will assume a vertical position. The south end of the needle will act similarly when going from the equator toward the south pole. In 1831, Sir John Ross discovered the north magnetic pole, or, rather, the area in which the dip amounted to 90°, in latitude 70° N and longitude 96° 40′ W, a position situated N N W of Hudson Bay. The same observer, in 1839, found the position of the south magnetic pole to be approximately in latitude 75° 30′ S and longitude 154° E, or to the south of Tasmania. Later investigations, however, modify these locations somewhat.

8. Isoclinic Lines.—The magnetic dip, like the variation, is subject to continual and progressive changes, both secular and periodical. Lines that are drawn intermediate to the poles and equator, connecting all points where the dip of the magnetic needle is the same, are called lines of equal inclination, or **isoclinic lines,** and charts on which such lines are plotted are called **isoclinic charts.**

These lines, as shown in Fig. 4, are somewhat analogous to the geographical latitude parallels, but do not coincide with them; they are, as a rule, nearly parallel to the magnetic equator, and the term *magnetic latitude* is used simply to denote the position of a place with reference to the magnetic dip.

9. Isodynamic Lines.—Another system of lines is created by joining all points on the earth's surface where the intensity of the earth's magnetic force is the same; these lines are called **isodynamic lines.**

To the navigator, the dip and the magnetic intensity is of no particular importance, but it is nevertheless fitting that he should have some knowledge of the nature of these subjects. In the mariner's compass, the effect of the dip is counteracted by small copper slides that are attached to the needle or by other suitable weights affixed to the under side of the compass card.

DEVIATION

10. As already stated, the deviation of the compass is caused by the magnetism of the iron and steel of which the ship is built; it is the deflection of the needle from the magnetic meridian, and is one of the most serious disturbances of the compass with which the navigator has to deal. In vessels where the compass is surrounded by masses of iron and steel, the deviation is at times so great as to render the compass almost useless, unless extraordinary precautions are taken to counteract the disturbing forces.

11. Soft and Hard Iron.—In order to explain the cause of deviation, it may be advisable, before going further into this subject, to state that in reference to magnetism there are two classes of iron; namely, *soft* and *hard iron*.

By **soft iron** is meant such iron as becomes magnetized immediately on being exposed to the influence of some magnetic body, but which has no power to retain the magnetism thus acquired when removed from the sphere of influence of the magnetic body. Malleable and cast iron belong to this class.

By **hard iron** is meant such iron as does not become magnetized by induction when exposed to the influence of a magnetic body, but which retains its magnetism permanently, or nearly so, when once magnetized. Artificial magnets are therefore necessarily made of hard iron.

12. Induced Magnetism.—When the pole of a magnet is brought in close proximity to a mass of soft iron, it will communicate to this mass a certain amount of magnetism

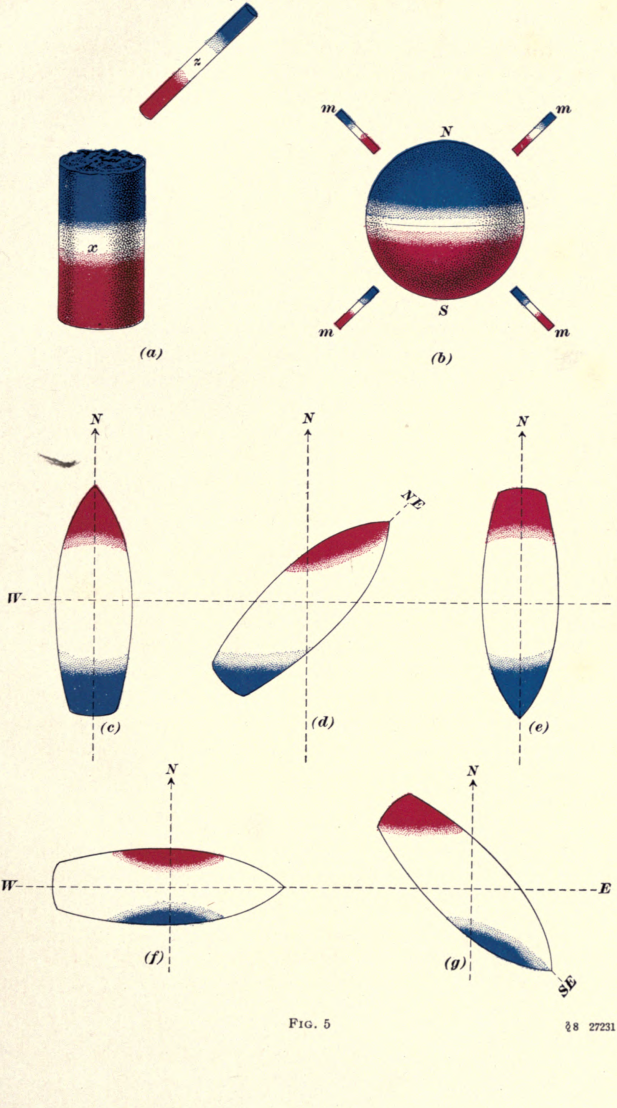


FIG. 5

without losing, in any measurable degree, any of its own magnetism. The magnetism thus acquired is known as **magnetism by induction,** or **induced magnetism,** and the polarity of the part exposed to the inducing pole is always opposite to that possessed by the pole. Thus, in Fig. 5 (a) if x is a soft-iron block and z a bar magnet, red pole downwards, the block will acquire blue polarity in its upper end and red polarity in its lower end. In other words, induced magnetism is of a character of polarity opposite to the kind inducing it.

13. Magnetism by Induction From the Earth. Bearing in mind the fact that the earth is a large magnet having blue polarity in the northern hemisphere, it is evident from the preceding articles that when a bar of soft iron in northern latitudes is held in a north-and-south direction, with its northerly end inclined downwards, it will be feebly magnetized from the earth and will have in its lower end a *red*, or positive, pole, which, according to the law of attraction and repulsion, will repel the north end of the magnetic needle. In its upper end, the bar will have a *blue*, or negative, pole, which will attract the north end of the magnetic needle. If, however, the bar is reversed, it will be found that the polarity of the bar is changed also, its lower and northerly end always being of red polarity, or opposite in character to the polarity of the earth. This is illustrated in Fig. 5 (b), where NS represents the earth and m soft-iron bars. It will be noticed that in the southern hemisphere the conditions are just the reverse. Again, referring to the northern hemisphere, if the bar is held horizontally in a north-and-south position, it will still retain red polarity in its north end and blue polarity in its south end, although the magnetism in this position is of less intensity. If the bar is held horizontally in an east-and-west position, it will lose its magnetism entirely.

Soft iron, however, may be transformed into hard iron by hammering, and it is therefore evident that a soft-iron bar of the kind referred to, after being hammered into hardness,

will retain its magnetism and will act as a magnet irrespective of its position, the nature of its polarity depending on the direction in which it was held when being hammered. Thus, if the bar is held in the direction of the meridian, its north end will have red, and its south end blue polarity, and this polarity will remain constant, no matter how the bar is turned.

14. Magnetic Polarity of Iron and Steel Vessels. Since the material of which a ship is constructed is composed of both soft and hard iron, it is evident that, while being built, the ship will acquire a magnetic character, partly through induction from the earth and partly through the great amount of hammering to which it is subjected when on the ways, and the polarity of this magnetism will depend on the position of the ship while being built. Thus, a ship built in a north-and-south direction, that is, with her bow toward the north, will have red, or positive, polarity in her forward part, as shown in Fig. 5 (*c*), and blue, or negative, polarity in her stern. The effect of this will be that the magnetism in the ship's forward part will repel the north end of the compass needle, and similarly, the magnetism in the after part will repel the south end of the needle. If the vessel is built in a northeasterly direction, as in Fig. 5 (*d*), red, or positive, polarity will be found on the port bow, and blue, or negative, polarity on the starboard quarter. Again, should the vessel be built in the direction of the meridian, with her bow toward the south, her stern will possess red polarity and the bow blue polarity, as shown in Fig. 5 (*e*). When built in east-and-west direction, as in (*f*), the side of the ship that faces the north magnetic pole will acquire red, and the other side blue polarity. When built with her bow to southeast, as in (*g*), or to southwest, the red polarity will be on the port quarter in the former case and on the starboard quarter in the latter case.

It is well to bear in mind that all red magnetism in a ship acts together from a common center, or focus, and the same is the case with the blue magnetism, exactly as if the polarity of each kind of magnetism was concentrated at one

point. These centers of polarity are technically known as the **magnetic poles** of the ship.

15. Subpermanent Magnetism.—The magnetic condition acquired by a vessel, as explained in the preceding article, is called **subpermanent magnetism,** because it remains permanent or nearly so in all latitudes and for a long period after the ship is launched. It is the magnetism that is communicated from the earth to the ship while building, and is rendered subpermanent by the hammering and riveting to which the various parts of the ship are subjected in the course of construction. Immediately after launching, if the ship is laid in a direction different from that occupied when on the ways, which is often done, the magnetism may as a result be reduced to a certain extent, but as a rule the effect is slight. It takes some time, however, before the magnetism of a ship becomes stable in amount, usually a year or two, but at the end of this period its amount and polarity may be considered as permanent, irrespective of direction and geographic position. An iron or steel vessel may therefore be considered as a large magnet having red and blue polarity, which affects the compass needle in exactly the same manner as an ordinary magnet.

The amount of subpermanent magnetism appears to be influenced by the size of the vessel or the quantity of iron and steel used in her construction. Thus, in vessels of large tonnage, this magnetism is comparatively greater than in vessels of smaller size.

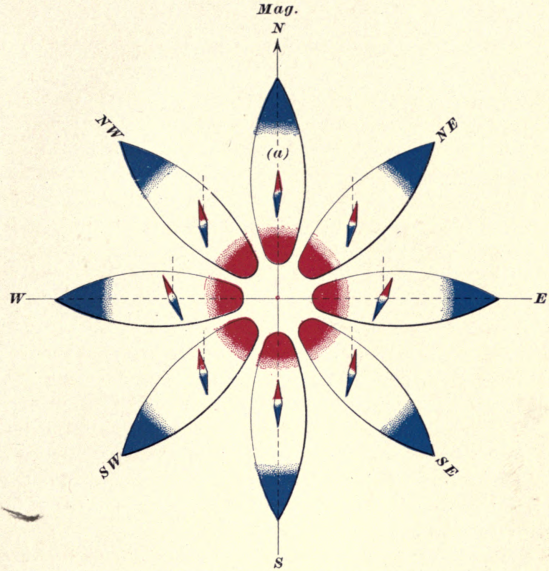
16. Retentive Magnetism.—The temporary magnetism that is sometimes communicated to an iron ship by induction from the earth when her head is kept in one and the same direction for some time is called **retentive magnetism.** Such magnetism may be acquired when, for instance, the ship is moored to a pier, or dock, for any length of time, or when steering a straight, continuous course for several days.

The effect of this magnetism is to increase or to decrease the subpermanent magnetism of the ship, according to its

polarity and the course on which the ship is running. Thus, assume a vessel to have blue polarity forward and red polarity in the stern. When such a vessel is heading for a considerable time on northerly courses, the tendency of the earth is to induce red polarity in the bow (see explanation, Art. **14**), and the effect of this will necessarily be to reduce, or weaken, the normal blue polarity. Again, when heading on southerly courses, the effect will be opposite; thus the blue and red polarity of the vessel will be augmented by the retentive magnetism. When running on westerly courses, for example, retentive magnetism will create a red pole on the starboard and a blue pole on the port side, and when the course is changed to north, this polarity will produce a westerly deviation; if changed to south, an easterly deviation. In transatlantic and transpacific steamers, the deviation thus produced by retentive magnetism may amount to a point or more, depending on how long the vessel has been headed east or west, as the case may be. Retentive magnetism frequently remains for days after the cause is removed.

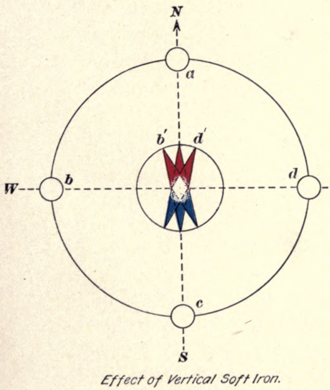
17. Transient induced magnetism is acquired by soft iron from any magnetic source, such as powerful magnets, electric currents, or the earth. As its name implies, this magnetism is transient and remains only so long as the inducing cause is present. It is evident that retentive magnetism and transient magnetism are identical, being acquired from the earth or other source by the soft iron of the ship.

18. Semicircular Deviation.—The effect of subpermanent magnetism on the compass may be illustrated as follows: Referring to Fig. 6 (x), suppose that a vessel has been built with her head magnetic south, and that as a consequence she has blue polarity in the bow and red polarity in the stern. Then, when heading in the direction of the magnetic meridian, as at (a), there will be no deflection of the needle because both magnetic poles of the ship are in line with the directive horizontal force controlling the needle. If the ship is swung with her head northwest, the north end



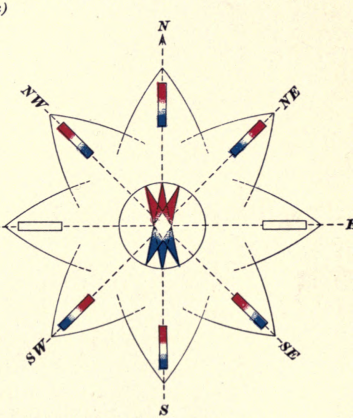
Effect of Subpermanent Magnetism

(x)



Effect of Vertical Soft Iron.

(y)



Effect of Horizontal Soft Iron.

(z)

FIG. 6

of the needle will be attracted by the blue polarity in the bow, and hence there will be a westerly deviation, which will gradually increase as her head is swung toward magnetic west, on which heading the error will attain its maximum, because the magnetic poles of the ship are then diametrically opposite the direction of the needle and are consequently exerting their greatest attraction. With the ship's head southwest, the deviation is still westerly, but decreases until heading south, when it disappears. As the ship's head is swung toward the east, an easterly deviation will occur, increasing gradually until its maximum is attained when heading east, as shown. In the northeast quadrant, the easterly deviation will gradually decrease as the ship is swung toward the north.

From the foregoing, it is evident that deviation due to subpermanent magnetism of hard iron has contrary names and attains its greatest value in opposite semicircles; for this reason, it is called **semicircular deviation.** For ships built with the bow north, the conditions are the reverse; then a maximum easterly error, or deviation, will occur when the ship is headed west, and a maximum westerly error when she is headed east.

In a similar manner all compasses are affected, and the nature of the error, or deviation, produced will depend on the magnetic polarity of the ship and, consequently, on the direction of the ship's keel when being built.

19. It should be remembered, however, that no matter where the blue and red magnetic poles of the ship are situated (they are, as a rule, diametrically opposite each other), they will, when both are on the line of the magnetic meridian, produce no error whatever; but at any intermediate position, they will produce an error that is maximum when the line connecting the two poles is perpendicular to the magnetic meridian.

20. The magnetism of the soft iron used in the construction of a ship remains to be considered. The soft iron entering into the make up of an iron vessel may be

conveniently divided into two classes: *vertical* and ***horizontal***. To the former belongs all iron running in a vertical direction, such as frames, stanchions, etc., and to the latter belongs all iron running horizontally, such as the keel, deck beams, etc. All vertical iron becomes magnetized by induction from the earth; this magnetism, however, is only transient, changing with the latitude. Thus, for a ship north of the magnetic equator the upper ends of all vertical soft iron will have blue polarity and the lower ends red polarity, while south of the magnetic equator the same vertical ironwork will possess red polarity in the upper and blue polarity in the lower ends, and will affect the compass accordingly. The amount of this magnetism, therefore, varies in the same proportion as the vertical magnetic force of the earth; in other words, it changes as the magnetic dip, being least at the magnetic equator and greatest at the magnetic poles.

21. How this magnetism affects the compass needle is illustrated in Fig. 6 (y), where the line NS represents the direction of the magnetic meridian. Assume that a bar of soft iron possessing blue polarity in its upper end is kept in a vertical position and moved around the compass with its upper end on about the same level as the suspended needle. When the bar is held in line with the needle, or on the magnetic meridian, as at a and c, there will be no deviation; but when held on a line perpendicular to the magnetic meridian, a maximum error b' or d' is produced that is westerly when the bar is to the west of the needle, as at b, and easterly when the bar is to the east of the needle, as at d. Thus, another semicircular error is produced by the vertical bar carried around the compass, and since all vertical soft iron in a ship is similarly influenced, it is evident that when the ship is turned about, this iron will affect the compass in exactly the same way as the subpermanent magnetism of the ship. Therefore, the semicircular deviation is not the effect of subpermanent magnetism alone, but is produced by the combined action of the subpermanent magnetism and the transient magnetism from the vertical soft iron of the ship.

22. Quadrantal Deviation.—Having considered the magnetic influence of vertical soft iron, the effect of horizontal soft iron will be investigated. By horizontal soft iron is meant not only that which is in the immediate neighborhood of the compass, but all horizontal iron that constitutes a part of the ship's hull, above, below, and in the plane of the compass card. In a preceding article it was pointed out that if a soft-iron bar is held in a horizontal north-and-south position, it will attain magnetism by induction from the earth, but will lose this magnetism when turned in an east-and-west position. The horizontal soft iron of a ship will receive and lose its magnetism in exactly the same way. The effect of this magnetism is shown in Fig. 6 (*z*), which represents the bow of a ship swung on different headings. When heading north in the direction of the magnetic meridian, no deviation will occur, because on that heading the magnetism of the horizontal bar will act in conjunction with the horizontal force of the earth. When swung to N W, a maximum error is produced by the blue pole of the bar attracting the red pole of the needle. As the bow is gradually swung toward the west the bar will lose its magnetism entirely, and when heading west, it will have no effect on the compass. Continuing the swinging process southwards, the magnetism of the bar will be reversed, as shown, and a maximum error, produced by the red pole of the bar acting at an angle on the blue pole of the needle, will occur when heading S W. This error will gradually decrease and disappear on south heading. A similar effect on the needle is manifested when the bow is swung from south toward east to north, as shown in the figure.

From the foregoing, it will be readily understood that when a ship is heading north or south and east or west, or when on any of the cardinal points, there will be no error, or deviation, from this kind of magnetism; but when heading on any of the other points, an error will be produced that is greatest when the ship is heading on the intercardinal, or quadrantal, points. For this reason, the deviation produced by the transient magnetism of horizontal soft iron is called the **quadrantal deviation.** This deviation has the same

name in opposite quadrants and contrary names in adjoining quadrants; thus, if it is easterly on N E headings, it will be easterly on S W and westerly on N W and S E headings. Generally, it is easterly in the N E and S W quadrants and westerly in the N W and S E quadrants, as shown in the figure. Quadrantal deviation is constant in all magnetic latitudes and is not appreciably affected by the lapse of time; for a particular heading, it is the same in all parts of the world.

As a rule, the amount of quadrantal deviation is not very great (rarely exceeding 3° or 4°, except in armored ships), because when the ship is heading on any of the quadrantal points, the magnetic forces of the fore-and-aft and the athwartship horizontal iron will, to a certain extent, neutralize each other, and thus minimize its effect on the compass.

23. Total Error of Compass.—From what has already been shown, it is evident that a compass placed on board of an iron or a steel vessel is subjected to various disturbances from the magnetism of the surrounding metal, and the errors thus produced are collectively known as deviation. At the same time, the magnetic needle is acted on by variation, and the combined effect of the two may properly be termed the **total error of the compass.**

24. Cautionary Remarks.—Variation and deviation must not be confounded with each other. Variation, being caused by the horizontal magnetic force of the earth, affects the compass alike on all courses, while deviation, being caused by the magnetism of the iron in the hull and fittings of the vessel itself, varies for different headings of the ship.

LOCAL ATTRACTION

25. Any disturbance, temporary or otherwise, due to iron, steel, dynamos, electric wiring, etc. in the immediate vicinity of the compass, and which is not included in the stationary equipment or the metal surrounding the compass, is called local attraction. In this expression is included also

the magnetic influence due to the locality in which the ship happens to be—for example, when in dock alongside of iron ships, cranes, pillars, or other iron structures, or when in close proximity to iron-bearing mountains or volcanic islands. The effect on the compass of a cargo including iron, such as iron ore, machinery, etc., may also be classed as local attraction.

Besides the disturbances already mentioned, there are at irregular intervals other mysterious movements of the magnetic needle, called *perturbations*, or *magnetic storms*, that are not fully understood. Auroral phenomena seem to affect the needle to a marked degree. In fact the greater the auroral display, the greater is the magnetic perturbation. Not only is the needle subject to unusual deflection during an aurora, but its movements seem to synchronize with the pulsations of the delicate auroral streamers in the sky.

The loss on June 28, 1904, of the steamship Norge, which struck the island of Rockall, 600 lives being lost, is attributed to a sudden deflection of the compass. According to her course, the Norge should have been 25 miles to the south of Rockall, and after a careful investigation it was impossible to account for the difference between the ship's real position and that of the reckoning without assuming a sudden and large change in the deviation of the compass. That some mysterious magnetic disturbances, possibly augmented by auroral phenomena, exist in the vicinity of Rockall is evident from reports made by other ships. The commander of the steamship Carl reports that on a transatlantic voyage he found by careful observations that in a very short time when in the neighborhood of Rockall "both compasses of the ship had acquired a hitherto unknown easterly deviation of 10° to 11°. A faint northern light was visible, and to this phenomenon was attributed the magnetic disturbance. Toward midnight the compasses were observed to return to their normal deviation." A second evidence is the report of the officer in command of the British ship Elixir, who states that a few days before the Norge struck, when in the vicinity of Rockall, "the compasses showed an increase of 9° in their normal deviation."

In addition to Rockall, well-authenticated observations show that the following places have a disturbing effect on the compass, and, therefore, when in their vicinity navigators will do well to be on their guard: Ascension; St. Helena; Cape St. Frances, Labrador; Solomon Islands; off the volcanic islands near Java; coast of Madagascar; coasts of Iceland; Isle de Los; west coast of Africa; and Cossack, North Australia.

COMPENSATION OF COMPASSES

GENERAL PRINCIPLES

26. The general principle of **compensating a compass** is to counteract the magnetic disturbance by means of magnets and soft iron placed in the immediate neighborhood of the compass, and in such positions as to cause a disturbance contrary to that caused by the iron of the ship. The magnetic needle will thus be left comparatively free. This may be illustrated as follows: Bearing in mind that the north-seeking end of the compass needle always possesses red polarity and that red polarity repels red and attracts blue, and vice versa, assume a needle to be deflected from magnetic north N to n, as in Fig. 7 (a). Then, in order to bring the needle back to its proper position N, or, what is the same thing, to counteract the effect of the surrounding iron and steel, compensating magnets may be placed in any of the positions shown at a suitable distance from the needle. It will be noticed, in each case, whether the magnets are used singly, in pairs, or in any other combination, that the whole operation of compensating is simply an application of the law of magnetic attraction and repulsion.

27. The methods of adjusting the different errors of the compass will now be considered. While the compensation of compasses, as a rule, is made, and should be made, by professional compass adjusters, it is nevertheless essential that a navigator should have a knowledge of

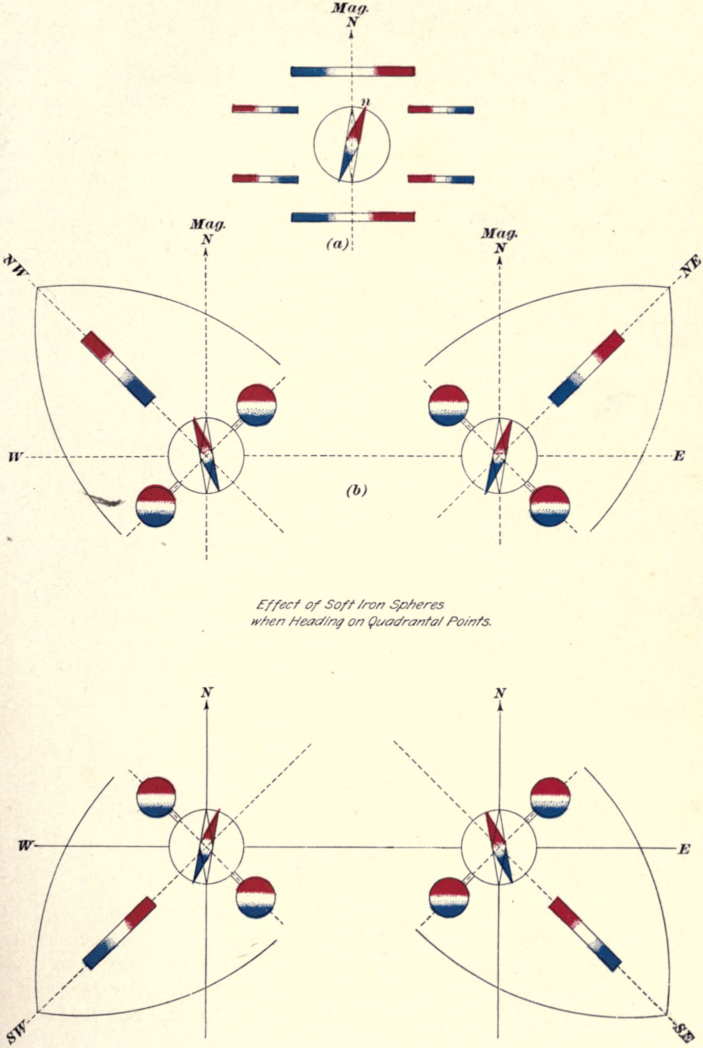

Mag. N
n
(a)
Mag. N
Mag. N
NW
NE
W
E
(b)
Effect of Soft Iron Spheres
when Heading on Quadrantal Points.
N
N
W
E
SW
SE

FIG. 7

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how compensation is performed, since occasions may arise where familiarity with this important subject will prove of great value.

28. Classification of Errors.—The two principal errors of a compass to compensate are the *semicircular deviation* and the *quadrantal deviation*. The semicircular error, as already explained, is the combined effect of the subpermanent magnetism of the ship and the induced magnetism of vertical iron; but, as a whole and for the purpose of compensation, it is convenient to divide this error into two parts, and consider each part as a separate force. The first part of this error, which is the effect of a magnetic force whose poles are in the fore-and-aft midship line, and which affects the compass needle when heading on easterly and westerly courses, is usually denoted by the letter B; while the second part, which represents a horizontal force whose poles are in a line at right angles to the fore-and-aft midship line, and which affects the needle when heading on northerly and southerly courses, is denoted by the letter C. The quadrantal deviation, resulting from transient induced magnetism in horizontal iron, and which attains its maximum value when the ship is heading on any of the quadrantal points, is denoted by D. These forces, B, C, and D are technically known as **coefficients.** When compensating a compass, it has been found good practice to correct the quadrantal deviation first, and then the two parts of the semicircular error.

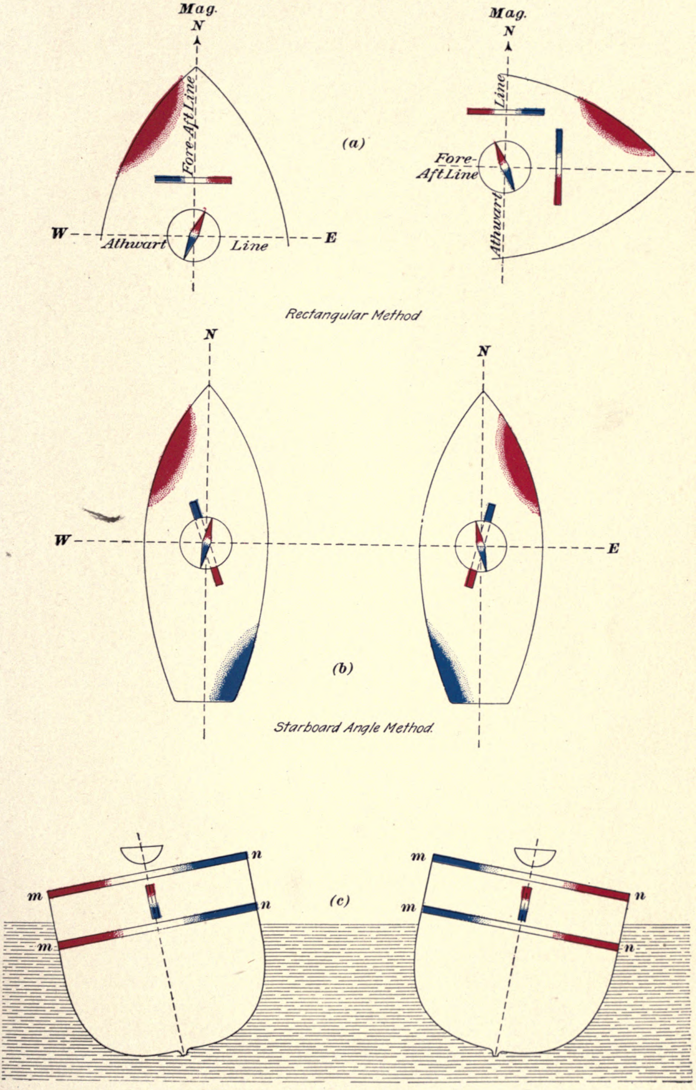
29. Compensating the Quadrantal Deviation. Since the quadrantal deviation, which is caused by the magnetism of horizontal soft iron, attains its maximum value on the quadrantal points, the ship is accordingly swung in the direction of one of these points—for example, N E, as shown in Fig. 7 (*b*); and since the error is caused by soft iron, it is necessary to compensate it by an arrangement of soft iron, usually hollow, soft-iron spheres or spheres filled with pieces of soft iron. These spheres are placed in brackets provided for that purpose in the plane of the compass

card so as to oppose the magnetism of horizontal iron. The error to be corrected being easterly in the N E and S W quadrants, and westerly in the N W and S E quadrants, in almost every ship, the spheres are placed athwartship on the same horizontal plane and at equal distances from the center of the compass, the distance being determined by trial, moving the spheres to and fro in their respective slits until the compass shows the correct quadrantal point on which the ship is headed.

The effect of the soft-iron spheres, as the ship is swung from one quadrantal point to the next, is shown in Fig. 7 (*b*). It will be noticed that the spheres always maintain their polarity unimpaired, no matter in which direction the ship is heading; in other words, the northern and southern hemispheres are always of the same magnetic character with their poles north and south, regardless of how the spheres are revolved. In this way, they counteract the magnetism of the horizontal soft iron, as shown in the illustration.

The quadrantal deviation being constant in all magnetic latitudes, its compensation remains practically constant everywhere, because the horizontal induction force of the spheres will vary in the same ratio as that of the horizontal iron that produces the quadrantal error.

30. To Compensate Coefficient *C*.—Swing the ship's head toward magnetic north according to a compass not influenced by the magnetism of the ship (for instance, by a compass on shore); or, better still, according to permanent marks on land, the bearing between which coincides with the magnetic meridian. If the compass in this position does not show exactly north, but is deflected to the east, as shown in Fig. 8 (*a*), center a magnet across the fore-and-aft line, with its red pole to the starboard, as shown. The distance must be determined by trial; begin by placing the magnet at some distance from the compass and gradually bring it nearer until the compass shows correct magnetic north, when the magnet is secured to the deck. If the needle had been deflected to the west, it is evident that the red end,



Mag.
N
Fore-Aft Line
(a)
W
Athwart
Line
E
Mag.
N
Line
Fore-
Aft Line
Athwart
E
Rectangular Method
N
N
W
E
(b)
Starboard Angle Method
n
m
n
m
(c)
m
n
m
n

FIG. 8

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or pole, of the magnet should have been placed to the port side. In the event of this error being large, the ship is swung toward magnetic south and a similar operation is performed on that heading, the magnet then being placed abaft the compass.

31. To Compensate Coefficient *B*.—The ship is swung either magnetic east or west. If swung to the east, and the compass north on that heading is deflected to the west, as in Fig. 8 (*a*), center a magnet across the athwartship line with its blue pole forward and at a distance from the compass sufficient to correct the error. The compass north being deflected to the east, the compensating magnet is reversed. A similar operation is then performed, if necessary, with the ship's head swung west. The foregoing applies to ships not equipped with a compensating binnacle. It becomes necessary then to have fore-and-aft and athwartship lines run out on the deck and intersect at a point vertically below the center of the compass to be compensated. The magnets are then placed perpendicular to and with their centers on these lines, as shown in Fig. 8 (*a*).

This method is known as the *rectangular method* of compensation. Another, though not so extensively used, is the *starboard-angle method*, in which one magnet compensates the entire errors of *B* and *C*; or, in other words the semicircular error. Briefly stated, the underlying principle of this method consists in locating the magnetic poles of the ship and then placing a compensating magnet on the line (beneath the compass) that connects the two poles, with the red pole of the magnet toward the blue pole of the ship and vice versa, as shown in Fig. 8 (*b*).

32. Storage of Magnets.—At the present time, and particularly in iron ships, compensating magnets are seldom, if ever, fastened to the deck, but are fitted instead to slide into horizontal fore-and-aft and athwartship receptacles within the binnacle. In most binnacles, the receptacles are arranged in such a manner as to be moved up or down, nearer to or farther from the compass, as may be required, and are then

secured by means of clamp screws that cannot be touched except by opening the door of the binnacle. In others, the movement of the magnet is controlled from the outside of the binnacle by means of a crank-key, thus enabling the adjuster to watch the compass while he is altering the position of the magnets and to move them the exact amount required. After the adjustment is completed, the crank-key is removed and the casing is locked, making it impossible for any one to tamper with the magnets. The principle of storing magnets within the binnacle is precisely the same as that of securing them to the deck, the magnets for both *B* and *C* being exactly parallel to the ship's deck or to the plane of the compass card when the ship is in an upright position.

33. When compensating magnets are fastened to the deck, their centers should be exactly on the chalk lines and they should be perpendicular to their respective lines. It matters not where the magnets are placed so long as this principle is observed. Also, if the magnets have been removed for any purpose, care should be taken that they are not reversed when replaced. Compensating magnets should be made of good steel, tempered and well magnetized; deck magnets, which may be from 12 inches to 24 inches in length and about $\frac{1}{4}$ inch by $1\frac{1}{2}$ inches in cross-section, should be covered with tallow and protected by a thin sheeting of metal. Before being used, their magnetism should be several months old.

34. The Flinders Bar.—As previously stated, the compensation of the quadrantal error is good for practically all latitudes. Such, however, is not the case with that part of the semicircular error caused by the induced magnetism of vertical iron. Since the amount of this magnetism depends on the magnetic dip, it is evident that the deviation resulting from it will vary as the magnetic dip. To distinguish this latter error from that produced by subpermanent magnetism and to apply it to a proper compensation is a difficult task, requiring skill, good judgment, and an intimate knowledge of the magnetic condition of the ship. The usual method of

correcting, or compensating, this error is by means of a vertical iron bar, called the **Flinders bar**, which is placed in a receptacle attached to the binnacle, either immediately before or abaft the compass. This bar, which receives its name from its inventor, Captain Flinders, of the British navy, is not a permanent magnet; it is made of soft iron and consequently receives its magnetism by induction from the earth. The object, therefore, in using the Flinders bar is to place it in such a position near the binnacle that the gradual change of its magnetism, produced by the change in latitude, will counterbalance the effect of the likewise varying magnetism of the vertical iron of the ship. This bar bears the same relation, therefore, to vertical soft iron as the spherical correctors do to the horizontal soft iron of the ship.

35. Heeling Error.—When, from any cause, the ship has a list to either side, a new error is created, which is generally known as the **heeling error.** The principal effect of this error may be explained as follows: When the ship heels over from pressure of wind, shifting of cargo, or unequal trimming of bunker, all horizontal iron, such as the deck beams, tends to assume an inclined position, and in doing so it receives magnetism by induction from the earth.

Thus, referring to Fig. 8 (c), for a ship in the northern hemisphere, the upper ends of the beams $m\,n$, whether the ship is heeling to port or to starboard, will acquire blue polarity and the lower ends red polarity. In the southern hemisphere, these conditions are reversed. As a consequence, the north end of the compass needle will be attracted by the upper ends of the beams in north magnetic latitudes and repelled by them in south magnetic latitudes. The presence of heeling error is usually indicated by undue vibrations of the card when the vessel rolls moderately. The more marked the vibrations, the greater is the force producing the heeling error.

As a general rule, the heeling error is greatest on northerly and southerly courses and least on easterly and westerly courses. The simplest method of compensating the heeling

error is to place a magnet vertically below the center of the compass bowl. Before compensating, the ship is swung to a north-and-south direction and heeled over at least 5°, for instance, to starboard. If in this position the compass north is deflected toward the uppermost, or windward, side (as is usually the case), the compensating magnet is placed with its red pole uppermost, and at a distance from the compass bowl that is determined by raising or lowering the magnet until the compass points correctly. Or, in a seaway, if the heeling error has not been attended to in port, when the vessel rolls from side to side on a north or south course, insert the magnet in its tube and raise it slowly from the bottom of the tube until the vibrations of the card almost disappear. In the very exceptional cases of the needle being deflected toward the lower, or leeward, side, the blue pole of the magnet is placed uppermost. The compensation for heeling error is good only for the latitude in which it is made, and, therefore, it must be renewed when the ship has changed her latitude considerably, usually for every change of 10° of latitude. At the magnetic equator, the error is at its minimum, but when entering the southern hemisphere it again increases in amount, although it is then of a different character. In southern magnetic latitudes, therefore, the vertical magnet will have to be reversed.

Professional adjusters usually correct the heeling error by means of a *dipping needle*, which measures the amount of vertical force at the locality where compensation is made. This does away with the rather cumbersome task of listing the ship. The heeling magnet is then raised or lowered in its tube until the vertical force is neutralized, which is indicated by the dipping needle being in a horizontal position.

36. Summary of Compensation.—The ordinary mechanical adjustment of a compass fitted in a compensating binnacle may be summed up briefly, as follows:

1. Place spheres on the side brackets and secure them temporarily by means of the nuts underneath. Insert the heeling magnet in its tube, red end up in north magnetic latitude.

2. Head the vessel magnetic N E (or S E, S W, N W), and if deviation is shown, move the spheres on the side brackets in or out until the compass is pointing correct N E (or S E, S W, N W). If the spheres overcorrect when placed at the ends of the brackets, remove them and use smaller spheres. If the spheres undercorrect when at the inner limits of the slits, use larger spheres.

3. Head the vessel magnetic north. If the compass shows easterly deviation, insert one or more magnets in the athwartship receptacle, red ends to starboard. Move the magnet tray up or down until the compass points north. If the compass shows westerly deviation, insert athwartship magnets with red ends to port and proceed as before.

Or, head the vessel magnetic south. Correct easterly deviation by placing red ends of athwartship magnets to port; for westerly deviation, to starboard.

4. Next head the vessel magnetic east. If easterly deviation is shown, insert fore-and-aft magnets with red ends forward; if westerly, place blue ends forward. Raise or lower the magnet tray until the compass shows east.

Or, head the vessel magnetic west. Insert fore-and-aft magnets with blue ends forward if deviation is easterly; with red ends forward if deviation is westerly.

5. Finally, head vessel north or south, and correct any heeling error the compass may have by giving the vessel a test of at least 5° on either side; raise the magnet in the tube until the error disappears, but do not overcompensate.

If inconvenient to heel ship in port, perform the compensation at sea when rolling moderately from side to side on a north or south course.

If, when raising the heeling magnet from the bottom of the tube, the error is aggravated, the wrong end of the magnet is uppermost.

In correcting the semicircular deviation, it is well to bear in mind the following: Divide the necessary compensating magnets equally on each side of the receptacle, so that the magnets are of equal numbers and at equal distance from the vertical axis of the binnacle.

Use a greater number of magnets at a distance from the compass in preference to a smaller number nearer the compass.

37. The foregoing remarks on compensation are general, and while the operations may appear easy of execution, they require a certain amount of skill and experience to meet all conditions that may arise. For this reason it is advisable always to employ a professional compass adjuster, the expense being insignificant when compared with the importance of the subject.

It is well to bear in mind that the main object of compensating a compass is not to entirely remove the deviation, which cannot be done, but to bring it within manageable limits; or, in other words, to reduce it to a minimum. After being compensated, a compass is more uniformly sensitive, whereas an uncompensated compass with large deviations has a tendency to alternate sluggishness and sensitiveness with certain directions of the ship's head.

COMPENSATING BINNACLES

38. Binnacles in which provision is made to receive magnets for compensating purposes are called **compensating binnacles.** Many different styles of more or less elaborate construction are now on the market. In Fig. 9 is shown the interior arrangements of the **Ritchie compensating binnacle.** The compass chamber, which is about 11 inches in diameter, is supported on three gun-metal legs, between which is affixed a drum $7\frac{1}{2}$ inches in diameter. In this drum is the adjusting apparatus, which consists of two magnet carriers *c* and *d*, both sliding on a center tube *a b*. Inside the tube hangs the magnet for adjusting the heeling error, which can be raised or lowered by the chain and clamped by the screw at *e*. The magnet carriers have four receptacles, each for the insertion of as many magnets, two on either side of the center tube, and each carrier may be fastened at any desired height by racks and side clamps. The compensating magnets, which are cylindrical in form,

fit snugly in the receptacles and are prevented from sliding out of their places by a spring, which locks the entrance to each receptacle. The ends of the magnets are painted red and blue, according to their polarity. The upper carrier *c* is generally used for the fore-and-aft magnets, and the lower one *d* for the athwartship magnets; or, if necessary, the upper one may be placed athwartship and the lower one fore-and-aft. Each carrier has a perpendicular range of about 15 inches, and is arranged in such a manner that it can be rotated horizontally about the center tube. The drum of the binnacle is closed by a long door with locks at top and bottom.

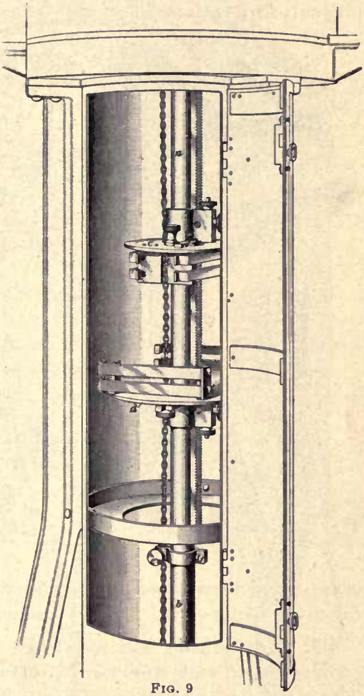


FIG. 9

39. A type of compensating binnacle manufactured by Bliss & Co., New York, is shown in Fig. 10. This binnacle is rectangular in form and is made of hardwood. Within the box are two horizontal partitions, the object of which is to strengthen the box and to support the center tube *a b* containing the magnet *c d* for the adjustment of the heeling error. On three sides of the binnacle are shallow closets *f g*, each of which is fitted with receptacles for the fore-and-aft

and athwartship magnets. The magnets *m*, two of which are shown in the figure, are made of hardened steel, and are about 8 inches long and ½ inch square, the ends being painted red and blue to indicate the polarity. After compensation is made, the doors are closed and locked, thus securing the magnets in position. The iron spheres *q*, *q* are attached to

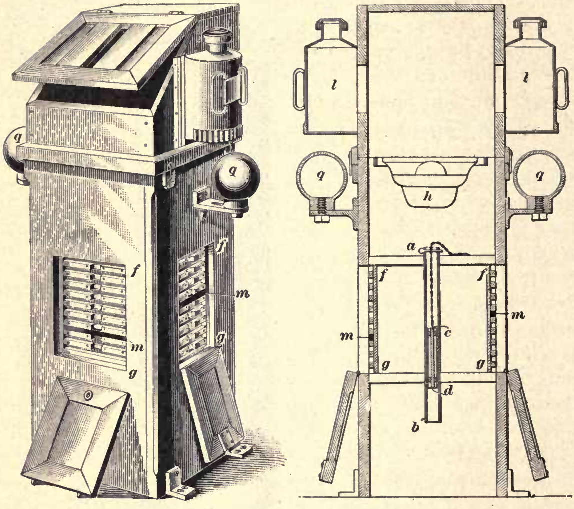


FIG. 10

brass brackets and are held in position by bolts, as shown; *l* and *l* are lamps, and *h* is the compass bowl.

40. Adjustable Magnet Rings.—In Fig. 11 is shown a compass fitted with **Morrison's adjustable magnet ring.** This device is intended as a substitute for the system of storing magnets beneath the compass. The tubes *a*, *b*, and *c* are receptacles for magnets to compensate the semicircular deviation, and *d* is a counterweight to keep the bowl level. The tubes can be raised or lowered by means of the thumbscrews attached to each. The forms of magnets used are

both straight and bent, being about the thickness of a knitting needle. By this arrangement, the magnets are always at the

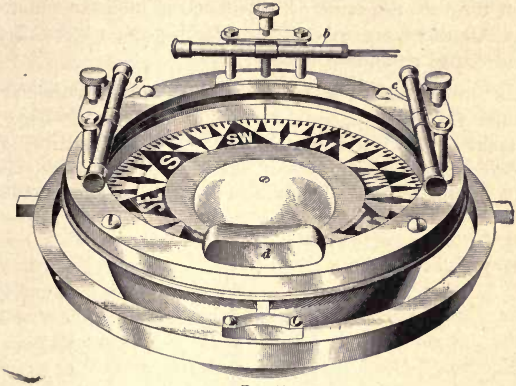
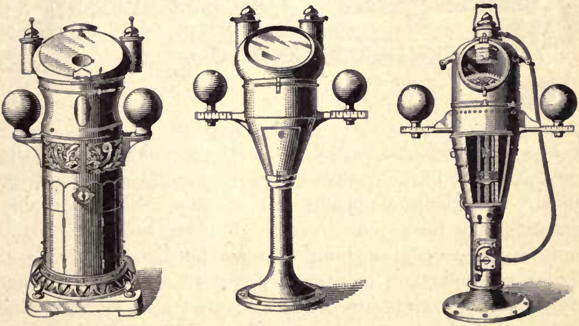


FIG. 11

same distance and parallel with the plane of the compass card, no matter what list the ship may have.

41. Standard Types of Binnacles.—Three of the best



(a) (b) (c)

FIG. 12

known types of standard compensating binnacles are shown in Fig. 12, (*a*) being the Lord Kelvin, (*b*) the Ritchie, and

(*c*) the Keuffel & Esser. Each type is provided with the latest innovation for moving and storing magnets, and altogether they are the acme of excellence in binnacle construction. All three are extensively used in the navies of the United States and Great Britain.

42. Lord Kelvin's Deflector.—A unique device known as **Lord Kelvin's patent deflector** is shown in Fig. 13. By means of this instrument it is possible to adjust the compass errors without the use of bearings of any kind. This device is thus of great usefulness in foggy weather. When

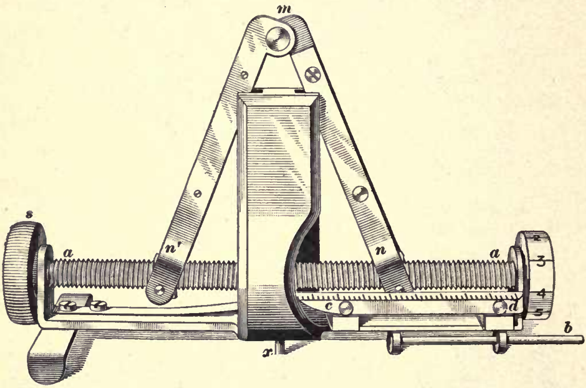


FIG. 13

in use, the deflector rests on top of the compass, with its central foot x in the conical hollow of the glass cover of the bowl. The slanting uprights $m\,n$ and $m\,n'$ are magnets that are hinged at the top and capable of being separated at the bottom by means of the long right- and left-handed screw a, a. These magnets are, of course, polarized at both ends, but only the lower ends are considered, that at n being a blue pole and that at n' a red pole. The rod b projecting from the right-hand end is a pointer for indicating the magnetic axis of the instrument. By turning the screw s, the lower ends of the magnets may be moved farther apart or

brought closer together, thus increasing or decreasing the magnetic effect of the instrument on the compass. The scale at *c d* is read by the index at the bottom of magnet *m n*.

43. This deflector is designed on the principle that if the directive force of the compass needles is the same on four different courses, two of which are opposite to the other two, then the compass should have no error on any heading; it is to assist in bringing about this magnetic condition that this instrument is used. The directive force influencing a compass needle is indicated by its tendency to remain in the direction that it assumes, and the amount of this directive force may be measured by finding the magnetic force required to deflect the needle so as to make a certain angle with its original direction.

The scale of the deflector is used to determine the amount of directive force by setting the magnets with the thumb-screw *s*, so that the compass needle will be deflected between 85° and 90° from its original position. This operation is usually performed with the ship heading north, south, east, and west, and the fore-and-aft and athwartship magnets and soft-iron correctors are manipulated so that the scale reading of the deflector on each of the headings named will be the same. When this is accomplished, the directive force of the compass on these headings has been equalized, and the amount of deviation remaining (which should be less than 2°) will depend on the accuracy with which the work has been performed. This is, in short, the object and manner of operating the instrument. Instructions for manipulating the deflector accompany each instrument.

TABULATION OF ERRORS

FINDING AND TABULATING THE DEVIATION

44. After having compensated a compass, there still remains a certain amount of deviation that must be known and tabulated; and since the amount of this deviation varies for different headings, it becomes necessary to determine the exact error on each point. To this end, the ship is swung with her head on each successive point, and the deviations thus found, according to methods that will be explained, are then tabulated and kept for reference in shaping courses.

45. The principal methods used for ascertaining this deviation are: (1) by the bearings of a distant object, the correct magnetic bearing of which is known; (2) by reciprocal bearings. These methods consist mainly of a process known as *swinging the ship* and are carried on when the ship is in a port, or harbor. At sea, other methods are used, which will be described in another Section.

46. Preliminary Precautions.—When about to determine the deviation, the ship should, in all respects, be ready for sea; stores and weights on board properly disposed; boats hoisted, and davits swung into proper position; anchors weighed; if a steamer, steam should be up. Any iron and steel, such as chains, bolts, tools, etc., should be removed and not allowed to remain near the compass. The ship is then moored to a buoy (preferably to a mooring made of timbers driven vertically in the bottom of the harbor and lashed together), as shown in Fig. 14, and swung around by means of a tug. In swinging the ship, the tug T used for this purpose should be given enough hawser so as not to influence the compass by the iron in its hull or its smokestack. Under no circumstances should the tug be allowed to lie alongside the ship while the swinging is in progress.

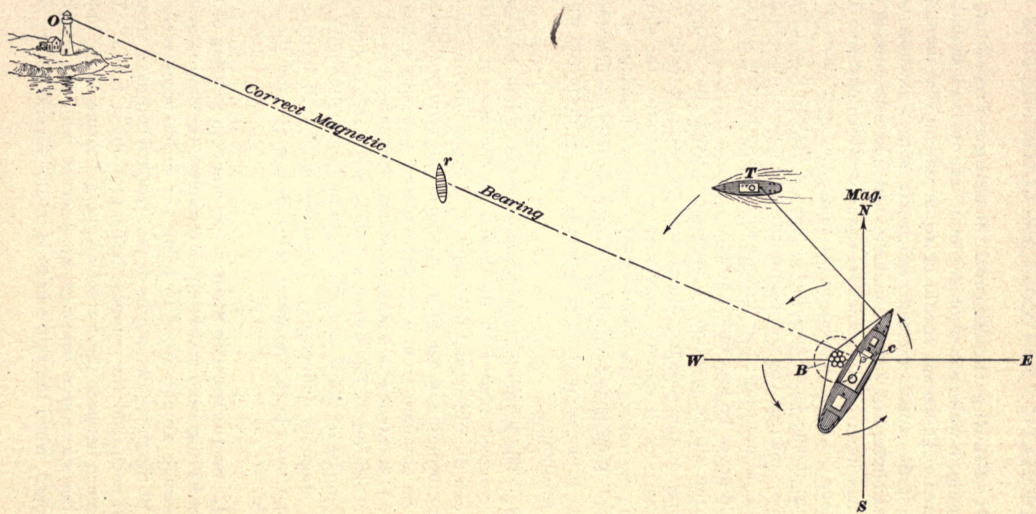


FIG. 14

47. Finding the Magnetic Bearing.—Preparatory to swinging, a well-defined object, or mark, on land should be selected. This mark should be so far distant that the diameter of the space through which the ship is swung (see dotted circle, Fig. 14) will make no sensible difference in the real bearing from the object *O* to the central point *B*. The distance *c O* must depend on the range the ship takes when swung; if she is at anchor in a bay, or harbor, 5 to 7 miles will be sufficient (by some authorities 10 miles); if swung in a dock, at mooring, 1 or 2 miles will suffice, but the distance should never be less than a mile.

If the ship's position is accurately fixed, the magnetic bearing of the selected object may be taken directly from the chart; or, it may be conveniently found after the ship is swung by taking the mean of all compass bearings of the object. Regularly established ranges, such as are now found in the principal ports, are, however, to be preferred whenever available.

48. The magnetic bearing may also be found by means of a compass placed on the beach on the line of bearing between the object and the ship; or, if the conditions do not permit bringing a compass on shore, the required bearing may be obtained from a compass placed in a ship's boat *r*, Fig. 14, located at a suitable distance from the ship on the line of bearing. The use of a boat is preferable to bringing the compass on shore, where it might be subjected to unknown local magnetic influences. It is evident that if bearing is taken from a boat, the water must be smooth and the boat steady.

49. Swinging the Ship.—The magnetic bearing of the object having been accurately determined, the ship is gradually swung around so as to bring her head successively on each of the thirty-two points of the standard compass, steadying at each, and the compass bearing of object taken. The difference between the correct magnetic bearing of the object and the successive bearings, as observed with the compass on board, when the ship's head is on the several points, will

show the error on each of these points; or, in other words, the deviation of the standard compass, according to the direction in which the ship's head was placed. The bearings are conveniently tabulated in a form similar to that shown in Table I, the second column containing compass bearings and the third the resulting deviations.

50. In the case shown, the correct magnetic bearing, as determined beforehand, was N 63° W, this bearing being taken from a chart. However, had this bearing not been known, it could have been readily obtained after swinging by taking the mean of all compass bearings, as shown at the foot of Table I. By adding the degrees of all bearings, reckoned from a common point of reference, and dividing the sum

TABLE I

CORRECT MAGNETIC BEARING OF DISTANT OBJECT FROM SHIP = N 63° W; DISTANCE = 6 MILES

Ship's Head by the Standard Compass	Bearing of Distant Object by Standard Compass	Deviation of Standard Compass	Ship's Head by the Standard Compass	Bearing of Distant Object by Standard Compass	Deviation of Standard Compass
North	N 59.8° W	3.2° W	South	N 66.2° W	3.2° E
N by E	N 65.6° W	2.6° E	S by W	N 63.1° W	0.1° E
N N E	N 71.2° W	3.2° E	S S W	N 60.0° W	3.0° W
N E by N	N 76.2° W	13.2° E	S W by S	N 56.5° W	6.5° W
N E	N 79.8° W	16.8° E	S W	N 53.3° W	9.7° W
N E by E	N 82.5° W	19.5° E	S W by W	N 50.0° W	13.0° W
E N E	N 83.5° W	20.5° E	W S W	N 46.8° W	16.2° W
E by N	N 84.1° W	21.1° E	W by S	N 43.8° W	19.2° W
East	N 83.3° W	20.3° E	West	N 41.8° W	21.2° W
E by S	N 82.3° W	19.3° E	W by N	N 39.7° W	23.3° W
E S E	N 81.1° W	18.1° E	W N W	N 39.0° W	24.0° W
S E by E	N 79.5° W	16.5° E	N W by W	N 39.4° W	23.6° W
S E	N 77.7° W	14.7° E	N W	N 41.0° W	22.0° W
S E by S	N 75.1° W	12.1° E	N W by N	N 44.0° W	19.0° W
S S E	N 72.7° W	9.7° E	N N W	N 48.2° W	14.8° W
S by E	N 69.0° W	6.0° E	N by W	N 53.8° W	9.2° W

Sum = 1,223.4 (for 16 pts.) Sum = 786.6 (for 16 pts.)

Correct magnetic bearing $= \frac{1{,}223.4 + 786.6}{32} =$ N 62.9° W, or N 63° W

thus obtained by the number of bearings taken, the correct magnetic bearing of the object will be found. It will be noticed in this case that the bearing so obtained agrees to within $\frac{1}{10}°$ of that found by direct plotting from chart.

51. Naming the Deviation.—To correctly name the deviations is an important matter, and in doing so the following rule should be remembered:

Rule.—*Deviation is* **east** *when the north point of compass is deflected to the* **east,** *or* **right,** *side of the magnetic north; and* **west** *when deflected to the* **west,** *or* **left,** *side of the magnetic north; or, if the correct magnetic bearing lies to the* **right** *of the compass bearing, the deviation is* **easterly;** *if to the* **left,** *the deviation is* **westerly.**

A first glance at this rule may convey the impression that the latter part of the rule contradicts the first part and vice versa. That such, however, is not the case is evident from what follows. In Fig. 15, let the outer circle and lines represent magnetic directions; the inner circle, compass directions. According to the first part of the rule just given, the deviation is westerly because compass north is deflected to west of magnetic north. Assume, for purpose of illustration, that the correct magnetic bearing of the object X is N 45° E, but by the compass it bears N 57° E, as shown. To find the deviation, disregard the magnetic card and lay out on the compass card the correct magnetic bearing N 45° E represented by the line $o\,a$. The correct magnetic bearing now lies to the left of the compass bearing and, therefore,

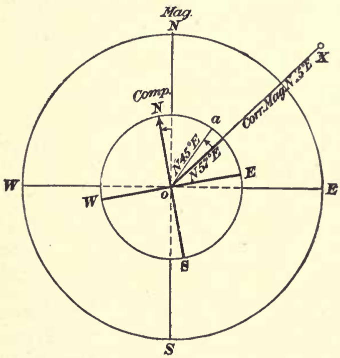


FIG. 15

according to the latter part of the rule, the deviation is west by an amount equal to $57° - 45° = 12°$ W; and by the first part of the rule the deviation is westerly also. The rule, therefore, as a whole, is evidently correct.

52. Application of Rule for Naming the Deviation. Referring to Table I, when the ship is heading, for instance, N E, as in Fig. 16, the selected object bears N 79.8° W by compass; but its correct magnetic bearing is N 63° W; according to the preceding rule, the deviation for this heading (N E) is therefore $79.8° - 63° = 16.8°$ E, because correct magnetic bearing lies to the right of compass bear-

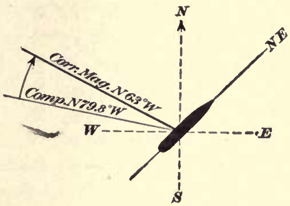


FIG. 16

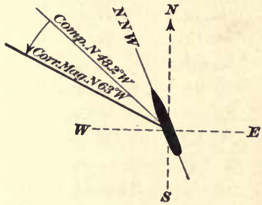


FIG. 17

ing, as shown in the figure. Again, when the ship is headed N N W, as in Fig. 17, the same object bears N 48.2° W by compass; the deviation on this heading N N W is consequently $63° - 48.2° = 14.8°$ W, because correct magnetic bearing falls to the left of compass bearing, as shown. In practice, the finding of deviation to the nearest degree or half degree is quite sufficient.

53. When deviations are small, as is usually the case in ships where compasses are carefully adjusted, it is sufficient to determine the deviation for every other point, or for the eight principal points, and then find the deviation for intermediate points by means of the various deviation diagrams in use, as will be explained later.

54. In cases where bearings have different names or do not lie in the same quadrant, as shown in Table II, it is advisable always to refer them to some convenient cardinal

point, as shown. This will prevent any mistakes from being made in finding the mean, or correct, magnetic bearing of the object. In Table I, it will be observed that all bearings have the same name; that is, they all lie in the N W quadrant.

TABLE II

Ship's Head by Standard Compass	Bearing of Distant Object by Standard Compass	Bearings Referred to East Point	Deviation of Standard Compass
North	N 41° E	E 49° N	36° 15′ W
N E	N 30° E	E 60° N	25° 15′ W
East	N 11° E	E 79° N	6° 15′ W
S E	N 12° W	E 102° N	16° 45′ E
South	N 33° W	E 123° N	37° 45′ E
S W	N 27° W	E 117° N	31° 45′ E
West	North	E 90° N	4° 45′ E
N W	N 28° E	E 62° N	23° 15′ W

Sum of bearings = 682°

Mean = $\frac{682}{8}$ = E 85.25° N = E 85° 15′ N

Correct magnetic bearing = N 4° 45′ E

55. Reciprocal Bearings.—When no suitable object is in sight by which a range may be established, the deviation may be found by what is known as **simultaneous reciprocal bearings.** This method consists of a compass being brought on shore and placed on a tripod in a carefully selected spot, where it will be free from the magnetic influence of any iron and where its location can be distinctly seen from the standard compass on board. As the ship is swung around, with her head successively on each of the thirty-two points of the standard compass, simultaneous observations, or bearings, are taken of each compass from the other, according to some prearranged signal. The bearings should be strictly simultaneous, and, in order to guard against any mistake regarding the exact instant at which bearings are taken, both observers should note the time of each observation by watches previously compared.

56. To obtain the deviation resulting from observations by this method, the bearings taken by the shore compass must be *reversed* and considered as correct magnetic. They are then compared with bearings taken by the ship compass and the deviation is found as in the previous method. The bearings may be conveniently tabulated as shown in Table III.

TABLE III

Time	Ship's Head by Standard Compass	Simultaneous Bearings		Deviation of Standard Compass
		By the Standard Compass	By the Shore Compass (Reversed)	
7h 56m	North	S 25.3° E	S 30.8° E	5.5° W
7h 59m	N by E	S 30.9° E	S 32.5° E	1.6° W
8h 3m	N N E	S 35.2° E	S 34.3° E	0.9° E
8h 5m	N E by N	S 38.7° E	S 35.4° E	3.3° E
8h 8m	N E	S 40.8° E	S 36.3° E	4.5° E

And so on for all points of the compass. It is advisable to reverse bearings by the shore compass before they are entered in the table, as shown.

57. Illustration of Reciprocal Bearings.—Referring to Fig. 18, suppose that when the vessel is heading N W by N the shore compass bears E N E and that the bearing of the ship by the shore compass, taken at the same time, is W by S. Reversed W by S is E by N, which is the correct magnetic bearing. The difference between this and the compass bearing is one point. Hence, the deviation for the heading N W by N is one point, or 11° 15′ E, because the magnetic bearing falls to right of the compass bearing, as shown in the figure.

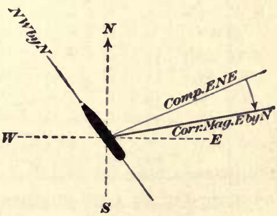


FIG. 18

58. Remarks.—It should be borne in mind that the deviations obtained by either of these methods belong only to the compass by which the observations were made, and cannot be applied to other compasses in the ship. Furthermore, the deviation is not applicable to that compass if removed and placed in some other part of the ship. It is evident that, while swinging a ship, the deviation of all the

TABLE IV

DEVIATION TABLE FOR STANDARD COMPASS
S. S. MATAPAN

1	2	3	1	2	3
Ship's Head (Course) by Standard Compass	Deviation	Correct Magnetic Course Made Good	Ship's Head (Course) by Standard Compass	Deviation	Correct Magnetic Course Made Good
North	14.0° W	N 14.0° W	North	14.0° W	N 14.0° W
N by W	16.5° W	N 28.0° W	N by E	11.0° W	North
N N W	18.5° W	N 41.0° W	N N E	7.5° W	N 15.0° E
N W by N	19.0° W	N 53.0° W	N E by N	4.0° W	N 30.0° E
N W	19.0° W	N 64.0° W	N E	1.0° W	N 44.0° E
N W by W	17.0° W	N 73.0° W	N E by E	1.5° E	N 58.0° E
W N W	14.5° W	N 82.0° W	E N E	4.0° E	N 71.5° E
W by N	11.0° W	West	E by N	5.5° E	N 84.0° E
West	7.0° W	S 83.0° W	East	7.0° E	S 83.0° E
W by S	2.5° W	S 76.0° W	E by S	8.0° E	S 71.0° E
W S W	1.5° E	S 69.0° W	E S E	9.0° E	S 58.5° E
S W by W	5.5° E	S 62.0° W	S E by E	10.0° E	S 46.0° E
S W	9.0° E	S 54.0° W	S E	11.0° E	S 34.0° E
S W by S	11.5° E	S 45.0° W	S E by S	12.0° E	S 22.0° E
S S W	13.0° E	S 35.5° W	S S E	13.0° E	S 9.5° E
S by W	14.0° E	S 25.0° W	S by E	13.5° E	S 2.0° W
South	14.0° E	S 14.0° W	South	14.0° E	S 14.0° W

compasses may be found if the direction of the ship's head, as indicated by each compass, is noted and tabulated.

59. It is usual, however, to determine the deviation only for the compass selected as the standard, and then, when about to set a course, to do so by that compass. For instance, if it is desired to steer N E, the ship's head is put

in a N E direction according to the standard compass, and if the steering compass does not agree, the course that it indicates when the standard shows correct N E should be kept.

60. Deviation Tables.—For purpose of navigation, the deviation tables need not necessarily contain or give the

DEVIATION CARD

For *Bridge* Compass of *Steam Yacht "Peregrine" Steel Vessel*

Adjusted at *Boston, Mass.*

June 11, 19*06.*

A. W. Rich. COMMANDER

Ship's Head by Compass	DEVIATION	Ship's Head by Compass	DEVIATION
NORTH	½° W	SOUTH	0
N by E		S by W	
N N E	1° W	S S W	½° E
N E by N		S W by S	
N E	½° W	S W	½° E
N E by E		S W by W	
E N E	0	W S W	1° E
E by N		W by S	
EAST	0	WEST	0
E by S		W by N	
E S E	1° W	W N W	½° W
S E by E		N W by W	
S E	0	N W	0
S E by S		N W by N	
S S E	½° E	N N W	0
S by E		N by W	

JOHN L. BLISS, Adjuster
128 Front Street, New York

FIG. 19

deviation closer than the nearest half degree, fractions below and above .5° being rejected. The form of deviation table shown in Table IV is recommended. Column 3 of this

table is obtained by applying the deviation in column 2 to the points of the compass in column 1; hence, column 1 indicates what course should be steered by the standard compass in order to make good any required magnetic course. Thus, if it is required to steer N N E $\frac{3}{4}$ E correct magnetic, look in column 3 and find N 30° E (which agrees very nearly with N N E $\frac{3}{4}$ E); the corresponding time in column 1 will then indicate the course to be steered by the compass, which in this case is N E by N.

61. A very simple form of deviation table used by a professional compass-adjusting firm of New York is shown in Fig. 19. This table also gives a clear illustration of how close a compass may be compensated. The original maximum deviation of this compass was about 30°, but by a judicious application of magnets, soft-iron spheres, and a large Flinders bar, it was reduced to the values shown.

62. Steering Card.—Another form of deviation table or "steering card," used by the firm just referred to is shown in Fig. 20. The compass, the deviations of which are recorded on this card, was located in the pilot house of the steamer Grand Republic, a wooden vessel, but with considerable iron and steel in boilers and machinery. The compass itself was a common liquid box compass, the card of which was graduated to points only. No quadrantal correctors were used in the compensation, and the magnets were fastened to shelves and woodwork surrounding the compass. It will be noticed that this card is different in arrangement from that shown in Fig. 19. To steer a given magnetic course N N E, for instance, the ship must be headed by compass $\frac{1}{8}$ point to the north of N N E, or N by E $\frac{7}{8}$ E; to steer S S E, magnetic, the course by compass must be S S E $\frac{1}{8}$ S, or S by E $\frac{7}{8}$ E; and so on. Steering cards like the one shown in Fig. 20 are much used in harbor and river steamers, where, as is generally the case, the only compass is located in the pilot house, and the ship has no equipment whatever for ascertaining the compass error by astronomical observations.

63. Remarks on Deviation Tables.—The accuracy of deviation tables should be tested whenever practicable or whenever there is reason to believe a change in the magnetic condition of the ship has taken place. After coming

STEERING CARD

For *Pilot House* Compass of
Str. "Grand Republic" Excursion Steamer
Adjusted at *L. I. Sound, off Glen Cove.*
June 26, 19*06.* *J. A. Pease,* COMMANDER

To Make Correct Magnetic	STEER	To Make Correct Magnetic	STEER
NORTH	*0*	SOUTH	*0*
N by E		S by W	
N N E	*1/8 N*	S S W	*0*
N E by N		S W by S	
N E	*1/8 N*	S W	*0*
N E by E		S W by W	
E N E	*1/8 N*	W S W	*0*
E by N		W by S	
EAST	*0*	WEST	*0*
E by S		W by N	
E S E	*1/8 S*	W N W	*0*
S E by E		N W by W	
S E	*1/8 S*	N W	*0*
S E by S		N W by N	
S S E	*1/8 S*	N N W	*0*
S by E		N by W	

JOHN L. BLISS, Adjuster
128 Front Street, New York

FIG. 20

out of dry dock, after considerable alterations in the fittings of the vessel, or after taking in or unloading some cargo of a magnetic character, such as machinery, iron ore, etc., a new deviation table should be made in case the tabulated

values do not conform to actual conditions as observed by azimuth instruments, to be described later. Too much reliance should not be placed on deviation tables compiled during the first 2 or 3 years after the ship is launched, because an iron or steel ship may change and lose much of her magnetism during that period.

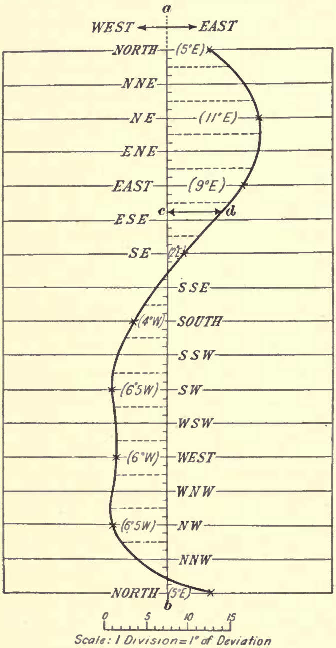


Fig. 21

64. Deviation Diagrams.—A **deviation diagram** is a graphic representation of the tabulated values of the deviation of a compass. There are several diagrams of this kind in use, one of the simplest forms being shown in Fig. 21. To construct a diagram similar to the one shown and to draw the curve representing the deviation for each point, take a sheet of ruled paper and mark the points of the compass on thirty-two consecutive lines. From a vertical line *a b* drawn through the center of the paper, lay off, on the horizontal lines, the deviations for the eight principal points to any desired scale, easterly deviations to the right and westerly to the left, and draw a flowing curve through the points thus obtained. The distance from the vertical line to the curve, according to the given scale, will give the deviation with considerable accuracy for any desired direction of the ship's head.

For instance, if it is required to plot a curve by the deviations given in Table V, the result will be as represented in Fig. 21. The deviation on any point can now be found by

TABLE V

Ship's Head	Deviation	Ship's Head	Deviation
North	5° E	South	4.0° W
N E	11° E	S W	6.5° W
East	9° E	West	6.0° W
S E	2° E	N W	6.5° W
South	4° W	North	5.0° E

measuring the perpendicular distance between the curve and the desired point on the vertical line. Thus, for example, to find the deviation for E S E ½ E, proceed as follows: From *c*, Fig. 21, representing the compass point E S E ½ E, draw a line perpendicular to *a b*; then, with the scale, measure the distance *c d*, which, in this case, is equivalent to 6½ divisions, or 6.5°. Since the curve at that point lies to the east, the deviation is easterly; hence, the deviation for E S E ½ E is 6.5° E.

In a like manner, the deviation for any intermediate point, quarter or half point, may be found directly from the diagram. It will be noticed in this diagram that there is no deviation on S E by S and N by W headings.

65. Napier's Deviation Diagram.—For larger deviations and greater accuracy, the diagram shown in Fig. 22, called **Napier's diagram,** after its inventor, is commonly used. The construction of this diagram is as follows: A vertical line is drawn near the center of a sheet of paper of suitable size, and on this line, at equal distances, are marked the thirty-two points of the compass, beginning with north at the top.* These points are named in the regular order in

*In Fig. 22, the diagram is divided into two parts, the top of the right-hand part being a continuation of the lower end of the left-hand part.

NAPIER'S DEVIATION DIAGRAM.

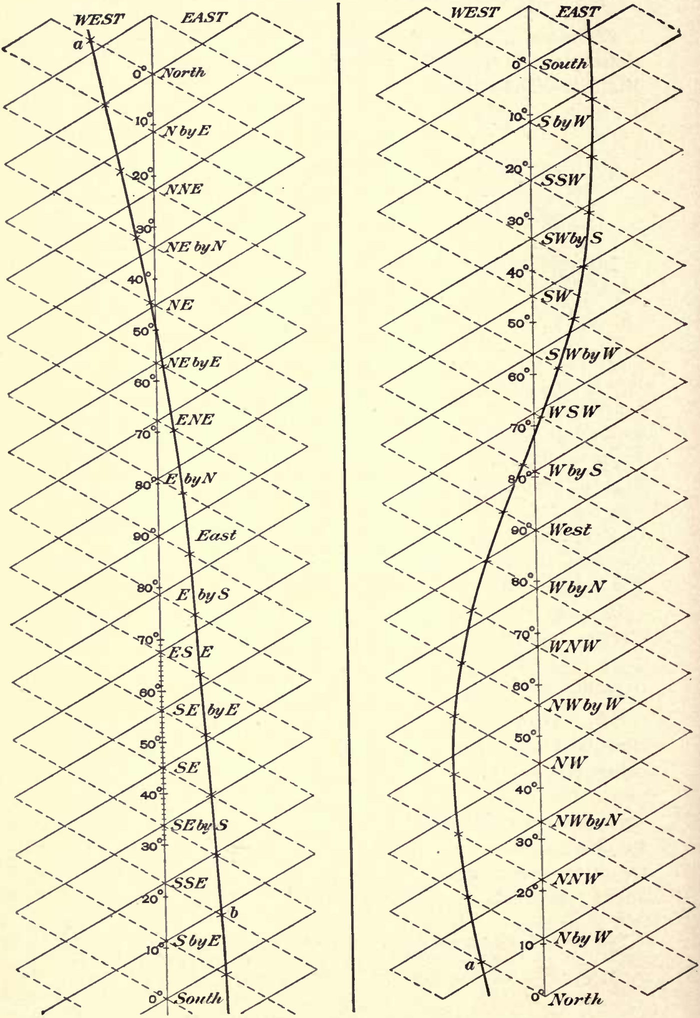


FIG. 22

which they occur on the compass card when read from north around toward the right. The vertical line is also divided into 360 equal parts, representing degrees, and are marked similarly to those on a compass card. Through each of the thirty-two points, two oblique lines are drawn as follows: a *continuous line*, making an angle of 60° toward the *right* of the central vertical line, also a *dotted line*, making an angle of 60° toward the *left* of the central vertical line. It will be seen that the diagram, when completed, represents a network of lines. Printed blank forms of this diagram may be obtained from dealers, thus obviating the necessity of drawing it, and on such forms the deviations recorded for the different headings of a ship may be readily plotted and connected by a curve.

66. How the Curve is Drawn.—The curve shown in Fig. 22 corresponds to the deviations given in Table IV; it is drawn in the following manner: The deviation for north is 14° W; hence, 14° is scaled according to the graduated vertical line, and this distance is then laid off to the *left* on the *dotted line* passing through the north point, thus locating the point *a*. If the deviation is *east*, as on the S S E heading, the corresponding distance is laid off to the right, and on the dotted line passing through the S S E point, locating *b*. After all the deviations have been similarly scaled off and marked with a light cross on the dotted lines, a curved line is drawn through the crosses, which is then the required deviation curve.

The best results are obtained when the deviation for at least eight equally distant points of the compass is determined, although an approximately correct curve can be drawn when the deviation for four points only is known; but these should be near, or preferably on, the cardinal points. In practice, however, eight or more points equally distributed should be used in constructing the curve.

67. It is evident that to find the deviation for any point from such a diagram it is simply necessary to scale the distance from that point along the dotted line to the deviation

curve. When it is desired to find the deviation for any intermediate point of the compass other than the points marked down, simply scale the distance, *parallel to the dotted lines*, from that point to the deviation curve. The deviation thus found is named east or west, depending on whether the curve at that point lies to the right or the left of the vertical line. Thus, in Fig. 22, the deviation for S 40° W is 11° E, nearly.

68. Use of Napier's Deviation Diagram.—The principal advantage of Napier's diagram is that the magnetic course, or bearing, corresponding to any compass course, or bearing, and vice versa can readily be determined by inspection. Thus, suppose it is desired to find the magnetic course for a certain compass course. First, place one leg of the dividers on that point of the vertical line representing the compass course considered, and measure the distance to the deviation curve either along or parallel to a *dotted line;* then, holding the leg of the dividers on the curve, swing the other leg around so that its direction coincides with or is parallel to a *continuous line*, and note the point at which it falls on the central vertical line. This reading will be the required magnetic course. For instance, the magnetic course for N W by compass, found in this manner from Fig. 22, is N 64° W.

If the compass course corresponding to a given magnetic course is to be found, the reverse process is employed. Thus, place one leg of the dividers on that point of the vertical line representing the magnetic course considered, and measure the distance to the deviation curve either along or parallel to a *continuous line*. Then, holding the leg of the dividers on the curve, swing the other leg around so that its direction coincides with or is parallel to a *dotted line*, and note the point at which it falls on the central vertical line. This reading will be the required compass course. For instance, the compass course for the magnetic course west, found in this manner from Fig. 22, is W by N.

It will be noticed that the results found by inspection, as just explained, agree exactly with those given in columns 1 and 3 of Table IV.

69. Proper Placing of Compass.—A navigator should ever be watchful about the proper performance of the compass and particularly so in modern steamships, where new forms of disturbances are likely to occur at any time. It should also be remembered that the directive force of a magnetic needle may be lessened, and the prime cause of this is vibration. Thus, if a compass is exposed or subjected to vibrations from the propeller or the engine room for any length of time, it will begin to act sluggishly, and the needles will have to be recharged, or remagnetized. Considerable care should therefore be given to the selection of a suitable place for the standard compass on board a ship. No iron or steel of any kind must be placed near or permitted to remain within a certain distance of the compass, and the selected place should be as free as possible from vibrations due to the working of engines. All vertical iron, such as stanchions, davits, etc., should, if possible, be at least 14 feet from the standard compass. To sum up, the compass should be fixed in a position selected not for the convenience of the helmsman or the builder, but in a place where the surrounding magnetic forces will least affect it, where vibrations are at a minimum, and where every facility exists for the examination of its error.

70. Protecting Compasses From Electrical Wiring. With the introduction of electricity on board ships, a new form of compass disturbance was created, inasmuch as the magnetism of large electromagnets used in dynamos and that of electric current in general may disturb a compass even at a considerable distance. The committee of Lloyd's Register of British and Foreign Shipping has suggested the following in reference to compasses and electricity on shipboard:

1. That dynamos and electric motors should be placed as far as possible from all compasses and at a distance of at least 30 feet from the standard compass.

2. That wires conducting electric currents should not come nearer than 16 feet to any compass, whereas wires conducting strong currents should be at a still greater distance.

3. That the compensation of compasses should be done when the dynamos are at rest, while the operation for determining the deviation should be performed when the dynamos are running.

In addition to these precautions, it is recommended that instead of single wires the system of double wiring be used in all electrical installations on shipboard.

PILOTING

NAVIGATING ALONG THE COAST

DETERMINING THE POSITION OF A SHIP WHEN IN SIGHT OF LAND

1. The position of a ship when navigating in sight of land may be found by several methods, the most important of which are explained in the following articles:

2. Single bearing and estimated distance consists of observing the bearing of some known object, for instance a light, by the compass and then estimating the distance by the eye. The method is practicable when the distance is small, but since distances measured by the eye, as a rule, are overestimated, probably to the extent of one-fifth or more of the actual distance, this method must not be relied on when the safety of the ship depends on an accurate knowledge of her position. When only one known object is in sight, the estimated distance should be checked, whenever possible, by a sounding.

3. Cross-Bearings.—The method of **cross-bearings,** which has been described in connection with the use of charts, is one of the simplest and most reliable used in fixing the position of a ship, provided the two bearings are taken so that the angle formed by them is sufficiently large to insure a defined point of intersection. The bearings should be taken in rapid succession and should be properly corrected for any deviation due to heading of ship that the compass may have, as explained in the latter part of this Section. If the chart

on which the bearings are to be plotted is not provided with magnetic-compass diagrams, the bearings must be corrected for variation of the locality also. Whenever convenient, the horizontal angle between the objects used for cross-bearings may be measured by a sextant, and the angle thus found should agree with that formed by the bearings. If these angles do not agree, an error must exist either in the bearings, in the application of the deviation or variation, or in the sextant angle.

4. Bow and Beam Bearings.—In the method of **bow and beam bearings**, a compass bearing is taken of a light, or other prominent known object, when it is 2, 3, or 4 points on the bow, and the time and log noted. The ship is then kept on a steady, continuous course, and when the bearing has doubled, the log and time are again noted. (If a patent log is used, it is not necessary to note the time, but simply the register of the log at both bearings.) The distance of the ship from the object is then *equal to the distance run in the interval between the first and second bearings*, or the difference of readings of the patent log at the two bearings.

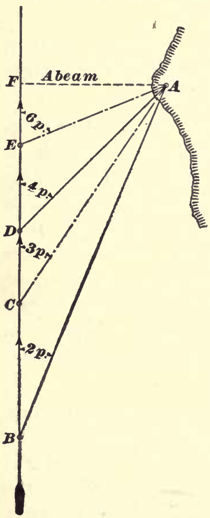


FIG. 1

The reason of this is evident by an inspection of the triangle BDA, Fig. 1. The sum of all angles in a triangle is equal to 180°, or 16 points; the angle at B is 2 points, at D 12 points, and consequently the remaining angle at A must be 2 points. The angles A and B are therefore equal, and hence the sides opposite are equal, or the side $BD = DA$. In triangle CEA, likewise, the side $CE = EA$; also, in triangle DFA, the sides DF and FA are equal, for the same reasons.

5. To find the ship's position on the chart, it is necessary only to lay off from the object the second bearing, properly corrected, and on this line, from the object, mark the distance run between observations. The point thus obtained will be the position of the ship.

To illustrate the method, suppose the reading of the patent log when at B, Fig. 1, to be 72.6 miles, and that when at D, or when the bearing is doubled, it indicates 75.8 miles. The distance of the vessel from A is then $75.8 - 72.6 = 3.2$ miles. In other words, AD is 3.2 miles long, or equal to the distance run.

6. This method is frequently used when the ship is at D, Fig. 1, or when the object bears 4 points off the bow; when the bearing has doubled, the ship is abeam the object, and at a distance equal to DF. Hence, the name *four-point bearing*, by which this method is commonly known among navigators. It is preferable, however, to utilize this method of bow bearings when the bearing of the object is 2 or 3 points off the bow; then. when either bearing is doubled, the position of the ship is fixed before the object is abeam, which insures safety should rocks or other obstacles be located abeam, or outside of the object.

In using this method with 2 and 4 points, the distance FA, Fig. 1, when abeam, is readily found by multiplying the distance run BD $(= DA)$ by .71 or, as is done in practice, by .7. Thus, if BD is equal to 5 miles, DF and FA will be, respectively, $5 \times .7 = 3.5$ miles. This will enable the navigator to know beforehand how far off the object he will be when abeam, assuming that the same course is steered. If, therefore, rocks or shoals should happen to be situated outside the object, they are easily avoided by changing the course. The distance when abeam is equal, also, to the distance run between bearings, taken at $26\frac{1}{2}°$ and $45°$, respectively; but the former method of multiplying the distance run from 2 to 4 points by *seven-tenths* is preferable, being simpler, and more readily committed to memory.

7. It should be noticed that this method is applicable, also, after the ship has passed the object. In such a case, the first observed angle on the stern should be twice the second angle. Thus, if the angle between the object and the ship's stern is 4 points, the second angle, or bearing, should be noted when the object bears 2 points. The distance run between the bearings, or readings, of the patent log is then equal to the distance of the ship from the object when it bore 4 points.

NOTE.—When practicing bow bearings, it is well to remember that in this method, as well as in any other method of coast navigation involving the *run of the ship*, due allowance must be made for any current or tide that is known to affect the ship's run.

8. Two Bearings of the Same Object.—It is evident that the foregoing method is subject to certain restrictions. The observer must wait and watch until the angle on the bow has doubled, or, if the ship has passed the object, until the angle on the stern is just one-half the first bearing. These inconveniences, involving considerable time and attention, are done away with in the method about to be described. A compass bearing is taken of some known object *at any instant*, and the number of points contained between this bearing and the ship's head, or course, is noted. A straight, continuous course is then kept until the bearing of the object has altered *at least 3 points*, when another bearing is taken and the number of points between it and the ship's head is again noted. The manner of finding the ship's position from these data is illustrated by the following example: Assume the true bearing of an object A, Fig. 2, to be N 53° W and the true course steered to be N 3° W. After a distance of 4.5 miles is covered, according to the patent log, a second bearing of the same object is found to be S 50° W; find the ship's position.

9. Graphic Solution.—Lay off from the object A, Fig. 2, the two bearings AE and AD, and draw a line gh in the direction of the ship's course (= N 3° W) so that it intersects the first bearing AD (it does not matter *where*). On this line, lay off from the intersecting point g, the

distance run in the interval, according to scale of chart, and from the point h thus obtained draw a line hm parallel to the first bearing. The point x at which this line intersects the second bearing AE will be the *position of the ship* at the time of the *second* observation, and by drawing from x a line xz parallel to the ship's course gh, the point of intersection z of that line with the first bearing AD will show the position of the ship at the instant the *first* bearing was taken. The ship's position at both bearings is thus quickly and conveniently determined.

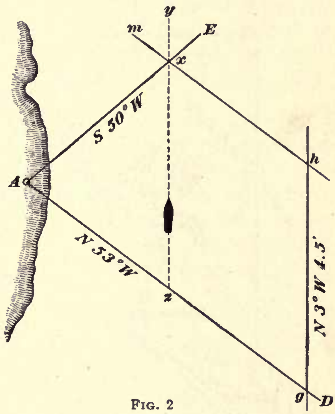


FIG. 2

10. Table Method. From the table on page 177 of the Nautical Tables, the solution is as follows: Enter the table with the first number of points in the top row of figures and the second number of points in the first column of figures, and take out the corresponding number; multiply this by the number of miles run in the interval. The product is the distance, in miles, at the time the *second* bearing was taken.

Referring to Fig. 2, the angle Azx between the ship's head and the first bearing is equal to 50°, or $4\frac{1}{2}$ points, nearly; the angle yxA at the second bearing is equal to $77° + 50° = 127°$, or $11\frac{1}{4}$ points, nearly. Now, under $4\frac{1}{2}$ in the top row and opposite $11\frac{1}{4}$ in the side column will be found the number .80. Multiplying this by the distance run in the interval will give the distance of the ship, in miles, from A at the second bearing; thus, $4.5 \times .8 = 3.6$.

To find the distance Az of the ship from the object at the *first* bearing, enter the same table with the *supplement* of

the second number of points in the top row and the supplement of the first number of points in the side column; take out the corresponding number and multiply by the distance run. The product will be the distance of the ship from the object at the instant of the first bearing.

11. Illustrative Example.—The light at Anglesea, N. J., on the south side of Hereford Inlet, bore N N W;

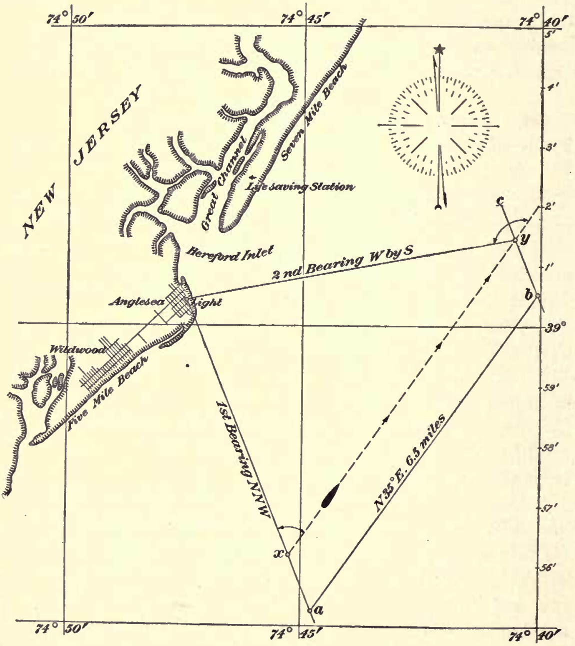


FIG. 3

after running N 35° E a distance of $6\frac{1}{2}$ miles, a second bearing of the same light was observed and found to be W by S.

To find, graphically, the distance of the ship from the light at the time of the second bearing, proceed as follows: Plot the respective bearings on the chart, as shown in Fig. 3, and from any point *a* on the first bearing draw a line *a b* in the direction of the course steered, N 35° E, and make this line equal in length to the distance run, or $6\frac{1}{2}$ miles taken from the scale. At the extremity *b* of the line, draw a second line *b c* parallel to the first bearing; the point *y* where this line intersects the second bearing is the position of the ship at the time of observing the second bearing, *x y* being the actual course run. Measuring this distance by means of the scale at the side of the chart, it is found to be about 5.6 miles.

12. The same result should be obtained by using the table on page 177 of the Nautical Tables. Thus, the angle between the course and the first bearing, measured by a protractor, is very nearly 58°, or $5\frac{1}{4}$ points; that between the ship's head and the second bearing is 136°, or 12 points. In the table, the number corresponding to these angles is found to be .88; multiplying this number by the distance, in miles, run between bearings gives $6.5 \times .88 = 5.7$ miles, which agrees very nearly with the result found by construction.

To find the distance of ship when the first bearing was taken, enter the table with the supplements of both angles, as already explained. The supplement of the second angle is $16 - 12 = 4$ points, and that of the first angle is $16 - 5\frac{1}{4} = 10\frac{3}{4}$ points. Using the former in the top row, and the latter in the side column, the corresponding number is found to be .73, which, when multiplied by the distance run, 6.5, will give the distance of point *x* from the light, or the ship's position at first bearing, as $6.5 \times .73 = 4.7$ miles. The correctness of this distance may be verified by measurement on the chart, Fig. 3, using, as before, the scale at the side.

13. To Find Distance of Ship From Base of Mountain Peak or Lighthouse of Known Height.—When the base of the known object is visible, its height *h*, Fig. 4,

above the surface of the sea may be considered as one of the perpendiculars in a right triangle, and the distance d of the ship from the object as one of the other perpendiculars, v being the angle subtended by h. Thus, the distance

$$d = \frac{h}{\tan v}$$

But since v is very small, $\tan v$ may be substituted by $v \times \sin 1'$, or

$$d = \frac{h}{v \times \sin 1'}$$

Dividing by 6,080, the length of a nautical mile in feet, the

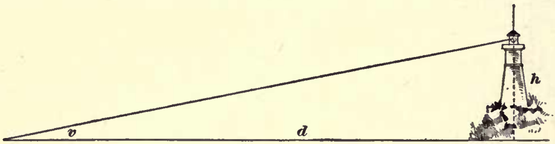


FIG. 4

result, or distance, will be expressed in *nautical miles;* or,

$$d = \frac{h}{v} \times \frac{1}{6{,}080 \sin 1'} = \frac{.56\,h}{v}$$

Again, dividing by 5,280, the length of a statute mile in feet, the distance will be expressed in *statute miles;* or,

$$d = \frac{h}{v} \times \frac{1}{5{,}280 \sin 1'} = \frac{.65\,h}{v}$$

14. From the preceding formulas the following rule may be deduced:

Rule.—*Measure, with a sextant or other instrument of reflection, the angle v subtended by the top and base of the object and apply a correction for index error, if necessary; this will give the value of v. Divide the height of the object by the number of minutes in the altitude and multiply the quotient by .56 or .65, according to the mile, nautical or statute, used. The result is the required distance.*

EXAMPLE 1.—The angle subtended by a lighthouse 144 feet high is found to be 13′; find the distance expressed in both nautical and statute miles.

SOLUTION.—In this case, $v = 13'$ and $h = 144$. Applying the rule just given, the distance

$$\left.\begin{aligned} d &= \frac{.56\,h}{v} = \frac{.56 \times 144}{13} = 6.2 \text{ naut. mi.} \\ d &= \frac{.65\,h}{v} = \frac{.65 \times 144}{13} = 7.2 \text{ stat. mi.} \end{aligned}\right\} \text{Ans.}$$

EXAMPLE 2.—The angle subtended by the lighthouse on Hog Island, to the north of Great Machipongo Inlet, Virginia, was measured by a sextant and found to be 10′ 30″; the height of the light above the sea level is 180 feet. What is the distance of the ship from the light, expressed in nautical miles?

SOLUTION.—In this case, $v = 10'\,30'' = 10.5'$, and $h = 180$. Hence,

$$d = \frac{.56 \times 180}{10.5} = 9.6 \text{ naut. mi.} \quad \text{Ans.}$$

15. Cautionary Remarks.—This method of locating the distance of a ship from a known object should be utilized only when the distance is inconsiderable and when the base of the selected object is distinctly seen. It should be borne in mind, also, that the distance obtained by this method is that of the ship from vertically below the observed object, and not that from the shore line. In cases where the lighthouse or other selected object stands at a considerable distance inland, it is evident that the angle measured will be the one subtended by the top of the object and the shore line. This angle, however, owing to the height of the eye, is greater than the required one, and since a greater angle corresponds to a shorter distance, the actual position of the ship will be farther off shore than the result by the method indicates. Owing to the interference of terrestrial refraction, which causes objects near the horizon to appear higher than they actually are, the result by this method should be considered only approximately correct. The heights of lights above mean high water, expressed in feet, are to be found in the List of Lights and Fog Signals of the United States published by the Government.*

*A copy of this list for the Atlantic and Gulf Coasts or for the Pacific Coast is sent free of charge to any shipmaster on application to the Office of the Lighthouse Board, Washington, D. C.

In coast navigation, this method may prove useful under various circumstances. For instance, when compelled to anchor during a fog, an occasional or partial lift of the fog may enable a navigator to measure the height of some known object, such as a lighthouse, a bluff, or a cliff, that happens to become visible, and thus, together with its bearing, afford a convenient means of approximately determining the position of the vessel.

16. To Find Distance of Ship From Known Fixed Object Seen on Sea Horizon.—The solution of this problem depends on the uniform curvature of the sea, in consequence of which all terrestrial objects appear or disappear from sight at certain distances from the observer. These distances, corresponding to different elevations of the eye and the object, are found on page 178 of the Nautical Tables (under the heading, Distances of Objects at Sea), expressed

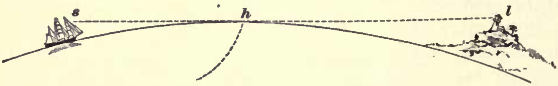


FIG. 5

in nautical and statute miles, respectively. In order to give a clear illustration of this method, let *l*, Fig. 5, represent a lighthouse situated on the summit of a mountain, *s* the lookout stationed at the masthead of an approaching vessel, and *h* the visible horizon of *s*. As soon as the top of the lighthouse appears above the horizon, it is evident that the distance of the ship from the light is equal to the sum of the distances *s h* and *h l*. These distances, depending on the heights of *s* and *l*, are recorded in the tables referred to.

Assume, for instance, that the height of the lighthouse above the level of the sea is 250 feet and that of the lookout 115 feet; what would be the distance of the ship from *l*, expressed in nautical miles, at the moment the lookout sees the top of the lighthouse on the horizon? Consulting the Table of Distances in Nautical Miles, it will be found that for a height of 250 feet the corresponding distance is 18.18 miles,

and for a height of 115 feet the distance of the visible horizon is 12.34 miles. Therefore, the distance of the ship from the lighthouse must be equal to the sum of these distances, or $18.18 + 12.34 = 30.52$ miles.

17. This method is very useful at night, in clear weather, when approaching or getting out of the range of visibility of a light, the height of which is known. It must be remembered, however, that the visibility of lights, as indicated on charts or in the List of Lights, is that distance corresponding to an elevation of the eye of 15 feet; hence, when the observer is at a greater or less height than 15 feet above the water-line of the vessel, his actual elevation should be taken into consideration and used in entering the table.

EXAMPLE 1.—The officer of the deck, having watched the Gay Head light situated on the western point of Martha's Vineyard Island, sees that it is about to disappear below the horizon. He takes its bearing and notes the time. The height of his eye above the water-line is 25 feet, and the height of the light, according to the List, is 170 feet. Find the distance, in nautical miles, of the ship from the light at the instant it disappeared.

SOLUTION.—Entering the table, it is found that

the distance corresponding to 25 feet = 5.75 naut. mi.
the distance corresponding to 170 feet = 14.99 naut. mi.

Hence, the distance of ship from light = 20.7 naut. mi. Ans.

EXAMPLE 2.—A steamer is running for the entrance to Havana, Cuba. At the time expected to get within sight of the light on Morro Castle, a man is sent aloft. His height above the water-line is 60 feet. After a while, he discovers the light and notifies the officer on the bridge to that effect. What is the distance of the ship from the light at that instant, expressed in nautical as well as in statute miles, the height of the light being 144 feet?

SOLUTION.—Entering the table, it is found that the distance corresponding to a height of

60 feet = 8.9 naut. mi., or 10.2 stat. mi.
144 feet = 13.8 naut. mi., or 15.8 stat. mi.

Hence, distance = 22.7 naut. mi., or 26.0 stat. mi. Ans.

NOTE.—A copy of these tables should be hung in some convenient place in the chart room or wheel house for ready reference. The distances given in the tables are corrected for ordinary atmospheric refraction, but may, at times, be greatly increased by abnormal refraction, especially in high latitudes.

EXAMPLES FOR PRACTICE

1. The bearing of a lighthouse taken at 3 P. M. was found to be N N W; at about 5:30 on the same afternoon it bore N E by N. The distance run in the interval between the observations was 12 miles, the course steered being W. Find the distance of the ship from the lighthouse at the first and second bearings by means of the proper tables, and also by construction.

Ans. { Dist. at first bearing = 12 mi.
Dist. at second bearing = 13.3 mi.

2. The angular height of a certain lighthouse as observed from aboard ship is 12′. Find the distance of the lighthouse (*a*) in nautical and (*b*) in statute miles, if the height as given in the List of Lights and Fog Signals is 140 feet.

Ans. { (*a*) 6.5 naut. mi.
(*b*) 7.6 stat. mi.

3. Find the distance, in nautical miles, of a vessel from Cape Charles lighthouse, if the angular height of the latter as measured with a sextant is 16′, its recorded height being 180 feet.

Ans. 6.3 naut. mi.

4. A bearing taken of the Cape Race light was N W by W. A distance of 8 miles was then covered in a W by S direction, and the bearing of the light from this point was N N E. Find, by construction and also by tables, the distance of the ship from the light at the second observation. Ans. 5.8 mi.

PILOTING IN FOGGY WEATHER

NAVIGATING WITH THE LEAD

18. To Find Position of Ship in Foggy Weather by Means of Soundings.—In misty or foggy weather, the following method of determining, approximately, the position of a ship is very valuable, and should never be neglected whenever circumstances call for its utilization: Take several casts with the lead or the sounding machine at frequent intervals and note the time, and the course and distance run between each cast; note, also, at each sounding, the character of the bottom, as shown by the arming of the lead. After a sufficient number of casts has been made, rule a line *a b*, Fig. 6, on a piece of tracing paper or tracing cloth to represent the direction of the meridian, and another line *c d* to

represent the true course (or courses) steered. On this line *c d*, lay off and mark according to the scale of the chart embracing the locality in which the ship is navigating, the distance run between each cast, and at each point thus obtained mark the respective soundings and character of the bottom, as shown in the figure. The result will be a *line*, or *chain, of soundings*. This done, place the tracing cloth on the chart with the end *c* of the line *c d* on the place where you conceived your position to be when the first cast was taken. If the soundings on the line *c d* agree with those on the chart, as seen through the tracing cloth, you will know at once where you are. But if they do not agree, move the tracing cloth (keeping the line *a b* parallel to the meridian), until you find a place where they do agree. This is the position of the ship.

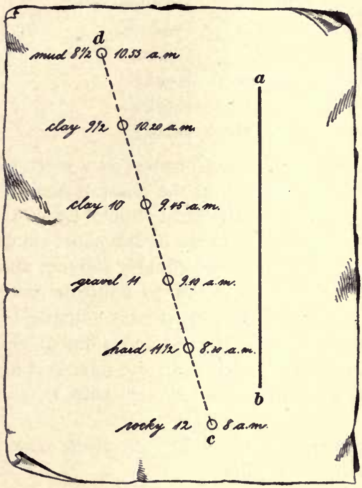


FIG. 6

It is evident that in cases where the ship has been affected by an unknown current or by a tide of uncertain velocity and direction, the line *a b* may deviate from the meridian by an angle depending on the effect of the current. In such cases, the navigator should be guided by the soundings, regardless of the direction in which they run when placed so as to agree with soundings printed on the chart.

19. Illustration of Method.—To illustrate the usefulness of this method, assume that a vessel bound for Halifax,

in going up the coast of Nova Scotia, runs into a fog north of Roseway Bank. When overtaken by the fog, her calculated position is about 20 miles southeast off Little Hope light, or at *A* on the chart, Fig. 7. From this position, she keeps a course N E by N true, and during her run northwards casts are taken with the sounding machine after each of the following distances are covered:

Distances	Depth	Bottom
$5\frac{1}{2}$ miles	71 fathoms	Pebbles
$6\frac{1}{2}$ miles	83 fathoms	Pebbles
$8\frac{1}{4}$ miles	68 fathoms	Gravel
5 miles	62 fathoms	Rocky
6 miles	58 fathoms	Rocky
$4\frac{1}{2}$ miles	45 fathoms	Rocky

These distances and soundings being plotted on a piece of tracing cloth, the sheet is spread out on the chart. According to the course run, N E by N, the ship should be at *C*; but the soundings do not agree with those on the chart along that direction. The tracing cloth is accordingly moved, and the soundings along the line *AB* seem to coincide very nearly with those recorded. The ship must consequently be at *B*, inside the 50-fathom line and about $6\frac{1}{2}$ miles out of her supposed position. An unknown current or tide setting toward the shore has evidently affected the ship to the extent shown.

The course is now changed to E N E $\frac{1}{4}$ E, along which direction the following casts are taken:

Distances	Depth	Bottom
6 miles	60 fathoms	Rocky
4 miles	65 fathoms	Rocky
$4\frac{1}{2}$ miles	68 fathoms	Fine sand
5 miles	53 fathoms	Fine sand
4 miles	51 fathoms	Rocky
4 miles	42 fathoms	Rocky
$2\frac{1}{2}$ miles	30 fathoms	Rocky

This second series of soundings, after being plotted on the tracing cloth, will agree with soundings marked along


NOVA SCOTIA
64°
44°
HALIFAX
Sackville
BEDFORD BASIN
Dartmouth
COLE HR.
Falkland
HERRING COVE
Camperdown
Chebucto Hd
Boll Rock
Sambro I
Sambro Ledges
Outer Bank
Prospect
Betty I
Mars Hd
BLIND B.
Indian Hd
HEAD HR.
FRENCH VILLAGE HARBOR
HUBBARD COVE
MILL COVE
MARGARET BAY
ASPOTAGOEN HR.
Chester
Gold River
MAHONE BAY
MAHONE HR.
E. Ironbound I
Green I
LUNENBURG
UPPER S. COVE
Bridgewater
LA HAVE R.
Cross I
Rose Pt
W. Ironbound I
Black Rock
C. La Have
GREEN BAY
NORTH WEST BAY
PORT MEDWAY
RUGGED HR.
Andrews Hd
LIVERPOOL BAY
Liverpool
Coffin I
Western Hd
ENE 1/4 E
VAR. 20° W.

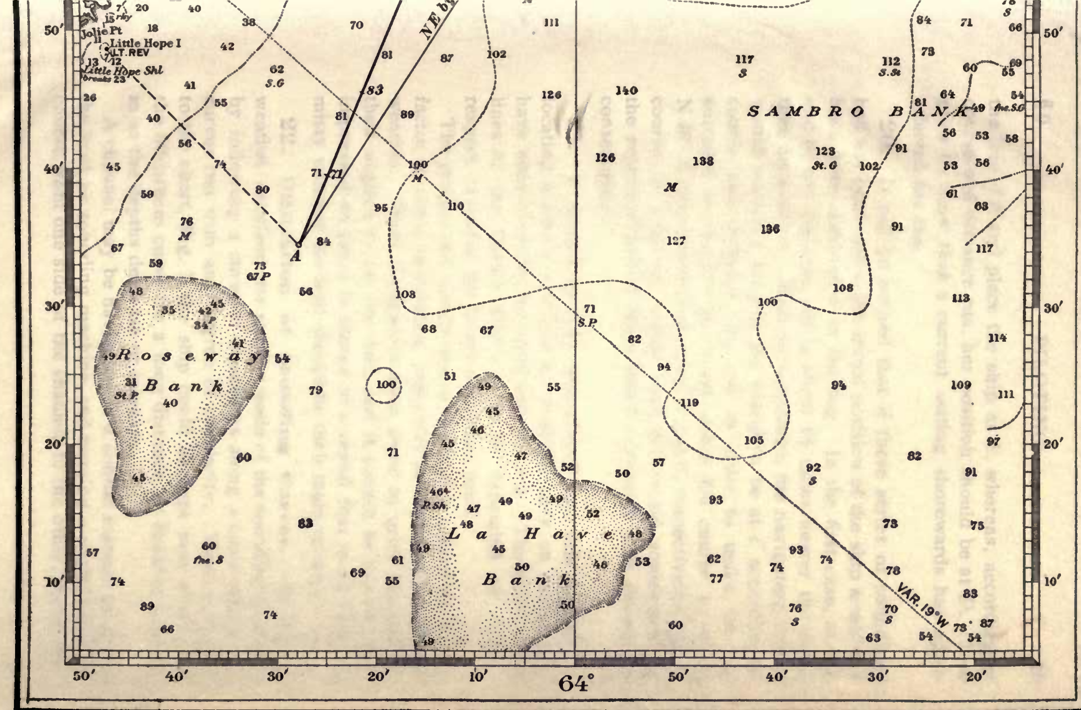

Jolie Pt
Little Hope I
LT. REV
Little Hope Shl
NE by
A
SAMBRO BANK
Roseway Bank
La Have Bank
VAR. 19° W
64°

FIG. 7

the line BE and place the ship at E, whereas, according to course and distance run, her position should be at D. This goes to show that a current setting shorewards has again affected her run.

20. It will be noticed that if these series of soundings had not been taken, the actual position of the ship would not have been discovered in the fog. In the first case, at the end of her first run, she is about 6½ miles nearer the shore than anticipated. Without soundings, the navigating officer would naturally assume his vessel to be at C according to course and distance run, and in order to make for the entrance to Halifax he would shape his course to about N 49° E (see dotted lines from B and C, respectively), which course, if kept up, would land his vessel somewhere in the neighborhood of Mars Head—probably with disastrous consequences.

21. It should be fully understood that this method of locating a ship's position is available only on coasts that have been carefully surveyed and charted. The entire coast lines of the United States may be depended on in this respect, provided the latest charts are used.

The proper and timely use of the lead is a very important factor in coast navigation, especially at night and in misty weather. Many navigators have come to grief because of their neglect to use the lead, and it cannot be too strongly impressed on those in charge of a vessel that in all kinds of misty weather the lead should be their main reliance.

22. Utilization of Sounding Curves.—In foggy weather, excellent use may be made of the sounding machine by following a curve of soundings along a coast where the curves run with any degree of regularity. Thus, referring to the chart, Fig. 7, the ship could be kept very closely on the 50-fathom curve for a long distance by heading out and in as the depths decrease or increase.

A channel may be navigated in a similar manner by using the lead, or sounding machine, and running a slightly zigzag course from one side of the channel to the other, the course

being changed whenever the water is found shoaling sufficiently to warrant going no farther to that side. It is better to "side" the channel in this way than to attempt to follow it directly, because when the water begins to shoal dangerously, it may not be possible to tell on which side of the channel the vessel happens to be.

SUBMARINE SIGNALING

23. Description of System.—A system of **submarine signaling** based principally on patents issued to Prof. I. Blake, A. J. Mundy, the late Prof. Elisha Gray, and others working in cooperation with them, has lately been developed. In this system, sound signals are transmitted through water, enabling a navigator to locate the position of his vessel and approximately determine, within a point, the bearing of buoys, lighthouses, and lightships in a fog, and to detect the presence of other vessels that are within the

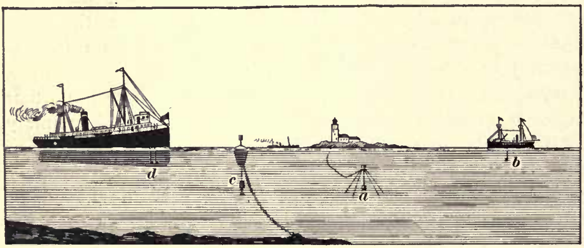


FIG. 8

operating area of the system. The velocity of sound in water is about four times as great as its velocity in air, the actual estimation being about 1,100 feet per second in air and 4,700 feet per second in water. In addition, sound signals transmitted through water are practically uniform in character, and are not subject to interference by the wind, freaks of echoes, silent spaces, or other noises, as is the case with sound signals sent through the atmosphere. The

mechanism of the system in its present form consists essentially of two parts: the *sound-producing* and the *sound-receiving apparatus*.

24. The **sound-producing apparatus** is made in various forms, according to its requirements. Thus, for stationary fog signals, the device is generally a submerged bell *a*, Fig. 8, of peculiar design, adapted to produce sounds of long range under water. This bell is usually anchored off some dangerous shoal or point where navigation is difficult, and is connected electrically with a shore station—for instance, a lighthouse. From this station the clapper is caused to strike the bell at certain intervals, producing distinct strokes separated by silent intervals. As a rule, the number of strokes and silent intervals is so arranged that the location of the bell may be identified by an approaching vessel equipped with a receiving apparatus. The diameter of such bells, a type of which is shown in Fig. 9, varies from 15 to 18 inches; they have a sound range under water of from 5 to 6 miles and have been heard at even greater distances.

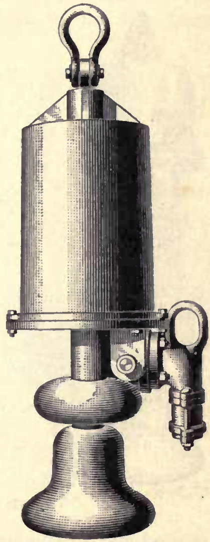


FIG. 9

25. In Fig. 10 is shown a type of submarine bell with which lightships are equipped. This bell is hung through a well in the center of the lightship or from its side at a depth of about 25 feet below the surface of the water (see *b*, Fig. 8), and is operated by compressed air supplied from the engine room of the ship. In another form *c*, Fig. 8, the submarine bell is suspended from a buoy and is rung automatically by the motion of the waves. The bell may also be attached to a whistling buoy without interfering with the

operation of the whistle, or it may be attached to an automatic gas-lighted and whistling buoy. This latter combination buoy is perhaps the most complete and efficient automatic marine-signaling apparatus in existence. Anchored in exposed positions off dangerous shoals or at the entrance of harbors, this buoy shows a brilliant flashing light and sounds a fog whistle, besides sounding the submarine bell by which vessels equipped with receiving apparatus may locate it miles away in the thickest weather.

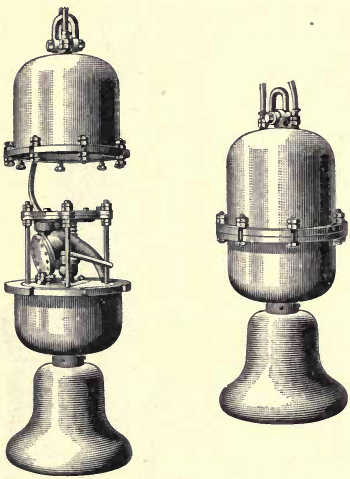


FIG. 10

26. The **sound-receiving apparatus** as applied to vessels consists of three parts: the *transmitters*, the *receiving telephones*, and the *direction indicator*, the two latter being placed usually in the pilot house.

27. The **transmitters** are located in the forehold of the vessel, one on each side, below the water-line, as shown at *d*, Fig. 8. Each transmitter consists of a cup-shaped metal cylinder, or tank, having the open end clamped against the inner side of the outer plating, or planking, of the hull, as shown in Fig. 11. The joint between the tank and the side of the ship is made tight by means of a gasket, and through a small opening at the top, the tank can be filled with a dense liquid. Through the covering that closes this opening is hung the microphone, or electric sound receiver, which is enclosed in a round, brass case about the size of a watch and is suspended in the liquid of the tank. The microphones in

each tank are connected by wires through a battery to the receiving telephones in the pilot house. It should be noted that the tanks are not located on the outside of the vessel; instead, the plating of the hull constitutes one end of the transmitter. Thus, when the sound vibration from a distant bell passes through the water, the sound is communicated to the plating of the ship and that in turn communicates it to the liquid in the tank, whence, by the medium of the microphone and wires, it is transmitted to the telephone receiver.

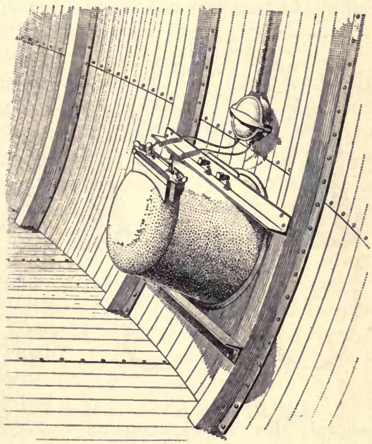


Fig. 11

The **direction indicator,** as shown in Fig. 12, consists of a round, metallic case about 9 inches in diameter, shaped like a ship's clock; also, it is fastened to the wall of the pilot house or the chart room in the same way as a ship's clock. On the dial of the indicator is a switch, by means of which either the port or the starboard microphone can be connected with the receiving telephone; and, in addition, the dial has an indicator showing in plain letters to which side the telephones are connected.

As will be seen from Fig. 12, the **receiving telephones** are similar to the common telephone receiver, and are hung on hooks on opposite sides of the instrument. They are always connected together, either to the port or to the starboard transmitter, according to position of switch, and may be used singly or together for listening to sound signals from below.

28. To Find Direction of Sounding Bell.—When the sound waves from a submerged distant bell reach the ship, an impulse is given to the liquid in the tank and is at once transmitted electrically to the receiving telephone in the pilot house. As sound travels through water in every direction from its source, it is evident that the impulse will be stronger and louder on the side of the ship nearer to the source. Hence, to head the bow toward the bell, the ship is swung toward the side from which the loudest sound comes. A sudden falling off in the intensity of sound indicates that the ship's fore-and-aft line is nearly in the direction of the bell, and by heading the bow so that the intensity of the sound on both sides is equal, the bearing of the bell is approximately the same as that of the ship.

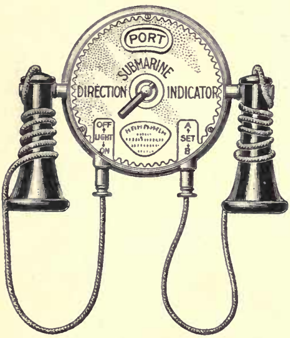


FIG. 12

29. For submarine communication between ships in motion, a sound of different character from the stationary submarine fog signals is used in order to avoid any confusion that may be caused by mistaking another ship's bell for that of a fog signal. Recent experiments and other observations have demonstrated the fact that if a vessel is equipped with submarine transmitting and receiving apparatus, it is possible to ascertain the approach of another vessel in which the bell machinery is at work and to locate her direction while she is still at a considerable distance.

30. General Remarks on Submarine Signaling. In 1906, a series of tests of submarine signals was instituted by the United States Lighthouse Board. During the course of these tests submarine bells on five lightships were rung continuously for 61 days and nights, and vessels equipped

with sound-receiving apparatus were requested to report the range and conditions under which the bells were heard. The result of the tests having proved entirely satisfactory to the Board, submarine signaling was adopted by the United States Government as an aid to navigation. As a consequence, nineteen lightships, extending from Portland, Maine, to Cape Hatteras, are equipped with submarine bells, and all lightships on the Great Lakes are similarly equipped. Orders have also been issued to the Coast and Geodetic Survey to mark on the charts all light vessels equipped with submarine signals; and notice has been sent to mariners that during storm and fog the signals will be rung during the same time that the fog whistle is blown.

The harbors of Liverpool, Cherbourg, Boulogne, Hamburg, Bremen, and Kiel are protected by submarine bells, and ships representing over 1,400,000 tons of mercantile marine are now (May 1, 1907) equipped to receive submarine signals. These include all the great transatlantic liners, United States coastwise steamers, passenger and freight steamers of the Great Lakes, besides numerous tugs, yachts, and other small craft. Altogether, the system bids fair to become an important factor in navigation, and all pilots and navigators should therefore familiarize themselves with the methods of manipulating the different instruments used in sending and receiving submarine signals.

RANGE LINES AND DANGER BEARINGS

RANGE LINES

31. An excellent method of fixing a ship's position when passing along a coast where known objects are in sight is shown in Fig. 13. The method consists in measuring the angle between an object *b* and the range line formed by the ship and two other objects *a* and *c*. A ship is said to be *in range* with two objects when she is on the straight line connecting the two objects. In order to insure a defined point of intersection, the angle between the third object (which

may be on either side) and the range line should not be less than 25°. The plotting of the position on the chart is then accomplished as shown in Fig. 14. The line connecting *c*

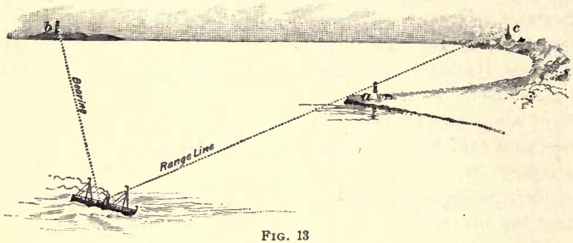


FIG. 13

and *a* is extended sufficiently seawards, and a point *d* is selected from which to lay out the measured angle. Suppose this angle to be 45°. The line *d f* is now drawn to form an angle of 45° with the range line, and another line *e b* is drawn parallel to *d f* from the object *b*; the point *e* is the position of the ship. It will be noticed that this method is entirely independent of the compass; even if the compass is used, any error it may have does not affect the result, because in this case it serves simply as an instrument to measure the apparent angle between bearing and range line.

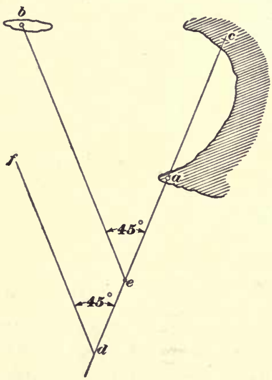


FIG. 14

THE DANGER ANGLE

32. The **danger angle,** which may be either *vertical* or *horizontal*, is the name given to a method that is used, when navigating along a coast, to avoid hidden dangers, such as rocks, shoals, sunken derelicts, or other obstructions situated

immediately at or below the surface of the water. By using this method, any such dangerous obstacle may be passed or rounded at any desired distance. For this reason, its proper name should be *safety angle* instead of *danger angle*, although the latter term is the one most commonly used.

33. Vertical Danger Angle.—The **vertical danger angle** is based on the principle that the distance to an object will remain the same as long as the angle subtended by the height of the object remains the same. Tables containing angles corresponding to different heights and distances, expressed in miles and fractions of a mile, have accordingly been prepared for the use of navigators.

Among such tables may be mentioned Captain Lecky's "Danger Angle and Off-Shore Distance Tables," and the tables of vertical danger angles incorporated in Captain Patterson's "Navigator's Pocketbook." If no tables are available, the required vertical angle is readily found by the following formula:

$$v = \frac{.57\,h}{d}$$

in which h = height of object, in feet;
d = distance, in miles;
v = required angle, in minutes of arc.

This formula is expressed in the form of a rule, as follows:

Rule.—*Multiply height of object, in feet, by .57; the product divided by the distance, in miles, will give the vertical angle expressed in minutes of arc.*

The value of the angle obtained by this rule is approximate only, though it is correct to the nearest minute, which in most cases is sufficiently close for practical purposes.

34. Illustrative Example.—Assume that a ship is about to round a point of land (see Fig. 15) on which a lighthouse l 170 feet high is situated. Outside of this point, and within a mile of the lighthouse, lie a number of rocks r immediately below the surface of the water; because awash,

they cannot be seen, but their position is indicated on the chart. It is desired to avoid these rocks by passing 1 mile outside of them or, what amounts to the same, by passing 2 miles outside of the lighthouse. In order to do this, it is necessary to know what angle the lighthouse will subtend at a distance

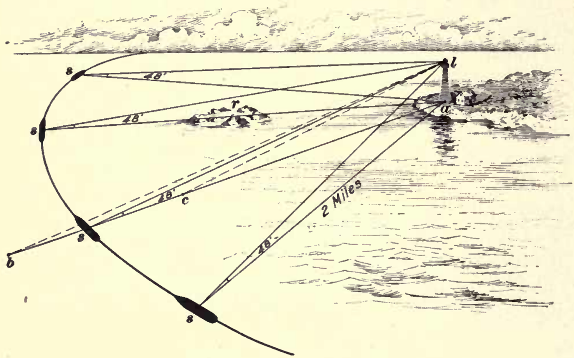


Fig. 15

of 2 miles; in other words, the vertical danger angle for that distance must be known. To determine this angle, enter the Tables of Vertical Danger Angles (Lecky), a specimen reproduction of which is shown below, with 170 feet in

Distance in Miles and Tenths	Vertical Danger Angle					
	170 Feet	175 Feet	180 Feet	185 Feet	190 Feet	195 Feet
1.9	0° 50′ 35″	0° 52′ 4″	0° 53′ 34″	0° 55′ 3″	0° 56′ 32″	0° 58′ 1″
2.0	0° 48′ 3″	0° 49′ 28″	0° 50′ 53″	0° 52′ 18″	0° 53′ 43″	0° 55′ 7″
2.1	0° 45′ 46″	0° 47′ 7″	0° 48′ 28″	0° 49′ 49″	0° 51′ 9″	0° 52′ 30″

the top row and 2 miles in the distance column, when directly below the former and opposite the latter is found 48′ 3″. This is the angle that the lighthouse should subtend at a distance of 2 miles. Using either the formula or the rule

given in the preceding article, approximately the same result is obtained; or,

$$v = \frac{.57 \times 170}{2} = \frac{96.90}{2} = 48.4'$$

All that has to be done now is to set the sextant to an angle of 48′ (due allowance being made for any index error the instrument may have), and to go ahead, keeping the course so that the angle will remain the same until the danger is passed. So long as the angle *l s a* between the top of the lighthouse *l* and the water-line *s a* is 48′, the ship is at the correct distance; if the angle becomes less, she is outside of the 2-mile limit, as at *b*; if the angle becomes greater, she is nearer the rocks than desired, as at *c*. That such must be the case is evident from the fact that the angle *l s a* is greater than *l b a* but smaller than *l c a*, and, of course, the greater the angle becomes the shorter will be the distance. It is not necessary to move the index bar of the sextant at all; simply have it clamped at the required angle, for, if the object *l* rises above the water-line *s a*, as seen in the horizon glass of the instrument, the angle is larger than that set and means that the ship is nearer the rocks than is desirable. If the object *l* drops below the water-line, the angle is smaller, and the ship is consequently outside of her intended course.

35. In observing vertical danger angles, it is advisable that the observer be as near to the surface of the water as possible; this will tend to minimize the error caused by the elevation of the observer's eye above the water-line. This error, however, will increase the angle subtended by the object, and, since a greater angle corresponds to a shorter distance, it is evident that the ship will actually be farther from the danger than the recorded distance and thus be in a safer position, unless a second danger lies outside and close to the ship's course.

This method may prove useful in cases where buoys and marks indicating the location of shoals and rocks have been destroyed or carried away by ice or otherwise.

36. Horizontal Danger Angle.—The method known as the **horizontal danger angle** is an application of one of the geometrical properties of the circle; namely, that angles inscribed in the same segment are equal. The following illustration will serve to explain the method:

Suppose that when steaming along a coast it is decided to avoid some hidden rocks *R*, Fig. 16, by passing ½ mile out-

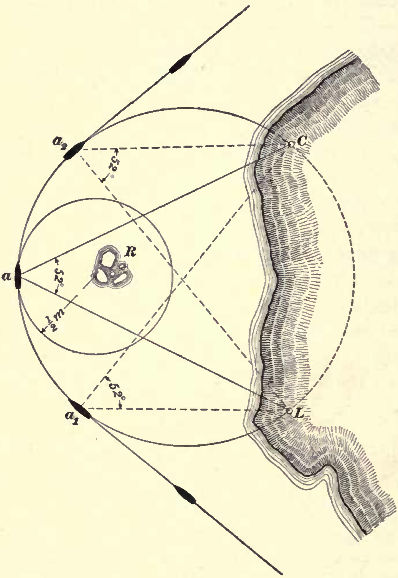


Fig. 16

side of them. On the shore, there are two known objects in sight, a lighthouse *L* and a church *C*, both being marked on the chart. Then, to find the danger angle corresponding to a distance of ½ mile from the rocks, proceed as follows: With the outermost rock as a center and a radius equal to ½ mile, describe a circle on the chart. Then, through the

most seaward point a of this circle and the points C and L, describe another circle; connect a with C and L; measure with a protractor the angle CaL formed by the lines aC and aL; assume it to be 52°, as in the figure. This is the required horizontal danger angle. Now, set that angle on the sextant (neglecting the index error if it is small) and watch the two selected objects C and L, holding the instrument in a horizontal position. When the two objects appear in the horizon glass, the ship is close to the *circle of safety* $a_1 a a_2$; and when they come in contact, the ship is *on* that circle; once on the circle, change the course of the ship so that the two images will remain in contact until the danger is passed. So long as this is being done, the ship will be on the circle of safety $a_1 a a_2$, since the angles Ca_1L, CaL, and Ca_2L are all equal, being angles in the same segment. If the angle increases, the ship is on the inside of the circle of safety and consequently nearer the danger than is desirable; if it becomes smaller, the ship is outside of the $\frac{1}{2}$-mile limit. However, by watching the angle closely and changing the course accordingly, one cannot fail to keep the ship at the required distance from the rocks.

37. Illustrative Example.—Suppose that a vessel is coming down Delaware Bay bound north and that the whistling buoy a, Fig. 17, that marks the location of McCries Shoal has been carried away by ice. It is deemed advisable to go outside of the shoal at a distance of, say, 1 mile. The weather is clear, and after consulting his chart the navigator decides to use the Cape May and the Hereford Inlet lights by which to form a horizontal danger angle. Referring to the chart, Fig. 17, he now proceeds as follows: With the point a as a center and a radius of 1 mile, according to scale of chart, he describes a circle, and through the outermost point b of this circle and through each light a second circle is drawn. Any point c or d of this circle is now connected, respectively, by straight lines with both lights, as shown. The angle at c or d formed by these lines is the required angle, and, on measuring by means of a protractor, it is

found to be 55°. Hence, by setting the sextant to that angle and keeping the two lights in contact, as explained in the preceding article, the shoal may be rounded at the required distance.

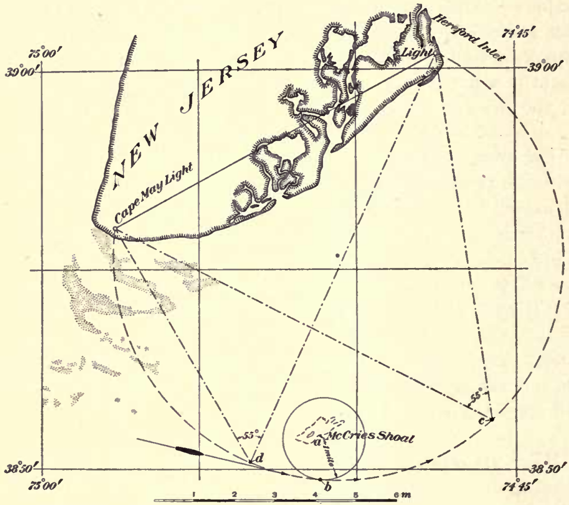


FIG. 17

38. Comparison of the Two Methods.—When circumstances permit the selection of either a vertical or a horizontal danger angle, the latter should always be preferred as being much more accurate than the former; primarily, on account of the horizontal angle being much larger than the vertical angle, and secondly, because the recorded heights of lights and other objects on shore are not always trustworthy and can be considered as being only approximately correct. This is particularly true in regard to the heights and altitudes recorded on foreign charts, especially those issued by Spanish authorities. Furthermore, it is evident that the horizontal danger angle may be used during both

night and day, whenever two lights are in sight; whereas, the use of the vertical angle is limited to the day only.

It must be remembered that, before using either of these methods in the actual practice of navigating a ship past a danger, one should be efficient in their use. Do not try to *experiment* in places where actual danger exists. Practice at first with imaginary danger until sure of being able to utilize the methods with ease and confidence.

DANGER BEARINGS

39. Bearings that by virtue of their direction may serve as a boundary between dangerous and navigable waters are known as **danger bearings**; they are of especial value in navigating between and in keeping clear of submerged reefs and shoal water. In the utilization of such bearings, much depends, of course, on the locality and on the charted objects in sight that may be used for the establishment of such bearings.

A case that will serve to illustrate the method will here be shown. Referring to Fig. 18, suppose that a vessel is at A steaming S W by S to round Culebrita and Culebra Islands and make Target Bay; she is to pass the channel between the red buoy at C and the black buoy at B. The prevailing currents in this region are not accurately known, and hence it is necessary to proceed with utmost caution, especially in bright sunlight when the colors of buoys may not be readily distinguished at a distance. Danger bearings are accordingly established, one running north to the lighthouse on Culebrita Island and another running S 85° W to Saldado Point, the southern promontory of Culebra Island (not shown on chart), that will give a point of intersection at D in the middle of the channel. A glance at the chart will now show that so long as the light on Culebrita Island bears west of north, the ship will be to the east of the north-and-south danger line and therefore well clear of the reef marked x. Similarly, by keeping the bearing of Saldado Point 86° or more to the west of south, the ship may safely pass the red buoy and

round the shoals at a proper distance. When at the intersecting point *D* of the two bearings, the red buoy should be clearly in sight, bearing about W by N, and the course may at that point be changed to W S W. It is evident that before

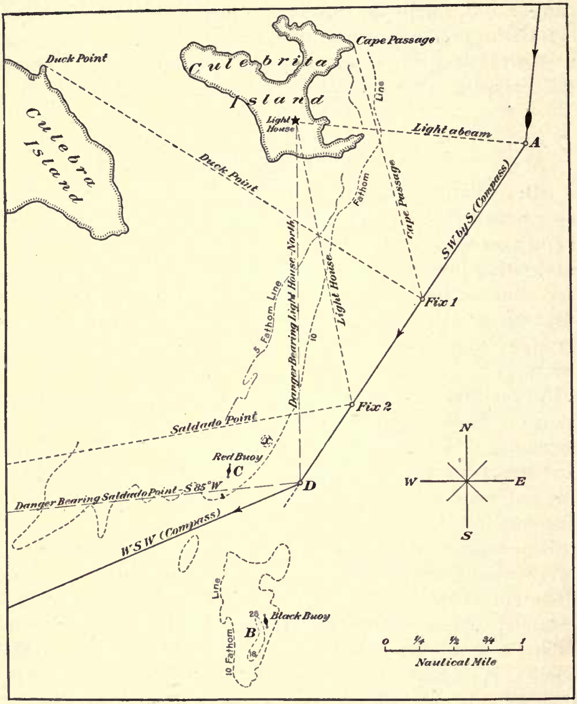


FIG. 18

the point *D* is reached, the exact position of the ship along the course *A D* may be fixed and checked by cross-bearings of the several objects in sight, as indicated on chart. All the intersections of such cross-bearings should lie on *A D*; if not, currents are interfering with the run of the ship.

STATION POINTERS

40. In connection with the use of charts and for determining the position of a ship while in sight of known objects, many instruments have been introduced to facilitate and simplify such operations. Among them the **station pointer,** or **three-arm protractor,** is important. This instrument, which does away with the possibility of committing errors in applying compass corrections, besides saving considerable time in fixing a position, is shown in Fig. 19. It consists of a graduated circle of metal having three arms attached to its center; of these, *e* is fixed, while the other two are movable. The latter are fitted with clamping screws *s*, *s′*, so that they can be set and secured to any required angle, the center line of the fixed arm serving as the zero point. The instrument is made in different sizes and is more or less perfect in detail; like the sextant, some are fitted with tangent screws and magnifying glass. As a whole, the instrument is easy to handle and adjust, and has no complicated mechanism to get out of order.

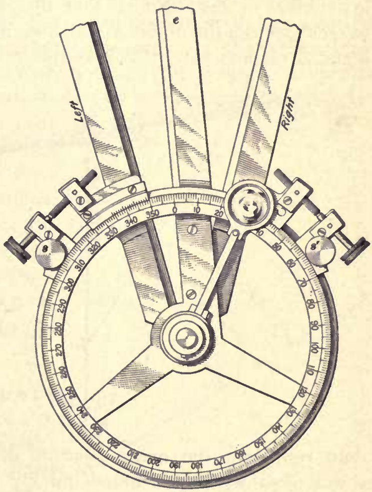


FIG. 19

41. How to Use the Station Pointer.—Select three objects on shore, the positions of which are known and which are to be found on the chart. Let these three objects be represented in Fig. 20 by *a*, *b*, and *c*; the angles between *a*

and *b* and *b* and *c* are then measured by a sextant, either in quick succession by one observer or separately by two observers at the same instant. This done, take the station pointer, holding the arms from you, and set the left arm so that the angle formed by it and the fixed leg will be equal to the measured angle between *a* and *b*. Similarly, set the right arm so that the angle between it and the zero arm is equal to the measured angle between *b* and *c*. Then place the instrument on the chart so that the center of the fixed arm passes through the object *b*, while the other arms pass through *a* and *c*, respectively. The center *p* of the instrument will

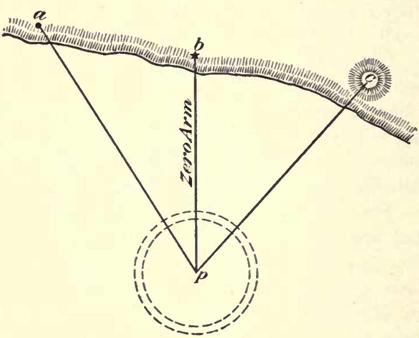


FIG. 20

then represent the exact position of the ship. Thus, the ship's position is determined quickly and accurately without the least aid of the compass.

42. Position Finder.—A still simpler form of the station pointer is an instrument recently introduced and called the **position finder**. This instrument is absolutely independent of both compass and sextant. Each of its three arms is fitted with a small steel pointer, and at the center of the instrument is a sight vane. The bearings are taken directly by the instrument, and are placed on the chart in exactly the same manner as with the station pointer, the time required for the whole operation being about 4 or 5 minutes.

CORRECTION OF COURSES

43. The correct conversion of one course into another, or the process of finding a required compass course from a true course, and vice versa, by allowing for known errors of the compass, is one of the most important duties allotted to a navigator. A mistake one way or the other may have a far-reaching effect, and for this reason a special effort should be made to study this subject very thoroughly.

As stated before, the course of a ship is the angle included by the ship's fore-and-aft line and a meridian, and since three distinct meridians exist, there are three kinds of courses: *compass*, *magnetic*, and *true course*, which will be explained in the order given.

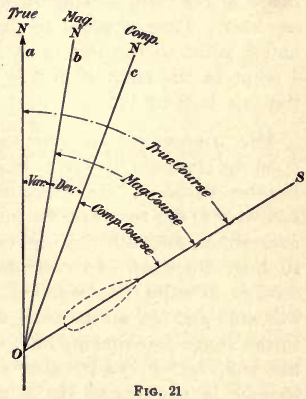


Fig. 21

44. The **compass course** is the angle cOS, Fig. 21, that the ship's track OS makes with the direction of the magnetic needle. This course may be affected by variation and deviation, and, consequently, must be corrected for both in order to obtain the true course.

45. The **magnetic course** is the angle bOS that the ship's track makes with the magnetic meridian. This course is affected only by variation, the application of which converts it into a true course. It is evident that where there is no deviation the compass and magnetic course will be identical.

46. The **true course** is the angle aOS that the ship's track makes with the true, or geographical, meridian, indicating true north and south. It is evident that at places on the earth's surface where no variation exists, and in ships having no deviation, the true course will be identical with the compass course.

47. Important Remarks.—In dealing with problems of correcting courses, always remember that, since the compass is the representation of the visible horizon, the position of the observer is considered to be at the *center* of the compass card. Hence, *when applying corrections, whether to the right or left, one must consider himself to be stationed at the center of the card, looking in the direction of the course to be corrected.* Thus, 1 point to the right of E N E is E by N, and 1 point to the left of E N E is N E by E; similarly, 1 point to the right of S S W is S W by S, and 1 point to the left is S by W.

48. Leeway.—In order to obtain the course made good from the compass course, it is sometimes necessary to apply, besides variation and deviation, a correction for *leeway*. **Leeway** is the result of the pressure of the sea or the wind exerted on the hull, rigging, or sails of a ship, causing her to drift sidewise. In reference to a steamship making a voyage in calm, fine weather, no leeway occurs; but with the wind and sea on the side, or in any direction not parallel to the ship's fore-and-aft line, they have a tendency to affect her run, and in such cases an appropriate correction for leeway is necessary. In a sailing vessel, with the wind astern or on either quarter, there is, as a rule, no leeway worth considering; but with the wind abeam, or before the beam, leeway is often quite considerable, depending on the velocity of the wind, on the sea, and most of all on the speed of the vessel. The higher the speed, the less is the leeway. With bad weather, a rough sea, and slow speed, the leeway may amount to as much as 4, 5, or more points.

Leeway may also be defined as the angle SOS', Fig. 22, that the ship's keel makes with her path through the water.

Thus, if NOS is the true course steered and OS' is the ship's actual path over ground—the arrow indicating the direction of the wind—the angle SOS' is the leeway. From this, it is evident that the correction for leeway must be applied *from* the wind. Thus, if a ship steers N E and a strong N N W wind is blowing, she will make a more easterly course than is indicated by the compass; hence, if the

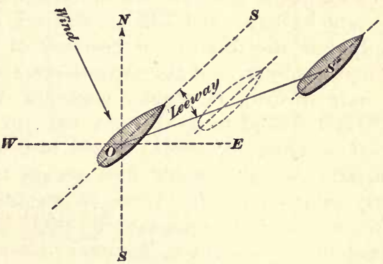


FIG. 22

leeway is 1 point, the course made good will be N E by E, although the ship is continually heading N E.

49. How Leeway is Estimated.—As stated before, the amount of leeway varies with the strength of the wind, the condition of the sea, the speed, and the shape of the wetted surface of the ship; therefore, no positive rule can be laid down for estimating its value. Experienced navigators usually estimate the leeway by eye, being guided by the angle between the ship's wake and her keel, or fore-and-aft line. A good plan, however, is to drop over the stern some floating object, and from its bearing, when a few fathoms away, estimate the amount of leeway, which is always expressed in points.

50. Use of Diagrams.—When correcting courses, the beginner should always make use of a diagram—no matter how roughly drawn—representing the cardinal points, and on it lay out the courses and corrections. Such a diagram will simplify the solving of the problems considerably and in a great measure prevent mistakes in applying corrections.

51. To find the true course from the compass course, observe the following rule:

Rule.—*Allow for leeway in the direction toward which the wind is blowing; apply easterly deviation and variation to the right, and westerly deviation and variation to left.*

In the application of this rule, fix in the mind the terms "apply to right or left," and add or subtract the corrections according to the increase or decrease of the angles in the different quadrants. If the circumference of the compass card were divided, in regular sequence, from north toward east and around to north, into 360°, or 32 points, the matter of applying corrections would be comparatively simple; easterly deviation would then always be additive, and westerly subtractive. But since, in the present card, courses are reckoned in opposite direction in any two adjacent quadrants, it sometimes becomes necessary to subtract and sometimes to add, and it is therefore better to *apply* to the right or to the left. The navigator should then entertain no hesitation as to when to add or when to subtract the corrections.

EXAMPLE 1.—A ship steers N N E $\frac{1}{2}$ E by compass, the wind is E by S, the variation 1 point E, the deviation 2 points W, leeway is estimated to $\frac{1}{4}$ point; find the true course made good.

SOLUTION.—

Compass course =	N N E $\frac{1}{2}$ E
Or =	N $2\frac{1}{2}$ points E
Leeway =	$\frac{1}{4}$ point
	N $2\frac{1}{4}$ points E
Variation =	1 point E
	N $3\frac{1}{4}$ points E
Deviation =	2 points W
True course =	N $1\frac{1}{4}$ points E
Or =	N by E $\frac{1}{4}$ E. Ans.

The dotted line at circumference of compass diagram, Fig. 23, shows plainly the application of the various corrections in this example, the arrow indicating the direction of the wind.

EXAMPLE 2.—The compass course is N N E variation 25° W, deviation 8° 10′ E; find the true course.

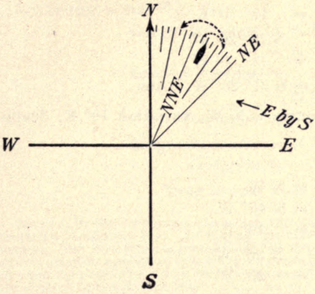


FIG. 23

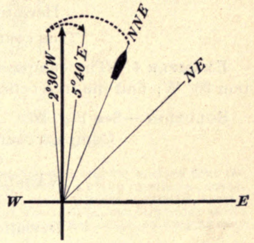


FIG. 24

SOLUTION.—See Fig. 24.

Compass course = N N E
Or = N 22° 30′ E
Variation = 25° 0′ W
N 2° 30′ W
Deviation = 8° 10′ E
True course = N 5° 40′ E
Or = N $\frac{1}{2}$ E, nearly. Ans.

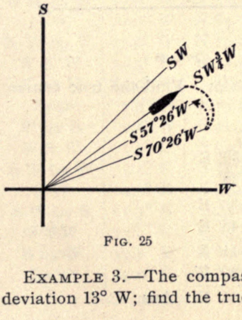


FIG. 25

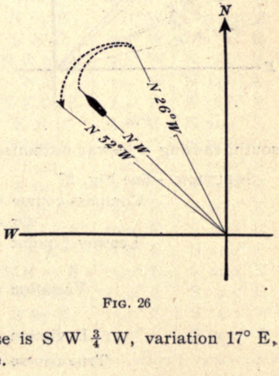


FIG. 26

EXAMPLE 3.—The compass course is S W $\frac{3}{4}$ W, variation 17° E, deviation 13° W; find the true course.

Solution.—See Fig. 25.

Compass course = S W $\frac{3}{4}$ W
Or = S 53° 26′ W
Variation = 17° 0′ E
S 70° 26′ W
Deviation = 13° 0′ W
True course = S 57° 26′ W. Ans

Example 4.—The compass course is N W, variation 19° E, deviation 26° W; find the true course.

Solution.—See Fig. 26.

Compass course = N W
Or = N 45° W
Variation = 19° E
N 26° W
Deviation = 26° W
True course = N 52° W. Ans

Example 5.—A ship steers E S E by compass, the variation is 26° 45′ E and the deviation 17$\frac{1}{2}$° W; a strong wind is blowing from the

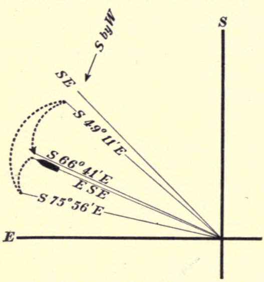


Fig. 27

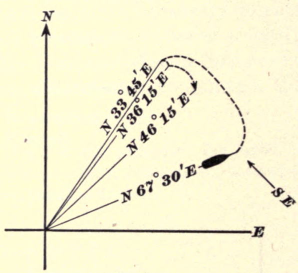


Fig. 28

south, causing a leeway estimated at $\frac{3}{4}$ point. Find the true course.

Solution.—See Fig. 27.

Compass course = E S E
Or = S 67° 30′ E
Leeway $\frac{3}{4}$ point = 8° 26′
S 75° 56′ E
Variation = 26° 45′ E
S 49° 11′ E
Deviation = 17° 30′ W
True course = S 66° 41′ E. Ans.

EXAMPLE 6.—The compass course is E N E, leeway 3 points, wind S E, deviation 2.5° E, variation 10° E. Required the true course.

SOLUTION.—See Fig. 28.

Compass course =	E N E
Or =	N 67° 30′ E
Leeway 3 points =	33° 45′
	N 33° 45′ E
Deviation =	2° 30′ E
Magnetic course =	N 36° 15′ E
Variation =	10° 0′ E
True course =	N 46° 15′ E. Ans.

NOTE.—In the foregoing examples, the last excepted, it will be noticed that the variation is applied first. In finding the true course from a given compass course, however, the deviation is usually applied first, thus obtaining the magnetic course, when variation applied to this will give the required true course; but this order is not an absolute necessity. The corrections may be applied in any order, so long as they are applied in the proper way.

TABLE I

DEVIATION TABLE

1	2	3	1	2	3
Ship's Head (Course) by Standard Compass	Deviation	Correct Magnetic Course Made Good	Ship's Head (Course) by Standard Compass	Deviation	Correct Magnetic Course Made Good
North	14.0° W	N 14.0° W	North	14.0° W	N 14.0° W
N by W	16.5° W	N 28.0° W	N by E	11.0° W	North
N N W	18.5° W	N 41.0° W	N N E	7.5° W	N 15.0° E
N W by N	19.0° W	N 53.0° W	N E by N	4.0° W	N 30.0° E
N W	19.0° W	N 64.0° W	N E	1.0° W	N 44.0° E
N W by W	17.0° W	N 73.0° W	N E by E	1.5° E	N 58.0° E
W N W	14.5° W	N 82.0° W	E N E	4.0° E	N 71.5° E
W by N	11.0° W	West	E by N	5.5° E	N 84.0° E
West	7.0° W	S 83.0° W	East	7.0° E	S 83.0° E
W by S	2.5° W	S 76.0° W	E by S	8.0° E	S 71.0° E
W S W	1.5° E	S 69.0° W	E S E	9.0° E	S 58.5° E
S W by W	5.5° E	S 62.0° W	S E by E	10.0° E	S 46.0° E
S W	9.0° E	S 54.0° W	S E	11.0° E	S 34.0° E
S W by S	11.5° E	S 45.0° W	S E by S	12.0° E	S 22.0° E
S S W	13.0° E	S 35.5° W	S S E	13.0° E	S 9.5° E
S by W	14.0° E	S 25.0° W	S by E	13.5° E	S 2.0° W
South	14.0° E	S 14.0° W	South	14.0° E	S 14.0° W

52. It should be borne in mind that in all problems of course corrections the *deviation applied must be that of the compass course*, no matter in what order the corrections are applied.

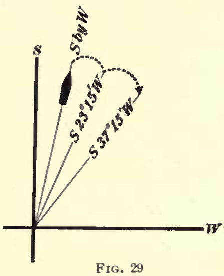


FIG. 29

EXAMPLE 1.—A steamer steers S by W by compass, the variation, according to chart, is 12° E, and the deviation is according to Table I; find the true course.

SOLUTION.—See Fig. 29.

Compass course =	S by W
Or =	S 11° 15′ W
Variation =	12° 0′ E
	S 23° 15′ W
Deviation (for S by W) =	14° 0′ E
True course =	S 37° 15′ W. Ans.

EXAMPLE 2.—The compass course is E S E, wind is south, estimated leeway $\frac{1}{2}$ point, variation by chart 5° W, deviation according to Table I; find the true course.

SOLUTION.—

Compass course =	E S E
Or =	S 67° 30′ E
Leeway $\frac{1}{2}$ point =	5° 37′
	S 73° 7′ E
Deviation (for E S E) =	9° 0′ E
Magnetic course =	S 64° 7′ E
Variation =	5° 0′ W
True course =	S 69° 7′ E
Or =	E S E $\frac{1}{4}$ E, nearly. Ans.

53. To Find the Compass Course When the True Course is Known.—When it is required to find the course to be steered by compass in order to reach a port or position the true bearing of which is known, the corrections must be applied in exactly the opposite direction to that used in finding the true course from the compass course.

Rule.—*Apply leeway toward the direction of the wind, westerly variation and deviation to the right, and easterly variation and deviation to the left.*

EXAMPLE 1.—The true course to a certain port is N 30° E, the variation is 28° W, and the deviation is 6° E; find what course to steer by compass.

Solution.—See Fig. 30.

True course =	N 30° E
Variation =	28° W
Magnetic course =	N 58° E
Deviation =	6° E
Compass course =	N 52° E. Ans.

Example 2.—The true course, or bearing, between two places is N 10° W, variation 4° E, deviation according to Table I; find the compass bearing.

Solution.—True bearing =	N 10° W
Variation =	4° E
Magnetic bearing =	N 14° W
Deviation (for N 14° W mag.) =	14° W
Compass bearing =	N 0° W, or North. Ans.

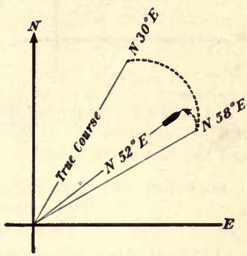


Fig. 30

54. In finding the compass course from the true course, the variation should always be applied *first* so as to obtain the magnetic course; the deviation corresponding to the magnetic course given in column 3 of Table I is then applied, the result being the compass course. In case the magnetic course does not agree with any of the values given in column 3, the deviation may, if small, be taken out for the value in column 3 corresponding most nearly to the magnetic course. With large deviations, the use of a deviation diagram is recommended as being a valuable aid for finding the value of the deviation between any two points.

Example 1.—The true course, according to chart, from one place to a certain other place is N 48° W; the mean value of variation is 5° E. Use deviation as given in Table I and find the compass course.

Solution.—See Fig. 31.

True course =	N 48° W
Variation =	5° E
Magnetic course =	N 53° W
Deviation (for N 53° W mag.) =	19° W
Compass course =	N 34° W
Or =	N W by N, nearly. Ans.

EXAMPLE 2.—The true course from position of ship to nearest port is S 8° E, mean value of variation 2° E, deviation according to Table I. Required, the compass course to steer, assuming there is no leeway.

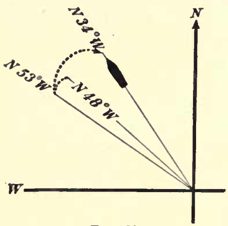


FIG. 31

SOLUTION.—

True course	=	S 8° E
Variation	=	2° E
Magnetic course	=	S 10° E
Deviation (for S 10° E mag.)	=	13° E
Compass course	=	S 23° E
Or	=	S S E, nearly. Ans.

EXAMPLE 3.—Required, the compass course, the true course being N 8° W, variation 17° 10′ W, deviation 3° 20′ E; the wind is easterly and the leeway estimated to $\frac{1}{2}$ point.

SOLUTION.—See Fig. 32.

True course	=	N 8° 0′ W
Variation	=	17° 10′ W
Magnetic course	=	N 9° 10′ E
Deviation	=	3° 20′ E
		N 5° 50′ E
Leeway $\frac{1}{2}$ point	=	5° 37′
Compass course	=	N 11° 27′ E
Or	=	N by E, nearly. Ans.

EXAMPLE 4.—The true course is S 47° E, wind S S W, leeway $\frac{1}{2}$ point, variation 27° 40′ W, deviation 3° 20′ E; find the compass course.

SOLUTION.—

True course	=	S 47° 0′ E
Variation	=	27° 40′ W
Magnetic course	=	S 19° 20′ E
Deviation	=	3° 20′ E
		S 22° 40′ E
Leeway $\frac{1}{2}$ point	=	5° 37′
Compass course	=	S 17° 3′ E
		Ans.

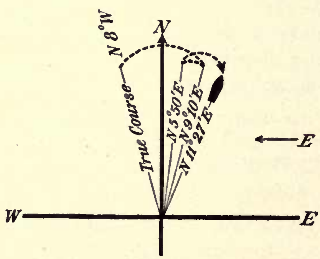


FIG. 32

55. Cautionary Remarks.—Learn to master the subject of course corrections thoroughly, as it is very important.

A mistake in the application of variation or deviation may possibly prove very costly. Not very long ago, the captain of a large transatlantic steamer, in shaping the compass course from the true course as indicated by the chart, applied the deviation the wrong way. After a few hours' run, the weather being misty, the steamer was grounded and wedged in between solid rocks that afterwards crushed through her plates, causing a loss of thousands of dollars to her owners and the reputation and position of her master; and all this simply because of the mistake made in applying westerly deviation to the left instead of to the right of the magnetic course. It is better to apply no deviation at all than to apply it the wrong way.

EXAMPLES FOR PRACTICE

Find the compass course from the following true courses:

1. True course is N 27° 30′ W, variation 18° 30′ E, deviation 3° 15′ W, wind westerly, leeway $\frac{1}{4}$ point. Ans. N 45° 34′ W

2. True course is S 75° W, variation 33° 10′ W, wind westerly, deviation 1° 50′ W. Ans. N 70° W

3. True course is S 17° E, variation 22° 40′ W, deviation 3° 30′ W, wind W S W, leeway $1\frac{1}{4}$ points. Ans. S 23° 14′ W

4. True course is N 58° W, variation 9° E, deviation 2° W. Ans. N 65° W

5. True course is N 48° W, variation 5° E, deviation according to Table I. Ans. N 34° W

6. True course is S 8° E, variation 2° E, deviation according to Table I. Ans. S 23° E

Find the true course from the following compass courses:

7. Compass course is north, wind W N W, leeway 1 point, deviation $22\frac{1}{2}$° W, variation $11\frac{1}{4}$° E. Ans. North

8. Compass course is S by E, wind E by S, leeway $1\frac{1}{2}$ points, variation 16° W, deviation 9° W. Ans. S 19° 23′ E

9. Compass course is east, wind N N E, leeway $1\frac{1}{4}$ points, variation 10° W, deviation 5° W. Ans. N 89° 4′ E

10. Compass course is N W by W, variation 8° E, deviation by Table I. Ans. N 65° 15′ W

11. Compass course is S 45° E, wind S W, leeway 19°, variation 11° W, deviation by Table I. Ans. S 64° E

12. Compass course is S W $\frac{1}{4}$ W, wind S S E, leeway $\frac{3}{4}$ point, deviation 30° 30′ W, variation 15° W. Ans. S 8° 45′ W

NOTE.—There is an error in one of the foregoing answers. The student that finds this error should give the correct answer in his work on this Section.

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56. Correction of Bearings.—When a bearing is corrected for deviation, remember to correct it, not for the deviation on the bearing, but for the deviation *due to the direction of the ship's head* at the moment the bearing was taken. This is very important.

EXAMPLE 1.—The bearing of a distant object by the standard compass is E S E $\frac{1}{2}$ E; the ship is heading N W by W; what is the correct magnetic bearing?

SOLUTION.— E S E $\frac{1}{2}$ E corresponds to S 73° E
Deviation (for N W by W, Table I) = 17° W
Correct magnetic bearing = S 90° E, or East. Ans.

Since the deviation in this case is westerly, or to the left, it is applied to the left of the compass bearing, as shown.

EXAMPLE 2.—The compass bearing of a certain lighthouse is N N W; find its true bearing, to the nearest degree, the variations being 6° E and the ship heading S S W.

SOLUTION.—
N N W corresponds to N 22° W
Deviation (for S S W, Table I) = 13° E
Magnetic bearing = N 9° W
Variation = 6° E
True bearing = N 3° W. Ans.

EXAMPLES FOR PRACTICE

1. When about to lose sight of land, the bearing of a lighthouse on shore, as observed by the standard compass, is S S E. At this instant the vessel is heading N E, the deviation for that heading being 17° E. What is the true bearing of the lighthouse, assuming the variation of the locality to be 4° W? Ans. S 9° 30′ E

2. A compass bearing taken of an object on shore is N W by W $\frac{1}{2}$ W. The deviation for the heading of vessel is 12° W, and the variation of the locality 5° W. Find: (*a*) the correct magnetic bearing; (*b*) the true bearing of the object.

Ans. $\begin{cases} (a) & \text{N } 73° \ 52' \text{ W} \\ (b) & \text{N } 78° \ 52' \text{ W} \end{cases}$

3. Find the true bearing of a distant object on shore if the bearing by the standard compass is N E $\frac{1}{4}$ E, the variation of the locality being 11° W. The ship's head at the instant of observation is N by W, the deviation being that recorded in Table I. Ans. N 20° 19′ E

DEAD RECKONING

(PART 1)

DETERMINING THE POSITION OF A SHIP BY COURSES AND DISTANCES RUN

INTRODUCTORY REMARKS

1. The methods used to fix the position of a ship by courses and distances run will now be considered. These methods are *plane*, *parallel*, *middle-latitude*, and *Mercator's sailing;* they are collectively known as *the sailings*, and also by the expression *fixing a ship's position by dead reckoning.*

The accuracy of the result attained by any of these methods depends on a careful estimate of the courses and distances run by the ship. In other words, when a navigator knows what courses have been steered and what distances have been run on each course, and when an appropriate allowance has been made for the influence of wind, tides, and currents, the exact position of his vessel may be determined at any required time by the methods about to be described.

PLANE SAILING

2. Plane sailing is usually defined as being the art of navigating a ship on the supposition that the earth is a flat surface. This definition should not be taken literally. In all sailings, the earth's surface is regarded to be what it really is—*spherical;* yet, the distance run during a day by an

ordinary vessel is so insignificant in comparison with the enormous surface of the earth that it may, for all practical purposes, be represented by a straight line on a *plane* surface.

This will be more clearly understood on examining an ordinary good-sized globe, representing the earth; a day's run, or the distance made good by a ship during a day, may be represented on such a globe by a straight line not more than perhaps $\frac{1}{2}$ or $\frac{3}{4}$ inch in length. If the space in which

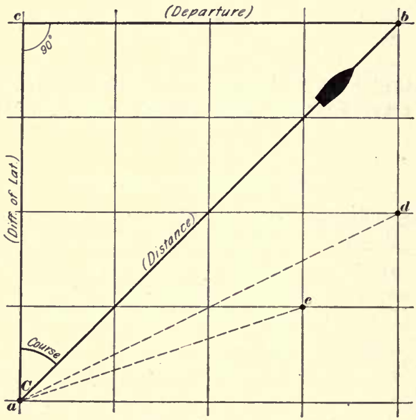

FIG. 1

this line is included were cut out and placed on a flat surface, the difference in length between the line on the globe and that on the flat surface would be so small as not to be perceptible to the naked eye, and the curvature of the meridians and latitude parallels, contained in the space, would be practically inappreciable.

Therefore, it is not assumed that the surface traversed is actually a plane, and that the meridians are parallel straight lines, but, for the reasons just mentioned, it is practicable to regard the small part of the spherical surface sailed over as a plane surface, and to consider the meridians

as parallel straight lines that intersect the latitude parallels at right angles, forming perfect squares, as shown in Fig. 1. Hence, the expression *plane sailing*.

3. From previous statements, it is known that when a ship steers a straight, continuous course, her track, or rhumb line, will intersect all meridians at the *same* angle. Therefore, in plane sailing, if a ship steams or sails, for instance, from *a* to *b*, Fig. 1, her track, or path, *a b* will form, with the meridian *a c* and the parallel *c b*, a right triangle *a b c*. The same thing will occur when the ship sails from *a* to *d*, or from *a* to *e*, or in any other direction except true north and south or true east and west. Hence, in plane sailing, only a plane right triangle has to be dealt with, and the most elementary acquaintance with plane trigonometry will suffice in solving problems relating to this method.

4. Formulas Relating to Plane Sailing.—Referring to the triangle *a c b*, Fig. 1, in which the angle *c a b* is the *course C*, the hypotenuse *a b* the *distance*, *c b* the *departure*, and *a c* the *difference of latitude*, then, by plane trigonometry, the following relations between the different parts of the triangle are established:

$$\text{Sin } C = \frac{cb}{ab} = \frac{\text{Dep.}}{\text{Dist.}} \qquad \text{Cot } C = \frac{ac}{cb} = \frac{\text{D. Lat.}}{\text{Dep.}}$$

$$\text{Cos } C = \frac{ac}{ab} = \frac{\text{D. Lat.}}{\text{Dist.}} \qquad \text{Sec } C = \frac{ab}{ac} = \frac{\text{Dist.}}{\text{D. Lat.}}$$

$$\text{Tan } C = \frac{cb}{ac} = \frac{\text{Dep.}}{\text{D. Lat.}} \qquad \text{Cosec } C = \frac{ab}{cb} = \frac{\text{Dist.}}{\text{Dep.}}$$

As previously stated, $\sin C = \frac{1}{\operatorname{cosec} C}$ and $\cos C = \frac{1}{\sec C}$, and, therefore, dividing by the cosecant is equivalent to multiplying by the sine of the same angle, or vice versa. Similarly, dividing by the secant is equivalent to multiplying by the cosine of the same angle, and so on.

5. From the foregoing, and by referring to the formulas and rearranging them into tabular form, the result shown in Table I is obtained.

TABLE I

FORMULAS RELATING TO PLANE SAILING

Given	Required	Formulas to be Used
Course and Distance	Diff. of Lat. Departure	D. Lat. = Dist. × cos C Dep. = Dist. × sin C
Course and Diff. of Lat.	Distance Departure	Dist. = D. Lat. × sec C Dep. = D. Lat. × tan C
Course and Departure	Distance Diff. of Lat.	Dist. = Dep. × cosec C D. Lat. = Dep. × cot C
Distance and Diff. of Lat.	Course Departure	Cos C = D. Lat. ÷ Dist. Dep. = Dist. × sin C
Distance and Departure	Course Diff. of Lat.	Sin C = Dep. ÷ Dist. D. Lat. = Dist. × cos C
Diff. of Lat. and Departure	Course Distance	Tan C = Dep. ÷ D. Lat. Dist. = Dep. × cosec C

6. Use of Traverse Tables.—Problems relating to plane sailing, besides being solved by trigonometry, may also be solved conveniently by the **Traverse Tables,** which are to be found in the collection of nautical tables accompanying this Course. These tables contain the true difference of latitude and departure corresponding to every course from 1° to 89°, and for distances of from 1 to 300 nautical miles. In the first sixteen pages of the tables, the courses are expressed in points and subdivisions of points; in the remaining pages, they are expressed in degrees. The manner of using these tables is very simple and is fully explained under the headings of Solution by Inspection in the different problems about to be given in connection with the methods of sailing. It should be remembered, however, that for

courses that are less than 45°, or 4 points, the tables should be entered at the *top*, and for courses greater than 45°, at the *bottom*. When entering the tables at the bottom, it should be noticed that while the *Dist.* column remains the same, the columns of *Lat.* and *Dep.* are *reversed*. The reason for this will be evident upon examining a right triangle, the acute angles of which are 2 and 6 points, respectively; in such a triangle, the departure for 2 points is equal to the difference of latitude for 6 points, the relation of the two elements simply being reversed. In general, the departure for one course is equal to the difference of latitude for the complement of that course. Thus, the departure for 3 points is equal to the difference of latitude for 5 points, or the difference of latitude for 35° is equal to the departure for 55°, and so on.

When distances are given in miles and tenths of a mile, they should be entered as whole numbers; the corresponding D. Lat. and Dep. are then found by shifting the decimal point of each one place to the left.

7. Graphic Representation of Cases.—In the following examples, it is advisable to sketch a right triangle in each case, considering the top of the paper as north, and the bottom as south, so that east will be on the right-hand side and west on the left-hand side. The north and south line being drawn to represent the portion of the meridian due to the difference of latitude, draw, from the latitude in, the base of the triangle to represent the departure (toward the right if the departure is east, and toward the left if the departure is west). The hypotenuse will then represent the distance sailed, and the angle included between it and the difference of latitude will be the course, the vertex of the angle being considered as the *center* of the compass.

8. *Course and distance given, to find the difference of latitude and the departure.*

EXAMPLE 1.—A ship from latitude 32° 15′ N sails N E by N, 115 miles. Required, her latitude in and departure.

Solution.—In this case the course and the distance sailed are known, and the D. Lat. and Dep. are to be found. Therefore, the following formulas are used:

$$\begin{array}{rl} \text{D. Lat.} &= \text{Dist.} \times \cos C \\ \log 115 &= 2.06070 \\ \log \cos 33^\circ 45' &= \underline{9.91985} \\ \log \text{D. Lat.} &= 1.98055 \\ \text{D. Lat.} &= 95.6' = 1^\circ 35.6' \text{ N} \end{array} \qquad \begin{array}{rl} \text{Dep.} &= \text{Dist.} \times \sin C \\ \log 115 &= 2.06070 \\ \log \sin 33^\circ 45' &= \underline{9.74474} \\ \log \text{Dep.} &= 1.80544 \\ \text{Dep.} &= 63.89 \text{ mi. E.} \quad \text{Ans.} \end{array}$$

$$\begin{array}{rl} \text{Lat. from} &= 32^\circ\ 15' \text{ N} \\ \text{D. Lat.} &= \underline{1^\circ\ 35.6' \text{ N}} \\ \text{Lat. in} &= 33^\circ\ 50.6' \text{ N.} \quad \text{Ans.} \end{array}$$

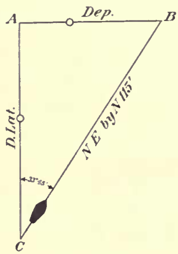

Fig. 2

Solution by Inspection.—See Fig. 2. Enter the Traverse Tables, page 94, and find the course N E by N (3 points), at the top of the table; in one of the columns marked *Dist.*, find Dist. = 115. Then, opposite 115 in the columns marked *Lat.* and *Dep.* will be found D. Lat. = 95.6′ and Dep. = 63.9′. Ans.

Example 2.—A ship leaving Cape Mendocino, in latitude 40° 26′ N and longitude 124° 24′ W, steers a true course S 56° W until a distance of 42.7 miles is covered. What is her latitude in and departure made good?

Solution.—

$$\begin{array}{rl} \text{D. Lat.} &= \text{Dist.} \times \cos C \\ \log 42.7 &= 1.63043 \\ \log \cos 56^\circ &= \underline{9.74756} \\ \log \text{D. Lat.} &= 1.37799 \\ \text{D. Lat.} &= 23.9' \text{ S} \end{array} \qquad \begin{array}{rl} \text{Dep.} &= \text{Dist.} \times \sin C \\ \log 42.7 &= 1.63043 \\ \log \sin 56^\circ &= \underline{9.91857} \\ \log \text{Dep.} &= 1.54900 \\ \text{Dep.} &= 35.4 \text{ mi. W.} \quad \text{Ans.} \end{array}$$

$$\begin{array}{rl} \text{Lat. from} &= 40^\circ\ 26' \text{ N} \\ \text{D. Lat.} &= \underline{23.9' \text{ S}} \\ \text{Lat. in} &= 40^\circ\ 2.1' \text{ N.} \quad \text{Ans.} \end{array}$$

Solution by Inspection.—In this case it will be noticed that the distance contains a decimal; but if this number is multiplied by 10, it will give 427 mi. with which to enter the Traverse Tables. Now, by an inspection of the tables, it will be seen that the numbers recorded in the distance columns do not exceed 300; therefore, proceed as follows: On the page for 56° is found

	opposite 300 mi.,	D. Lat. 167.8′,	Dep. 248.7′
and	opposite 127 mi.,	D. Lat. 71.0′,	Dep. 105.3′
	Sum = 427 mi.	238.8′	354′

Therefore, the values corresponding to 427 mi. are D. Lat. 238.8′ and Dep. 354′.

Since the actual distance was multiplied by 10 and the D. Lat. and the Dep. corresponding to this value were found, each is therefore 10 times as large as it should be, or the correct D. Lat. and Dep. for 42 7 mi. are, respectively, 23.9′, nearly, and 35.4′; whence,

$$\begin{aligned} \text{Lat. from} &= 40° \ 26' \ \text{N} \\ \text{D. Lat.} &= \quad 23.9' \ \text{S} \\ \hline \text{Lat. in.} &= 40° \ 2.1' \ \text{N} \\ \text{Dep. made good} &= 35.4 \text{ mi. W} \end{aligned} \Bigg\} \text{Ans.}$$

9. *Course and difference of latitude given, to find the distance and the departure.*

EXAMPLE.—A ship sails S E $\frac{1}{2}$ E from latitude 15° 55′ S, until by observation she is in latitude 18° 49′ S. Required, her distance run and departure.

SOLUTION.—In this case, the Dist. and Dep. are to be found; therefore, the following formulas are used:

$$\begin{aligned} \text{Lat. from} &= 15° \ 55' \ \text{S} \\ \text{Lat. in} &= 18° \ 49' \ \text{S} \\ \hline \text{D. Lat.} &= 2° \ 54' = 174' \ \text{S} \end{aligned}$$

$$\begin{aligned} \text{Dist.} &= \text{D. Lat.} \times \sec C \\ \log 174 &= 2.24055 \\ \log \sec 50° \ 37.5' &= 10.19764 \\ \hline \log \text{Dist.} &= 2.43819 \\ \text{Dist.} &= 274.3 \text{ mi. Ans.} \end{aligned} \qquad \begin{aligned} \text{Dep.} &= \text{D. Lat.} \times \tan C \\ \log 174 &= 2.24055 \\ \log \tan 50° \ 37.5' &= 10.08583 \\ \hline \log \text{Dep.} &= 2.32638 \\ \text{Dep.} &= 212 \text{ mi. E. Ans.} \end{aligned}$$

SOLUTION BY INSPECTION.—See Fig. 3. Enter the Traverse Tables with the course S E $\frac{1}{2}$ E (= $4\frac{1}{2}$ points) at the bottom of the table; opposite 173.8 in the *Lat.* column will then be found Dep. 211.8′ and Dist. 274 mi. Ans.

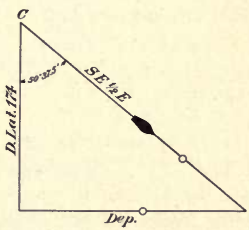

FIG. 3

10. *Course and departure given, to find the difference of latitude and the distance.*

EXAMPLE.—From a place in latitude 1° 36′ N, a ship sails S 42° W until she has made a departure of 250 miles. What is her present latitude and what is the distance run?

SOLUTION.—In this case, the Dist. and D. Lat. are sought; hence, the following formulas are used:

$$\text{Dist.} = \text{Dep.} \times \operatorname{cosec} C \qquad \text{D. Lat.} = \text{Dep.} \times \cot C$$

$$\begin{array}{rlrl}
\log 250 = & 2.39794 & \log 250 = & 2.39794 \\
\log \operatorname{cosec} 42° = & 10.17449 & \log \cot 42° = & 10.04556 \\
\hline
\log \text{Dist.} = & 2.57243 & \log \text{D. Lat.} = & 2.44350 \\
\text{Dist.} = & 373.6 \text{ mi. Ans.} & \text{D. Lat.} = & 277.7' \text{ S}
\end{array}$$

$$\begin{array}{rl}
\text{Lat. from} = & 1° 36' \text{ N} \\
\text{D. Lat. } 277.7' = & 4° 37.7' \text{ S} \\
\hline
\text{Lat. in} = & 3° \ 1.7' \text{ S. Ans.}
\end{array}$$

Solution by Inspection.—See Fig. 4. Enter the Traverse Tables with the course 42° and find half the given Dep. 125 (the whole being too great) in the *Dep.* column, opposite to which, in the *Lat.* and *Dist.* columns, will be found 139′ and 187′, respectively. Multiply these by 2. The results will give the whole D. Lat. 278, and the Dist. 374 mi. Ans.

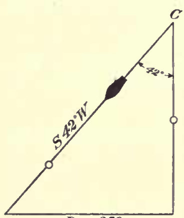

Fig. 4

11. *Distance and difference of latitude given, to find the course and the departure.*

Example.—From latitude 37° 3′ N, a ship sails between the north and west points 430 miles, until her difference of latitude is 214 miles. Required, her course steered, departure, and latitude in.

Solution.—In this case, the course and Dep. are sought; hence, the following formulas are used:

$$\cos C = \text{D. Lat.} \div \text{Dist.} \qquad \text{Dep.} = \text{Dist.} \times \sin C$$

$$\begin{array}{rlrl}
\log 214\ (+10) = & 12.33041 & \log 430 = & 2.63347 \\
\log 430 = & 2.63347 & \log \sin 60° 9' = & 9.93819 \\
\hline
\log \cos C = & 9.69694 & \log \text{Dep.} = & 2.57166 \\
\text{Course} = & \text{N } 60° 9' \text{ W. Ans.} & \text{Dep.} = & 373 \text{ mi. W. Ans.}
\end{array}$$

$$\begin{array}{rl}
\text{Lat. from} = & 37° \ 3' \text{ N} \\
\text{D. Lat.} = & 3° 34' \text{ N} \\
\hline
\text{Lat. in} = & 40° 37' \text{ N. Ans.}
\end{array}$$

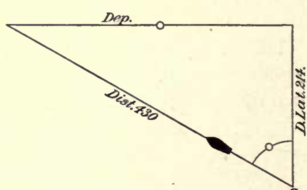

Fig. 5

Solution by Inspection. See Fig. 5. Find in the columns of the Traverse Tables the Dep. corresponding to one-half the Dist. 215, and one-half the D. Lat. 107. Multiplying this Dep. 186.2′ by 2, the product will be the whole Dep., or 372.4 mi., nearly. Since the column

where the D. Lat. is found is marked *Lat.* at the bottom, the course is to be taken from the bottom of the table also, which in this case is 60°, or N 60° W. Ans.

12. *Distance and departure given, to find the course and the difference of latitude.*

EXAMPLE.—Suppose a ship sails 200 miles between south and east from latitude 21° 32′ S, until her departure is 35 miles. What course did she steer, and what is her difference of latitude and latitude in?

SOLUTION.—In this case, the course and D. Lat. are sought; therefore, proceed as follows:

$$\sin C = \text{Dep.} \div \text{Dist.}$$
$$\log 35\ (+10) = 11.54407$$
$$\log 200 = \underline{\ \ 2.30103}$$
$$\log \sin C = \ \ 9.24304$$
$$\text{Course} = \text{S } 10°\ 5'\ \text{E.} \quad \text{Ans.}$$

$$\text{D. Lat.} = \text{Dist.} \times \cos C$$
$$\log 200 = 2.30103$$
$$\log \cos 10°\ 5' = \underline{9.99324}$$
$$\log \text{D. Lat.} = 2.29427$$
$$\text{D. Lat.} = 196.9'\ \text{S.} \quad \text{Ans.}$$

$$\text{Lat. from} = 21°\ \ 32'\ \text{S}$$
$$\text{D. Lat.} = \underline{\ \ 3°\ 16.9'\ \text{S}}$$
$$\text{Lat. in} = 24°\ 48.9'\ \text{S.} \quad \text{Ans.}$$

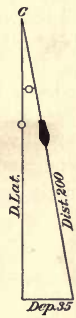

FIG. 6

SOLUTION BY INSPECTION.—See Fig. 6. Seeking in the Traverse Tables, opposite *Dep.* 35′ and *Dist.* 200′, a D. Lat. of 197′ is found; the corresponding course is 10°, or S 10° E. Ans.

13. *Difference of latitude and departure given, to find the course and the distance.*

EXAMPLE.—From latitude 27° 15′ S, a ship sails to latitude 24° 39′ S, making 79.5 miles of easterly departure. Required, the course steered and the distance made by the ship.

SOLUTION.—In this case, the course and Dist. are sought; hence, the following formulas are used:

$$\text{Lat. from} = 27°\ 15'\ \text{S}$$
$$\text{Lat. in} = \underline{24°\ 39'\ \text{S}}$$
$$\text{D. Lat.} = \ \ 2°\ 36',\ \text{or } 156'\ \text{N}$$

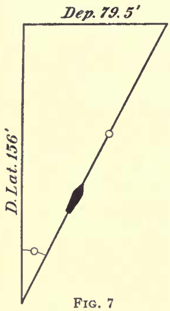

FIG. 7

$$\tan C = \text{Dep.} \div \text{D. Lat.}$$
$$\log 79.5\ (+10) = 11.90037$$
$$\log 156 = 2.19312$$
$$\log \tan C = 9.70725$$
Course = N 27° E. Ans.

$$\text{Dist.} = \text{Dep.} \times \text{cosec } C$$
$$\log 79.5 = 1.90037$$
$$\log \text{cosec } 27° = 10.34295$$
$$\log \text{Dist.} = 2.24332$$
Dist. = 175.1 mi. Ans.

SOLUTION BY INSPECTION.—See Fig. 7. The nearest D. Lat. and Dep. in the Traverse Tables are 155.9 and 79.4; the corresponding course and the distance, therefore, are N 27° E and 175 mi., respectively. Ans.

14. The method of ascertaining a ship's position by plane sailing is sufficiently accurate in low latitudes, but only partially so in higher latitudes, unless the distance sailed is small and the course made good lies along the meridian or nearly so.

EXAMPLES FOR PRACTICE

1. A ship sails from latitude 37° 30′ N to latitude 46° 8′ N, steering a course N E by N. Required, the distance run and the departure made.
Ans. { Dist. = 623 mi.; Dep. = 346.1 mi. E

2. A ship from latitude 50° 13′ N sails between S and E 98 miles until her departure is 82 miles. Find the course steered and the latitude in.
Ans. { Course = S 56° 48′ E; Lat. in. = 49° 19′ N

3. A ship from St. Helena, in latitude 15° 55′ S, sails S S E $\frac{1}{2}$ E until she has made 115 miles of departure. Find her latitude in and the distance run.
Ans. { Lat. in = 19° 30° S; Dist. = 244 mi.

4. A ship from latitude 55° 30′ N steers a course S W by S during 20 hours and then finds by observation that her latitude in is 53° 17′ N. (*a*) How many knots did she make per hour, and (*b*) what is the departure made?
Ans. { (*a*) 8 knots per hr.; (*b*) Dep. = 88.9 mi.

5. From Cape Hatteras, in latitude 35° 15′ N, a ship sails between N and E 226 miles until her departure is 198 miles. Find her course and latitude in.
Ans. { Course = N 61° 11′ E; Lat. in = 37° 4′ N

6. A ship runs S E by E from latitude $1° 45'$ N, and then by observation is in latitude $31'$ S. Required, her distance and departure.

$$\text{Ans.}\begin{cases}\text{Dist.} = 244.8 \text{ mi.}\\ \text{Dep.} = 203.5 \text{ mi.}\end{cases}$$

TRAVERSE SAILING

15. In the preceding examples, it will be noticed that only a *single* course has been considered. But, since a vessel may change her course several times during a trip, or at intervals of a few hours, it becomes necessary to have a method, or process, by which the several courses and distances run for a certain interval of time may be reduced to a single course and distance that the vessel would have made had she gone directly from the place left to the place arrived at. This method, or process, known among seamen as **working a traverse,** is explained in the articles that follow.

16. Explanation of Method.—Assume that a schooner is about 6 miles from Cape Canaveral, or at A in Fig. 8, and that she sails from this place to B, from B to C, from C to D, and from D to E, being bound northwards. By laying out the difference of latitude and departure for each course run, a number of small triangles are formed; and by connecting A and E by means of a straight line and erecting the perpendiculars Ag and Eg, a large right triangle AEg is formed, in which the side Ag represents the total difference of latitude, and the side gE the total departure corresponding to the total distance AE and course gAE made good by the vessel. Now, by examining these triangles it will be found that the total difference of latitude Ag and the total departure gE are, respectively, the algebraic sum of the several differences of latitude and departures made on each separate course run by the ship. Thus, the total difference of latitude Ag is equal to the sum of $Ab + Bc + Cd + De$. Inserting the numerical values of these, the total D. Lat. $= 34.7 + 33 + 28 + 10 = 105.7$ miles. The same is true of the departures. If the difference between easterly and

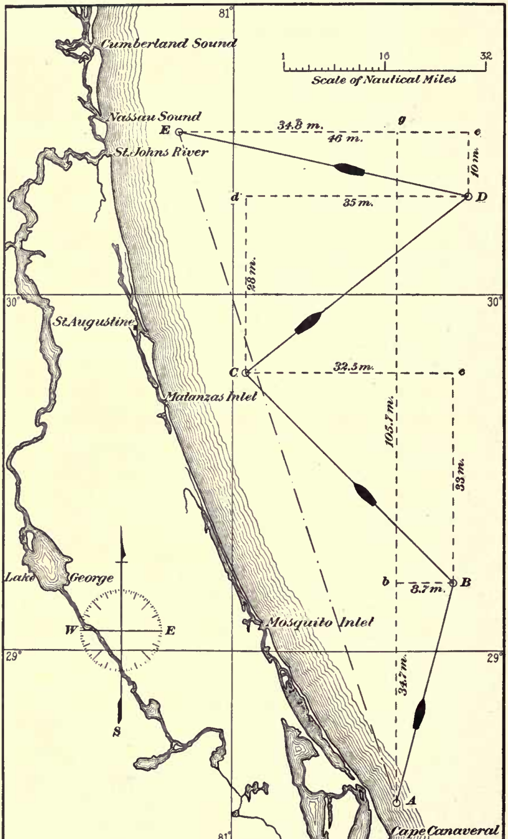
Cumberland Sound
Scale of Nautical Miles
Nassau Sound
St. Johns River
St. Augustine
Matanzas Inlet
Lake George
Mosquito Inlet
Cape Canaveral
34.8 m.
46 m.
10 m.
35 m.
28 m.
32.5 m.
105.7 m.
33 m.
8.7 m.
34.7 m.

Fig. 8

westerly departures is taken, the result will be the total departure made good. Thus,

$$\begin{array}{rlrl} bB = & 8.7 \text{ mi. E} & cC = & 32.5 \text{ mi. W} \\ dD = & 35. \text{ mi. E} & eE = & 46. \text{ mi. W} \\ \hline \text{Dep.} = & 43.7 \text{ mi. E} & \text{Dep.} = & 78.5 \text{ mi. W} \\ & & & 43.7 \text{ mi. E} \\ \hline & & \text{Diff., or total Dep.} = & 34.8 \text{ mi. W} \end{array}$$

By actual measurement on Fig. 8 according to the scale in the upper part of the chart, it will be found that the total difference of latitude Ag is 105.7 miles, and the total departure gE, 34.8 miles.

17. Constructing a Traverse.—From the foregoing it is evident that in order to find the total departure and difference of latitude made good by a ship that has sailed on several courses, it is first necessary to find the departure and difference of latitude corresponding to each separate course and distance. This is conveniently found by inspection of the Traverse Tables, and the several departures and differences of latitude thus found are then entered into a table, called a **traverse**, consisting of four columns headed N, S, E, W, respectively, similar to the one shown below.

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W

The figures in each column are then added together, when the difference of the sums of the N and S columns will be the total difference of latitude made good; this is given the same name as the greater sum. Likewise, the difference between the sums of the E and W columns will be the total departure made good, and this is named the same as the greater sum. From this total difference of latitude

and departure is then calculated the course and the distance made good according to formulas of Plane Sailing, or they are found, by inspection, from the Traverse Tables.

18. Illustration of a Traverse.—A vessel has run the following courses and distances: S W by S, 24 miles; N N W, 57 miles; S E by E ½ E, 84 miles; and south, 35 miles. To find the course and distance made good, proceed as follows: Construct a traverse similar to the one shown, the first column to contain the courses; the second, the distances corresponding to each course; the third and fourth, the differences of latitude; and the fifth and sixth, the departures. The table having been constructed, enter the Traverse Tables and take out the difference of latitude and the departure for each course and distance run, and insert them in their proper columns. Thus, when the course is southerly, the difference of latitude must be entered in the column headed S; when northerly, in the column headed N. The departure when westerly must be entered in the column headed W; and when easterly, in the column headed E. For the beginner, a good idea is to make a horizontal dash in all places where no figures are to be entered, as shown in the table. All the data having been properly inserted in their respective columns, the traverse will appear thus:

Traverse

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S W by S	24	——	20.0	——	13.3
N N W	57	52.7	——	——	21.8
S E by E ½ E	84	——	39.6	74.1	——
South	35	——	35.0	——	——
		52.7	94.6	74.1	35.1
			52.7	35.1	

D. Lat. made good = 41.9′ S 39.0′ E = Dep. made good

Then add the differences of latitude in each column and subtract the smaller sum from the greater; the remainder will be the total difference of latitude made good, with the same name as the greater. Proceed similarly with the departures.

The result by the traverse in this case shows that the whole difference of latitude made good is 41.9 miles *south*, and the departure 39 miles *east*. From these data the course and distance made good are computed as follows:

$\tan C = \text{Dep.} \div \text{D. Lat.}$	$\text{Dist.} = \text{Dep.} \times \text{cosec}\, C$
$\log 39\ (+10) = 11.59106$	$\log 39 = 1.59106$
$\log 41.9 = 1.62221$	$\log \text{cosec}\, 42° 57' = 0.16662$
$\log \tan C = 9.96885$	$\log \text{Dist.} = 1.75768$
Course made good = S 42° 57′ E.	Dist. made good = 57.2 mi.

Or, the course and distance made good are found by seeking in the Traverse Tables for the place where the total D. Lat. (41.9) and Dep. (39) stand opposite each other in their respective columns. This occurs on the page of 43°, where 41.7 and 38.9 are the nearest to the given values. Hence, by inspection, the course made good is S 43° E, or nearly S E ¼ S, and the distance made good is 57 miles, which agrees with result obtained by calculation. It should be noted that the course takes its name from the difference of latitude and the departure made good during the run.

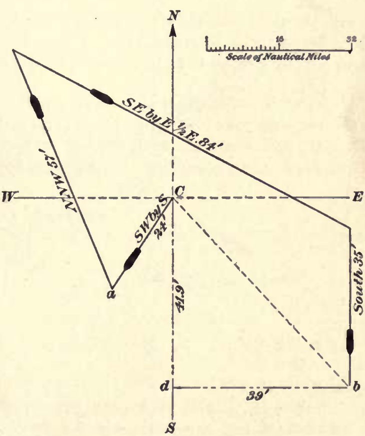

Fig. 9

19. Graphic Solution.—The preceding problem may also be solved graphically as follows: On a sheet of paper draw two lines *N S* and *W E*, Fig. 9, to represent the direction of the cardinal points. From the center *C*, lay out the first course S W by S and the distance 24 miles according to an adopted scale; then, from *a* lay off the second course N N W, 57 miles; thence S E by E ½ E, 84 miles; and, finally, the last course south, 35 miles. The point *b* is then the place at which the ship has arrived. Connect *b* with *C*; also draw *d b* perpendicular to *N S*. *C d* is then the difference of latitude; *d b*, the departure; *C b*, the distance; and the angle *d C b*, the course made good by the ship. By measuring the different parts of the triangle formed with the same scale used in plotting the courses, they will be found to agree exactly with results obtained by the traverse and by the use of formulas.

Example 1.—A ship has sailed the following courses: S S E ¼ E, 16 miles; E S E, 23 miles; S W by W ½ W, 36 miles; W ¾ N, 12 miles; and S E by E ¼ E, 41 miles. Find the course and distance made good.

Solution.—Construct a traverse similar to the one in the preceding example and take out from the Traverse Tables the corresponding D. Lat. and Dep. for each course. Then calculate the course and Dist. made good according to the proper formulas, thus:

Traverse

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S S E ¼ E	16	——	14.5	6.8	——
E S E	23	——	8.8	21.2	——
S W by W ½ W	36	——	17.0	——	31.7
W ¾ N	12	1.8	——	——	11.9
S E by E ¼ E	41	——	21.1	35.2	——
		1.8	61.4	63.2	43.6
			1.8	43.6	
		D. Lat. made good =	59.6′ S	19.6′ E	= Dep. made good

tan C = Dep. ÷ D. Lat.	Dist. = Dep. × cosec C
log 19.6 (+ 10) = 11.29226	log 19.6 = 1.29226
log 59.6 = 1.77525	log cosec 18° 12′ = 10.50538
log tan C = 9.51701	log Dist. = 1.79764
Course made good = S 18° 12′ E. Ans.	Dist. made good = 62.7 mi. Ans.

Solution by Inspection.—Looking in the Traverse Tables for a place where the D. Lat. and Dep. stand together in their respective columns, the course and the distance made good are found at once. In this case, the course is 18° and the distance 63 mi. Hence, the course made good by the ship is S 18° E, and the direct distance covered from the place left to the place arrived at is 63 mi.

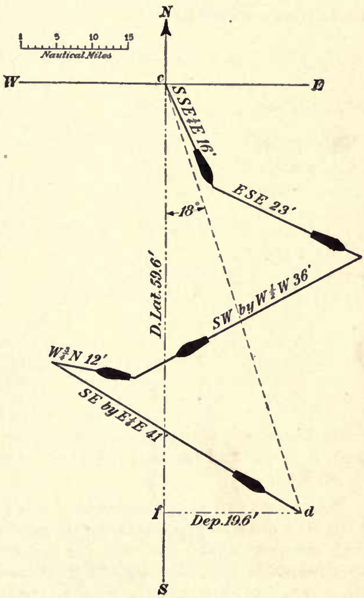

Fig. 10

Solution by Construction.—Proceed as before by drawing lines representing the direction of north, south, east, and west, Fig. 10. From their point of intersection c lay off the successive courses with their respective distances; the point d will then mark the position of the ship. Connect d with c and draw a perpendicular from d to the line NS; fd is then the Dep; fc, the D. Lat.; and cd, the Dist. made good.

It should be noted that, if the courses and distances are accurately plotted according to the scale used, the results should agree exactly with those found by the traverse. In this case, therefore, the length of cf, or the D. Lat. made good, should be exactly 59.6 mi., the Dep. fd should be 19.6 mi., and the Dist. cd 63 mi. If they do not agree, some error must have been made in the plotting. Any scale can be used that is suitable to the size of the paper on which the construction is made.

It is evident that, when the courses are plotted directly on the chart, the scale of that chart must be used.

Example 2.—From the Frying-Pan Shoals Lightship, which is moored in latitude 33° 34′ N and longitude 77° 49′ W, a ship sails the following courses: S W by W, 45 miles; E S E, 50 miles; S W, 30 miles; S E by E, 60 miles; and S W $\frac{3}{4}$ S, 63 miles. Required, her latitude in, the departure, and the course and distance made good.

Solution.—Arrange a traverse as in the preceding examples and find the corresponding D. Lat. and Dep.; thus,

Traverse

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S W by W	45	——	25.0	——	37.4
E S E	50	——	19.1	46.2	——
S W	30	——	21.2	——	21.2
S E by E	60	——	33.3	49.9	——
S W $\frac{3}{4}$ S	63	——	50.6	——	37.5
	D. Lat. made good = 149.2′ S			96.1	96.1
				96.1	
	Dep. made good =			0	

Lat. from = 33° 34′ N
D. Lat. = 2° 29.2′ S
Lat. in = 31° 4.8′ N. Ans.

In this case, the Dep. made good is zero; hence, the course made good is south, and the total Dist. made good is equal to the D. Lat., or 149.2 mi.

Example 3.—A ship in latitude 1° S is bound to a port in latitude 1° 10′ N, a distance of 220 miles in the northwest quarter; but, meeting with contrary winds, she runs the following courses: N E by N, 63 miles; N W $\frac{1}{4}$ W, 85 miles; north, 96 miles; and N N W, 87 miles. Required, the latitude of the place arrived at, and the course and the distance from there to the port of destination.

Solution.—To begin with, find the D. Lat. eg, Fig. 11, between the two places, which, in this case, is equal to the sum of their latitudes, or 60 + 70 = 130 mi. Then compute the Dep. gc according to formulas of Plane Sailing. The result will be 177.5 mi. Next find the Dep. df and the D. Lat. cf made good by the ship, either by arranging a traverse or by laying off from the starting point c the courses and distances sailed.

TRAVERSE

Courses	Dist.	D. Lat. N	D. Lat. S	Dep. E	Dep. W
N E by N	63	52.4	——	35.0	——
N W $\frac{1}{4}$ W	85	57.1	——	——	63.0
North	96	96.0	——	——	——
N N W	87	80.4	——	——	33.3

D. Lat. made good = 285.9′ N   35.0   96.3

Lat. from = 1° 0′ S   35.0

D. Lat. = 4° 45.9′ N   Dep. made good = 61.3′ W

Lat. in = 3° 45.9′ N = Lat. of the point *d*. Ans.

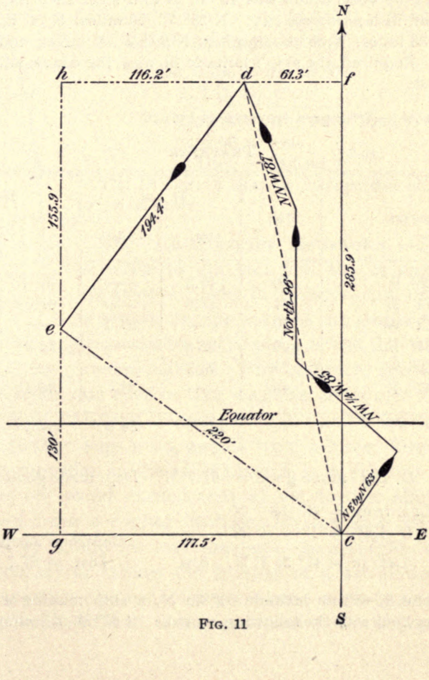

FIG. 11

To find what course the ship must steer and what distance she must run in order to reach her destination *e*, lay off from *d* on a line parallel to *W E* 116.2 mi., the difference between the departures *g c* and *d f*. This will be equal to the departure between *d* and *e*. Then from *h*, on a line perpendicular to *h d*, lay off 155.9 mi., which is the difference between the two differences of latitude *c f* and *g e*. Having found the Dep. and D. Lat., compute the course and the Dist. to be run in order to reach *e*. Thus,

$$\tan C = \text{Dep.} \div \text{D. Lat.} \qquad\qquad \text{Dist.} = \text{Dep.} \times \operatorname{cosec} C$$

$$\begin{array}{rl} \log 116.2(+10) = & 12.06521 \\ \log 155.9 = & \underline{\ 2.19285} \\ \log \tan C = & 9.87236 \end{array} \qquad \begin{array}{rl} \log 116.2 = & 2.06521 \\ \log \operatorname{cosec} 36^\circ 42' = & \underline{10.22357} \\ \log \text{Dist.} = & 2.28878 \end{array}$$

Course to steer = S 36°42′ W. Ans. Dist. to run = 194.4 mi. Ans.

EXAMPLE 4.—At noon on a certain day, the latitude of a ship determined by observation was 46° 18′ N and since then the following courses and distances were run: N 25° W, 50 miles; N 74° E, 64 miles; S 52° W, 36 miles; N 35° E, 40 miles; N 69° W, 75 miles; and S 47° E, 48 miles. Required, the ship's latitude in, also, the course and distance made good.

SOLUTION.—Arrange a traverse as before.

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N 25° W	50	45.3	——	——	21.1
N 74° E	64	17.6	——	61.5	——
S 52° W	36	——	22.2	——	28.4
N 35° E	40	32.8	——	22.9	——
N 69° W	75	26.9	——	——	70.0
S 47° E	48	——	32.7	35.1	——
		122.6	54.9	119.5	119.5
		54.9			119.5

D. Lat. made good = 67.7′ N Dep. made good = 0

$$\begin{array}{rl} \text{Lat. from} = & 46^\circ\ 18'\ \text{N} \\ \text{D. Lat.} = & \underline{\ 1^\circ\ \ 7.7'\ \text{N}} \\ \text{Lat. in} = & 47^\circ\ 25.7'\ \text{N. Ans.} \end{array} \qquad \left.\begin{array}{l} \text{Course} = \text{north} \\ \text{Dist.} = 67.7 \text{ mi.} \end{array}\right\}\text{Ans.}$$

EXAMPLE 5.—From latitude 51° 30′ N, a ship making a speed of 8 knots per hour sails the following courses: S 67° W, 3 hours; N 45° W,

$2\frac{1}{2}$ hours; W, 4 hours; S 33° W, $2\frac{1}{4}$ hours, and N W $\frac{1}{2}$ W, 2 hours. Required, the course and distance made good and, also, the latitude in.

SOLUTION.—

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S 67° W	24	——	9.4	——	22.1
N 45° W	20	14.1	——	——	14.1
West	32	——	——	——	32.0
S 33° W	18	——	15.1	——	9.8
N W $\frac{1}{2}$ W	16	10.2	——	——	12.4

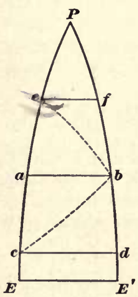

FIG. 12

24.5 Dep. made
24.3 good = 90.4′ W

D. Lat. made good = 0.2′ S

Lat. from = 51° 30′ N
D. Lat. = 0° 0′

Lat. in = 51° 30′ N. Ans.

Course = west
Dist. = 90.4 mi. } Ans.

The D. Lat. .2′ may be disregarded without any practical error.

20. Cautionary Remarks.—Attention is called to the fact that the balance of the departures made in a succession of courses is not strictly the same as the single departure made in the single course from the place left to that ultimately arrived at by traverse sailing. For instance, if a ship sails in any latitude due east or due west, as from *a* to *b* or from *b* to *a*, Fig. 12, the total distance *a b* will also be the departure. But, if another ship were to sail from a lower latitude *c* on the same meridian to the same place *b*, it is obvious that her departure *c d* would exceed that of the former ship; and if she sailed from a higher latitude, for instance from *e*, her departure *e f* would be less than *a b*. In a single day's run, the inaccuracy of taking the balance of a set of departures as the departure due to the single course made good is, however, too small to lead to any practical error of importance.

21. Courses Should Be True.—Hitherto, in the examples given, the courses have been true; in actual practice, however, the courses are *compass courses*, and must be corrected for leeway, deviation, and variation. In cases where the distances sailed are small and the variation the same for all courses, the magnetic courses, corrected for leeway, may be used in entering the Traverse Tables, and the result will be the magnetic course made good, but this must then be corrected for variation. *To avoid mistakes, however, each course should be reduced to true before it is entered in the traverse.* This should be made an invariable rule.

EXAMPLES FOR PRACTICE

1. A ship runs as follows: N ½ E, 75 miles; E ¾ S, 66 miles; S W by W ½ W, 115 miles; S by E ¾ E, 98 miles; W N W, 47 miles. Required, the course and distance made good.

Ans. { Course = S 31° 35′ W; Dist. = 74.6 mi.

2. A ship in latitude 34° 22′ S sails S W ½ S, 27 miles; S E by E, 45 miles; S W by S, 48 miles; W, 32 miles; and S S W ½ W, 18 miles. Find the course and distance made good and the latitude in.

Ans. { Course = S 24° 45′ W; Dist. = 112 mi.; Lat. in = 36° 4′ S

3. A cruiser doing patrol duty steams the following courses and distances: S 69° W, 4 miles; S 58° E, 15 miles; S 66° E, 8 miles; S 66° W, 12 miles; S 1° E, 6 miles; S 55° W, 2 miles; N 21° E, 2 miles; S 55° W, 4 miles; S 32° E, 14 miles; S 55° W, 28 miles. Find what course and distance the cruiser made good and, if her starting point was 44° 10′ N, the latitude in.

Ans. { Course = S 15° W; Dist. = 55 mi.; Lat. in. = 43° 17′ N

4. From latitude 4° 40′ S, a ship sails as follows: S W ¾ W, 62 miles; S by W, 16 miles; W ¼ S, 40 miles; S W ¾ W, 29 miles; S by E, 30 miles; S ¾ E, 14 miles. Required, the course and distance made good and the latitude in.

Ans. { Course = S 43° 14′ W; Dist. = 158 mi.; Lat. in = 6° 35′ S

PARALLEL SAILING

22. Explanation.—From the examples in plane sailing, it should be noticed that this method takes into account only the latitude, the difference of latitude, and the departure. How much the ship had changed her *longitude*, however, was not ascertained; in fact, longitude did not enter at all into the examples given.

As soon as the idea of longitude is introduced, the part of the earth's surface sailed over can no longer be considered as a plane nor the meridians as being parallel straight lines. The sphericity of the globe must then be taken into consideration; also the fact that the meridians of each degree of longitude are 60 minutes apart at the equator and meet at the poles. It must also be borne in mind that the latitude parallels, being small circles parallel to the equator, are none of them equal in circumference, radius, or area to the equator. Hence, the *length* of an arc of a latitude parallel at some distance from the equator, although intercepted by the same meridians, is not the same as its corresponding arc on the equator. This goes to show that it is only on the equator, or rather in a region bounded approximately by parallels 5° N and S, that a ship sailing 60 miles true east or west will alter her longitude just 1°.

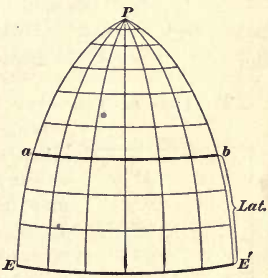

FIG. 13

Sailing along a latitude parallel from a to b, Fig. 13, or from b to a, that is, true east or west, the distance run $a\,b$ is equal to the departure made good, but in length, this distance, or departure, is not the same as the difference of longitude EE'. Therefore, in order to find the difference of longitude made when sailing east or west on any of the parallels, the relation existing between the two arcs $a\,b$ and EE' must be known.

23. Relation Between Latitude, Departure, and Difference of Longitude.—The relation existing between the latitude $E' b$, Fig. 13, of any parallel, the departure along that parallel, and the corresponding difference of longitude may be expressed as follows:

$$a\,b : E E' = \cos E' b : 1$$

or, *an arc of a parallel is to its corresponding arc of the equator as the cosine of the latitude of the parallel is to 1, or unity* (the radius of the earth being taken as the unit). From this proportion it is found that

$$E E' = \frac{a\,b}{\cos E' b},$$

$$\text{or} \quad \text{D. Long.} = \frac{\text{Dep.}}{\cos \text{Lat.}} = \text{Dep.} \times \sec \text{Lat.}; \qquad (1)$$

$$\text{and} \quad \text{Dep. or Dist.} = \text{D. Long.} \times \cos \text{Lat.}; \qquad (2)$$

$$\text{and} \quad \cos \text{Lat.} = \text{Dep.} \div \text{D. Long.} \qquad (3)$$

24. Use of Traverse Tables in Solution of Problems.—The preceding formulas solve all questions relating to parallel sailing, and as in plane sailing the data involved may be embodied in a right triangle. Thus, in Fig. 14, the side $A\,C$ may represent the distance, or departure, sailed; the hypotenuse $B\,C$, the difference of longitude (in linear measure); and the angle $A\,C\,B$, included between $A\,C$ and $B\,C$, the latitude of the parallel.

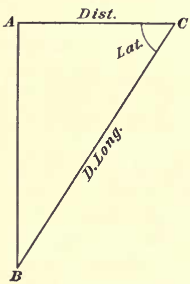

FIG. 14

By this arrangement it is evident that any problem in parallel sailing, like problems in plane sailing, can be solved by consulting the Traverse Tables, the stipulation now being to enter the tables with the latitude of the parallel as *course* and the distance run as *difference of latitude*, when the corresponding *distance* found in the tables will be the required difference of longitude. The perpendicular $A\,B$ in the triangle has no significance; it serves merely to connect the other parts.

25. *To find the difference of longitude when the distance between two places on the same parallel is given.*

Example.—A ship from latitude 68° 11′ N and longitude 12° 15′ W sails due east 143 miles. Find her longitude in.

Solution.—In this case the D. Long. is sought; therefore, formula **1**, Art. **23**, is used. Thus,

$$
\begin{aligned}
\text{D. Long.} &= \text{Dep.} \times \text{sec Lat.} \\
\log 143 &= 2.15534 \\
\log \sec 68°\ 11' &= \underline{10.42988} \\
\log \text{D. Long.} &= 2.58522 \\
\text{D. Long.} &= 384.8' = 6°\ 24.8'\ \text{E} \\
\text{Long. from} &= 12°\ 15'\ \text{W} \\
\text{D. Long. } 384.8' &= \underline{6°\ 24.8'\ \text{E}} \\
\text{Long. in} &= 5°\ 50.2'\ \text{W. Ans.}
\end{aligned}
$$

Solution by Inspection.—Entering the Traverse Tables with 68° as course and with the distance 143 in a latitude column, the corresponding D. Long. is found in the distance column. Thus,

corresponding to 112.4 is 300

and corresponding to 30.6 is 82

hence, corresponding to 143.0 is 382′ = 6° 22′ E = D. Long.

Long. from = 12° 15′ W

Whence, Long. in = 5° 53′ W. Ans.

Note.—If the latitude of the parallel is very near a half degree, as 42° 31′ N, a more accurate result may be obtained by finding the values corresponding to 42° and 43°, respectively; the *mean* of these will be that corresponding to 42½° of latitude.

26. *To find the distance between two places on the same parallel when the difference of longitude is given.*

Example.—A ship in longitude 64° 30′ W and latitude 40° N sails due east to a place in longitude 47° 19′ W. Required, the distance run.

Solution.—In this case, the Dist. is required; hence, formula **2**, Art. **23**, is used.

$$
\begin{aligned}
\text{Long. from} &= 64°\ 30'\ \text{W} \\
\text{Long. in} &= \underline{47°\ 19'\ \text{W}} \\
\text{D. Long.} &= 17°\ 11' = 1{,}031'\ \text{E} \\
\text{Dist.} &= \text{D. Long.} \times \text{cos. Lat.} \\
\log 1{,}031 &= 3.01326 \\
\log \cos 40° &= \underline{9.88425} \\
\log \text{Dist.} &= 2.89751 \\
\text{Dist.} &= 789.8\ \text{mi. Ans.}
\end{aligned}
$$

Solution by Inspection.—Enter the Traverse Tables with the latitude (40°) as course and the D. Long. as distance, when the required Dist. is found in the corresponding latitude column. Thus,

	corresponding to	300 is 229.8
	corresponding to	300 is 229.8
	corresponding to	300 is 229.8
and	corresponding to	131 is 100.4

Whence, corresponding to the sum 1,031 is 789.8 mi. Dist. Ans.

27. *When the difference of longitude and the distance between two places on the same parallel are given, to find the latitude of that parallel.*

Example.—A ship from longitude 15° 30′ W sails due west 156 miles and then finds that she is in longitude 20° 44′ W. What is the latitude of the parallel on which she sailed?

Solution.—In this case, the latitude of the parallel being sought, formula **3**, Art. **23**, is used.

$$\begin{aligned}\text{Long. from} &= 15^\circ\ 30'\ \text{W}\\ \text{Long. in.} &= 20^\circ\ 44'\ \text{W}\\ \hline \text{D. Long.} &= \ \ 5^\circ\ 14' = 314'\ \text{W}\end{aligned}$$

$$\begin{aligned}\cos \text{Lat.} &= \text{Dep.} \div \text{D. Long.}\\ \log 156\ (+10) &= 12.19312\\ \log 314 &= \ \ 2.49693\\ \hline \log \cos \text{Lat.} &= \ \ 9.69619\\ \text{Lat. of parallel} &= 60^\circ\ 13'.\quad \text{Ans.}\end{aligned}$$

Solution by Inspection.—Seek in the Traverse Tables until half the D. Long. and Dist. (157 and 78) are found opposite each other in the distance and latitude columns, respectively, when the corresponding number of degrees (60°) at the bottom of the table will be the required latitude of the parallel. Ans.

28. Utility of Parallel Sailing.—The method of finding the latitude by parallel sailing, as shown in the last example, gives only an approximate value of the latitude, and its employment should therefore be very limited. In actual practice, this method is very seldom, if ever, used.

Before the general adoption of the chronometer as a means of determining the longitude, parallel sailing was particularly useful in making small islands and other ports, in which case it was customary to get into the desired latitude and then steer a true east or west course.

In very low latitudes (5° or below), when the distance does not exceed 300 miles, it may, without any practical error, be considered as the difference of longitude.

EXAMPLES FOR PRACTICE

1. A ship in latitude 32 N is bound to a port in the same latitude but situated 6° 24′ of longitude to the east. What distance must she run in order to reach her destination? Ans. 325.6 mi.

2. A ship in latitude 32° 22′ N and longitude 52° 20′ W sails true west 365 miles. Required, her distance from the islands of Bermuda in the same latitude but in longitude 64° 43′ W. Ans. 262.6 mi.

3. From latitude 57° 32′ N, a ship sails east (true) 560 miles. Supposing that her longitude left was 13° 5′ W, what is her longitude in? Ans. 4° 18′ E

4. From a point situated in latitude 52° 10′ N and longitude 0° 29′ W, a ship steers due east 492.5 miles. Required, (*a*) her latitude and (*b*) her longitude in. Ans. $\begin{cases} (a) \text{ 52° 10′ N} \\ (b) \text{ 12° 54′ E} \end{cases}$

5. From latitude 34° 57′ S, a ship sails due east 981 miles. Her longitude in is 59° 44′ E. Required, the longitude of her starting point. Ans. 39° 47′ E

6. Find (*a*) what course to steer from a place in latitude 12° 17′ S and longitude 34° 29′ E to another in latitude 12° 17′ S and longitude 79° 16′ E, and also (*b*) the number of miles to be run. Ans. $\begin{cases} (a) \text{ East} \\ (b) \text{ Dist.} = 2{,}625.5 \text{ mi.} \end{cases}$

7. From a place in latitude 35° 30′ N and longitude 6° 15′ W, a ship sails as follows: West, 250 miles; north, 525 miles; east, 250 miles. What is (*a*) her latitude and (*b*) her longitude in? Ans. $\begin{cases} (a) \text{ 44° 15′ N} \\ (b) \text{ 5° 33′ W} \end{cases}$

MIDDLE-LATITUDE SAILING

29. In the preceding articles, it has been shown how the difference of longitude may be determined when sailing along a parallel; now, the more general problem of how to determine the difference of longitude when sailing on any other course than true east or west will be considered. For the solution of this problem, navigation by dead reckoning offers two distinct methods; namely, *middle-latitude sailing* and *Mercator's sailing*. The former will be discussed first.

30. Explanation.—**Middle-latitude sailing** is in reality a combination of plane and parallel sailing, which is evident from what follows. When a ship sails from A to B, Fig. 15, it will be noticed that the departure $mo + dB$ is less than Af, the distance between the meridians in the latitude of A, and greater than the distance CB in the latitude of B. There must therefore be some latitude between A and B at which the distance between the meridians is equal to the departure. It is assumed that this is the mean of the latitudes between A and B; in other words, that the departure is equal to the length mn just midway between A and B. This latitude mn is therefore called the *middle latitude*, and is usually denoted by *M. Lat.* Assuming, then, that mn is equal to the departure made by a ship sailing from A to B, the difference of longitude EE' is readily found either by calculation or by inspection of the Traverse Tables, as shown in the articles that follow.

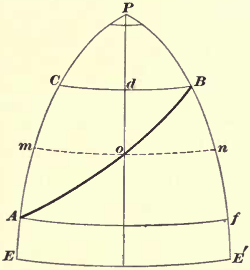

FIG. 15

31. Application of Traverse Tables.—In plane sailing it was shown that the different data considered in connection with that method could be conveniently represented in a right triangle, as abc, Fig. 1, or ABC, Fig. 16. Now, by assuming that the departure is equal to the length of the middle-latitude

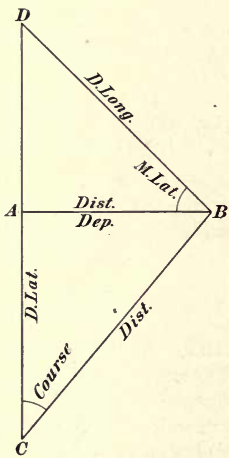

FIG. 16

parallel, the side $A B$, Fig. 16, representing the departure, will in this case be equivalent to the distance sailed on the middle-latitude parallel. If, therefore, another triangle $A B D$ is constructed, containing the side $A B$, a case of parallel sailing is established, where $A B$ represents the distance; the angle $A B D$, the latitude of the parallel (the middle latitude); and the hypotenuse $B D$ the difference of longitude. (Compare with Fig. 14.) The triangle $D B C$ therefore contains all elements of middle-latitude sailing; the lower triangle $A B C$ is constructed on the principles of plane sailing, and the upper one on those of parallel sailing; and what is departure in the *lower* triangle is regarded as distance made good, along the middle-latitude parallel, in the *upper* triangle. For this reason, it is evident that problems in middle-latitude sailing can be solved also by the use of Traverse Tables, which will be shown as occasion requires in later paragraphs.

32. Derivation of Formulas.—According to formula **1**, Art. **23**, parallel sailing,

$$\text{D. Long.} = \text{Dep.} \times \sec \text{Lat.}$$

Or, in this case (triangle $A B D$, Fig. 16),

$$\text{D. Long.} = \text{Dep.} \times \sec \text{M. Lat.} \qquad (1)$$

But, according to plane sailing (triangle $A B C$, Fig. 16),

$$\text{Dep.} = \text{Dist.} \times \sin C = \text{D. Lat.} \times \tan C$$

Substituting these values in formula **1**, gives, respectively,

$$\text{D. Long.} = \text{Dist.} \times \sin C \times \sec \text{M. Lat.} \qquad (2)$$

and $$\text{D. Long.} = \text{D. Lat.} \times \tan C \times \sec \text{M. Lat.} \qquad (3)$$

From these equations Table II is formed. This table contains all formulas necessary for solving the various cases of middle-latitude sailing.

33. How to Find the Middle Latitude.—The **middle latitude** of any two places is equal to half the sum of the two latitudes when the places considered are on the *same* side of the equator. Thus, if

	Lat. from =	42° 40′ N
and	Lat. in =	36° 36′ N
then	the sum =	79° 16′
	divided by 2 =	39° 38′ N = M. Lat.

TABLE II

FORMULAS RELATING TO MIDDLE-LATITUDE SAILING

Given	Required	Formulas to be Used
Both latitudes and longitudes	Dep. Course Dist.	Dep. = D. Long. × cos M. Lat. Tan C = Dep. ÷ D. Lat. Tan C = cos. M. Lat. × D. Long. ÷ D. Lat. Dist. = D. Lat. × sec C Dist. = Dep. × cosec C
Both latitudes and departure	Course Dist. D. Long.	Tan C = Dep. ÷ D. Lat. Dist. = Dep. × cosec C D. Long. = Dep. × sec M. Lat.
One latitude, course, and distance	D. Lat. Dep. D. Long.	D. Lat. = Dist. × cos C Dep. = Dist. × sin C D. Long. = Dep. × sec M. Lat. D. Long. = D. Lat. × tan C × sec M.Lat.
Both latitudes and course	Dep. Dist. D. Long.	Dep. = D. Lat. × tan C Dist. = D. Lat. × sec C D. Long. = Dep. × sec M. Lat. D. Long. = D. Lat. × tan C × sec M. Lat.
Both latitudes and distance	Course Dep. D. Long.	Cos C = D. Lat. ÷ Dist. Dep. = Dist. × sin C D. Long. = Dep. × sec M. Lat.
One latitude, course, and departure	D. Lat. Dist. D. Long.	D. Lat. = Dep. × cot C Dist. = Dep. × cosec C D. Long. = Dep. × sec M. Lat.
One latitude, distance, and departure	Course D. Lat. D. Long.	Sin C = Dep. ÷ Dist. D. Lat. = Dist. × cos C D. Long. = Dep. × sec M. Lat.

When the latitudes have different names, subtract the smaller from the greater; the remainder divided by 2 is the middle latitude. Thus, if

	Lat. from	$= 8° \; 30' \text{ N}$
and	Lat. in	$= 2° \; \; 0' \text{ S}$
then	the difference	$= 6° \; 30'$
	divided by 2	$= 3° \; 15' \text{ N} = \text{M. Lat.}$

34. In very low latitudes or in a region bounded by parallels 5° N and S, the difference of longitude is very nearly equal to the departure made by the ship. Therefore, when navigating in this locality, there will be no occasion for using the middle-latitude method nor for finding the middle latitude, since in such a case, the difference of longitude made good may be considered as the departure, and the formulas of plane sailing can be used to obtain the desired result, provided the distance sailed does not exceed 400 miles.

35. Classification of Cases.—In the actual practice of deep-sea navigation, there are, broadly speaking, only two cases in which the middle-latitude method is made use of; they are as follows:

1. *When the latitude and the longitude of two places are known, to find the course, distance, and departure from one place to the other.*

2. *When the latitude and the longitude of the place sailed from, together with the course and the distance run, are known, to find the latitude and longitude of the place arrived at.*

Of these cases, the first may or may not be applied in practice, because it is easier and more convenient to find the course and distance between two places directly from the chart than to calculate or search for them in the Traverse Tables.

36. *To find the course and distance between any two places, the latitude and longitude of each being known.*

Example 1.—A ship in latitude 37° 3′ N and longitude 9° 1′ W is bound for a port situated in latitude 32° 38′ N and longitude 16° 56′ W. Required, the course to steer and the number of miles to be covered.

SOLUTION.—

Lat. from = 37° 3′ N
Lat. in = 32° 38′ N
D. Lat. = 4° 25′ = 265′ S
Sum of Lats. = 69° 41′
½ sum, or M. Lat. = 34° 50′ N

Long. from = 9° 1′ W
Long. in = 16° 56′ W
D. Long. = 7° 55′ = 475′ W

$$\tan C = \cos \text{M. Lat.} \times \text{D. Long.} \div \text{D. Lat.}$$

$$\begin{aligned}
\log \cos 34^\circ 50' &= 9.91425 \\
\log 475 &= 2.67669 \\
\text{a. c. } \log 265 &= 7.57675 \\
\hline
\log \tan C &= 10.16769 \\
\text{Course} &= \text{S } 55^\circ 48' \text{ W. Ans.}
\end{aligned}$$

$$\text{Dist.} = \text{D. Lat.} \times \sec C$$

$$\begin{aligned}
\log 265 &= 2.42325 \\
\log \sec 55^\circ 48' &= 10.25020 \\
\hline
\log \text{Dist.} &= 2.67345 \\
\text{Dist.} &= 471.5 \text{ mi. Ans.}
\end{aligned}$$

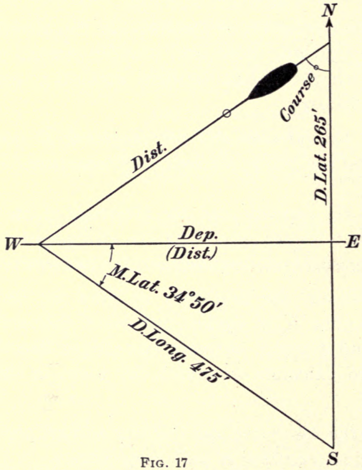

FIG. 17

SOLUTION BY INSPECTION.—Enter the Traverse Tables with M. Lat. 34° 50′ (or 35°, nearly) as course (see Fig. 17) and D. Long. 475′ in the distance column, when the Dep. will be found in a latitude column.

Thus,	corresponding to 300 is 245.7	
and	corresponding to 175 is 143.4	
Whence,	corresponding to 475 is 389.1	= departure

Now, having found the side *WE*, or Dep., enter the tables again with 132.5 (half D. Lat.) and 194.5 (half Dep.) in a latitude and a departure column, respectively, and find the corresponding course and distance. The course thus found is nearly 56°, or 5 points, and half the distance is 235, which, when multiplied by 2, gives the distance as 470 mi. Ans.

EXAMPLE 2.—Find the course and distance from Pensacola, Fla., latitude 30° 20.8′ N and longitude 87° 18.5′ W, to Havana, Cuba, latitude 23° 9.3′ N, and longitude 82° 21.5′ W.

SOLUTION.—

Pensacola, Lat. =	30° 20.8′ N	Long. =	87° 18.5′ W
Havana, Lat. =	23° 9.3′ N	Long. =	82° 21.5′ W
D. Lat. =	7° 11.5′	D. Long. =	4° 57′
Or =	431.5′ S	Or =	297′ E
Sum of Lats. =	53° 30.1′		
$\frac{1}{2}$ sum M. Lat. =	26° 45′ N		

$$\tan C = \cos \text{M. Lat.} \times \text{D. Long.} \div \text{D. Lat.}$$

$$\begin{aligned} \log \cos 26^\circ 45' &= 9.95084 \\ \log 297 &= 2.47276 \\ \text{a. c.} \log 431.5 &= 7.36502 \\ \hline \log \tan C &= 9.78862 \\ \text{Course} &= \text{S } 31^\circ 35' \text{ E. Ans.} \end{aligned}$$

$$\text{Dist.} = \text{D. Lat.} \times \sec C$$

$$\begin{aligned} \log 431.5 &= 2.63498 \\ \log \sec 31^\circ 35' &= 10.06962 \\ \hline \log \text{Dist.} &= 2.70460 \\ \text{Dist.} &= 506.5 \text{ mi. Ans.} \end{aligned}$$

SOLUTION BY INSPECTION.—Enter the Traverse Tables with M. Lat. 27° as course and the D. Long. 297 as distance, and find the Dep. in the latitude column. In this case, it is 264.6, or 265 nearly. The D. Lat. and Dep. being, respectively, 432 and 265, the tables are searched for a place where these are found together. The numbers, however, are too large, and both are therefore divided by 10, which gives 43.2 and 26.5. This division does not in any way disturb the relative proportion between the original numbers; it is done simply to facilitate the use of the Traverse Tables. Looking over the tables, on the page for 32°, the numbers 43.3 and 27 are found to agree nearest with those given. Hence, the course from Pensacola to Havana is

S 32° E. The distance opposite these numbers is 51, but since the D. Lat. and the Dep. used were one-tenth of the actual values, this distance of 51 is only one-tenth of the actual distance; therefore, to get the real distance, it is multiplied by 10. Hence, the distance from Pensacola to Havana is 51 × 10 = 510 miles, nearly. Ans.

The result obtained agrees very nearly with the course and distance found by using a protractor and a parallel ruler on the chart. It is evident that the method of consulting the chart directly is preferable to that of finding the course and distance in the manner shown.

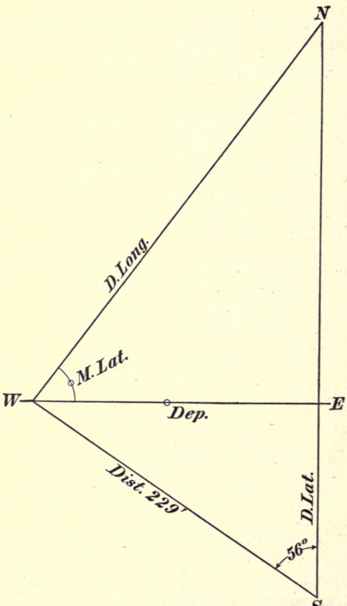

FIG. 18

37. *To find the latitude and longitude in when the latitude and longitude from and the course and distance run are known.*

EXAMPLE 1.—From latitude 52° 6′ N and longitude 35° 6′ W, a steamer runs N 56° W 229 miles. Required, her latitude and longitude in, also the departure made good.

SOLUTION.—

D. Lat. = Dist. × cos C
log 229 = 2.35984
log cos 56° = 9.74756
log D. Lat. = 2.10740
D. Lat. = 128.1′ N

Lat. from = 52° 6′ N
D. Lat. = 2° 8.1′ N
Lat. in = 54° 14.1′ N. Ans.
Sum of Lats. = 106° 20.1′
M. Lat. = 53° 10′ N

D. Long. = D. Lat. × tan C × sec M. Lat.
log 128.1 = 2.10740
log tan 56° = 10.17101
log sec 53° 10′ = 10.22222
log D. Long. = 2.50063
D. Long. = 316.7′ W

Dep. = Dist. × sin C
log 229 = 2.35984
log sin 56° = 9.91857
log Dep. = 2.27841
Dep. = 189.8 mi. Ans.

Long. from = 35° 6′ W
D. Long. = 5° 16.7′ W
Long. in = 40° 22.7′ W. Ans.

SOLUTION BY INSPECTION.—Enter Traverse Tables with course 56° and Dist. 229′ (see Fig. 18), and find the corresponding D. Lat. 128.1 and Dep. 189.8 in their respective columns. Then, with the M. Lat. as course and the Dep. just found in a latitude column enter the tables again, when the required D. Long. will be found in the distance column. Thus,

corresponding to 180.5 is 300
and corresponding to 9.3 is 15.5

Whence, corresponding to the sum 189.8 is 315.5′ = D. Long.

EXAMPLE 2.—A steamer leaves Point Loma, Cal., latitude 32° 40.2′ N and longitude 117° 14.6′ W, and runs a distance of 88 miles, steering a true course N W. Find her latitude and longitude in.

SOLUTION.—

D. Lat. = Dist. × cos C
log 88 = 1.94448
log cos 45° = 9.84949
log D. Lat. = 1.79397
D. Lat. = 62.2′ N

Lat. from = 32° 40.2′ N
D. Lat. 62.2′ = 1° 2.2′ N
Lat. in = 33° 42.4′ N. Ans.
Sum of Lats. = 66° 22.6′
M. Lat. = 33° 11′ N.

D. Long. = D. Lat. × tan C × sec M. Lat.
log 62.2 = 1.79397
log tan 45° = 10.00000
log sec 33° 11′ = 10.07731
log D. Long. = 1.87128
D. Long. = 74.3′ W

Long. from = 117° 14.6′ W
D. Long. 74.3′ = 1° 14.3′ W
Long. in = 118° 28.9′ W. Ans.

Solution by Inspection.—Enter the Traverse Tables with course and Dist. run and find the corresponding D. Lat. and Dep.; both are equal to 62.2 mi., because, in this case, the course being N W, the ship has run an equal distance north and west. The latitude in, therefore, as already shown, is 33° 42.4′ N. Ans.

Again enter the Traverse Tables, with M. Lat. as course and Dep. 62.2 in a latitude column, when opposite, in the distance column, is found the required D. Long., in this case 74′. Hence,

Long. in = 118° 28.6′ W. Ans.

Consulting the chart, this position is found to be about 4 mi. southeast off Point Vincente, San Pedro Hill, Cal.

Example 3.—A vessel steers a true N 53° E course from a place in latitude 31° 30′ N and longitude 75° 10′ W. After covering a distance of 2,107.4 miles, what is her latitude and longitude in?

Solution.—

D. Lat. = Dist. × cos C

log 2,107.4 = 3.32374
log cos 53° = 9.77946
log D. Lat. = 3.10320
D. Lat. = 1,268.2′ N

Lat. from = 31° 30′ N
D. Lat. 1,268.2′ = 21° 8.2′ N
Lat. in = 52° 38.2′ N. Ans.
Sum of Lats. = 84° 8.2′
M. Lat. = 42° 4.1′ N

D. Long. = D. Lat. × tan C × sec. M. Lat.

log 1,268.2 = 3.10320
log tan 53° = 10.12289
log sec 42° 4′ = 10.12938
log D. Long. = 3.35547
D. Long. = 2,267.1′ E

Long. from = 75° 10′ W
D. Long. = 37° 47.1′ E
Long. in = 37° 22.9′ W. Ans.

Solution by Inspection.—Enter the Traverse Tables, with the given course 53° and distance 2,107.4 mi., in the following manner:

		D. Lat.	Dep.
For course 53° and Dist:	300 mi. =	180.5	239.6
	300 mi. =	180.5	239.6
	300 mi. =	180.5	239.6
	300 mi. =	180.5	239.6
	300 mi. =	180.5	239.6
	300 mi. =	180.5	239.6
	300 mi. =	180.5	239.6
	7.4 mi. =	4.5	5.9
D. Lat. and Dep. for	2,107.4 mi. =	1,268.0	1,683.1

Lat. from = 31° 30′ N
D. Lat. 1.268′ = 21° 8′ N
Lat. in = 52° 38′ N. Ans.
M. Lat. = 42° 4′ N

To find the D. Long., enter the Traverse Tables with the M. Lat. 42° as the course, and Dep. 1,683 in a latitude column. Since the Dep. is so large, however, divide by 9, which gives 187. Then, the D. Long. corresponding to 42° and 187 is found to be 252′. Multiplying this value by 9 gives 2,268′ for the required D. Long. Whence,

Long. from = 75° 10′ W
D. Long. = 37° 48′ E
Long. in = 37° 22′ W. Ans.

It will be noticed that the results found by means of the Traverse Tables agree very closely with those found by computation. Sometimes, however, when the distance is large, the results obtained by these two methods may differ by a considerable amount (as in example 2, Art. **36**), since the tables are expressed in whole degrees only. Hence, in such cases, the results obtained by calculation are considered more reliable.

In the solution of this example, the Traverse Tables are used in such a way as to illustrate two methods of entering them when the given elements are very large.

Example 4.—A steamer leaves a place in latitude 42° 29.7′ N and longitude 59° 21′ W, and covers a distance of 30 nautical miles, steering a true course W S W ½ W. Find her latitude and longitude in.

Solution by Inspection.—In this case, the distance run being small, the Traverse Tables only are used for finding the required data. Thus, find the D. Lat. and Dep. by entering the Traverse Tables with the course and the distance run. The D. Lat. thus found is 8.7′, and the Dep. is 28.7′. Apply the former to the latitude from. Thus,

Lat. from = 42° 29.7′ N
D. Lat. = 8.7′ S
Whence, Lat. in = 42° 21′ N. Ans.

Then find the M. Lat. as usual. Thus,

Lat. from = 42° 29.7′ N
Lat. in = 42° 21.0′ N
Sum = 84° 50.7′
M. Lat. = 42° 25.3′ N

Enter the Traverse Tables again with the M. Lat. as course (selecting the nearest whole degree, in this case 42°) and the Dep. 28.7 in a latitude column, when the corresponding number in the distance column will be the D. Long. made. The nearest number to 28.7 in the latitude column is 29, and the corresponding D. Long. is therefore 39′. Apply this to the longitude from. Thus,

Long. from = 59° 21′ W
D. Long. = 39′ W
Whence, Long. in = 60° 0′ W. Ans.

38. Correction of the Middle Latitude.—The assumption that the middle latitude is half the sum of the two latitudes is not quite correct. In cases where the difference of latitude is great—over 3° or 4°—a correction should be added to the middle latitude. This correction is found in Table III, and by applying it, the middle-latitude method may be used with safety to find the ship's position even after a run of several days.

Table III is used in the following manner: Reduce the difference of latitude made good to the nearest number of

TABLE III

CORRECTION TO BE ADDED TO THE MIDDLE LATITUDE IN ORDER TO OBTAIN THE TRUE MIDDLE LATITUDE

Middle Latitude	Difference of Latitude													Middle Latitude
	3°	4°	5°	6°	7°	8°	9°	10°	12°	14°	16°	18°	20°	
15°	2′	3′	5′	7′	9′	12′	15′	18′	26′	36′	47′	59′	72′	15°
18°	1′	3′	4′	6′	8′	10′	13′	16′	23′	32′	41′	52′	64′	18°
21°	1′	2′	4′	5′	7′	9′	12′	15′	21′	29′	37′	47′	58′	21°
24°	1′	2′	3′	5′	7′	9′	11′	14′	20′	27′	35′	44′	54′	24°
30°	1′	2′	3′	5′	6′	8′	10′	13′	18′	25′	32′	41′	50′	30°
35°	1′	2′	3′	4′	6′	8′	10′	12′	18′	24′	32′	40′	49′	35°
40°	1′	2′	3′	5′	6′	8′	10′	13′	18′	25′	32′	41′	50′	40°
45°	1′	2′	3′	5′	6′	8′	11′	13′	19′	26′	34′	43′	53′	45°
50°	1′	2′	4′	5′	7′	9′	11′	14′	20′	28′	36′	46′	57′	50°
55°	1′	3′	4′	6′	8′	10′	13′	16′	22′	31′	40′	51′	63′	55°
58°	2′	3′	4′	6′	8′	11′	14′	17′	24′	33′	43′	55′	68′	58°
60°	2′	3′	4′	6′	9′	11′	14′	18′	26′	35′	46′	58′	72′	60°
62°	2′	3′	5′	7′	9′	12′	15′	19′	27′	37′	49′	62′	77′	62°
64°	2′	3′	5′	7′	10′	13′	16′	20′	29′	40′	52′	67′	83′	64°
66°	2′	4′	5′	8′	11′	14′	18′	22′	32′	43′	57′	72′	90′	66°
68°	2′	4′	6′	8′	12′	15′	19′	24′	34′	47′	62′	79′	99′	68°
70°	2′	4′	6′	9′	13′	16′	21′	26′	38′	52′	68′	88′	110′	70°
72°	3′	5′	7′	10′	14′	18′	23′	29′	42′	58′	76′	98′	124′	72°

degrees, and enter the table at the top; vertically beneath and opposite the given middle latitude will be found the required correction. Add this correction to the middle latitude; call the sum the *true middle latitude*, and use it instead

of the uncorrected middle latitude as used in the preceding examples.

39. In the example that follows, where the difference of latitude is large, the difference of longitude will be calculated with the *true* middle latitude.

EXAMPLE.—From a place in latitude 51° 18′ N and longitude 9° 50′ W, a ship sails S 33° 8′ W, 1,024 miles. Find the latitude and longitude in.

SOLUTION.—

D. Lat. = Dist. × cos C	Lat. left = 51° 18′ N
log 1,024 = 3.01030	D. Lat. = 14° 17.5′ S
log cos 33° 8′ = 9.92293	Lat. in = 37° 0.5′ N. Ans.
log D. Lat. = 2.93323	Sum of Lats. = 88° 18.5′
D. Lat. = 857.5′ S	M. Lat. = 44° 9.2′ N
	Corr. = + 26′ (Table III)
	True M. Lat. = 44° 35.2′ N

Dep. = Dist. × sin C	D. Long. = Dep. × sec M. Lat.
log 1,024 = 3.01030	log 559.7 = 2.74796
log sin 33° 8′ = 9.73766	log sec 44° 35′ = 10.14738
log Dep. = 2.74796	log D. Long. = 2.89534
Dep. = 559.7 mi. W. Ans.	D. Long. = 785.8′ W

Long. left = 9° 50′ W
D. Long. = 13° 5.8′ W

Long. in = 22° 55.8′ W. Ans.

NOTE.—If in working out this example the correction for the middle latitude had been omitted, the difference of longitude would have been 780.1, which is 785.8 − 780.1 = 5.7 miles in error.

40. Efficiency of the Middle-Latitude Method. Theoretically, the results obtained by middle-latitude sailing are not correct in high latitudes, because, since the cosines and secants of large angles change rapidly, a small error in the middle latitude will accordingly produce a larger error in the departure, difference of longitude, or course. Furthermore, the inaccuracy of this method reveals itself when the difference of latitude is great or the course small, because the greater the difference of latitude, the farther is the true middle latitude from the mean of the latitudes, which is evident from Table III. By applying the correction, however, the error in the result will be greatly minimized, though not wholly removed. Moreover, the

results by middle-latitude sailing should in most cases be determined by computation, since the method by inspection of the Traverse Tables does not admit of strict accuracy, as they are calculated to whole degrees only.

No error of any practical importance, however, is caused by the use of the ordinary method of middle-latitude sailing for an ordinary day's run, and the correction will render it safe even for 3 or 4 days' run; but, for long distances, the more accurate and vigorous method of Mercator's sailing should be used.

In the following Examples for Practice it is advisable to make use of rough sketches for each solution; this will render a clearer view and greatly assist in finding the answers.

EXAMPLES FOR PRACTICE

1. A ship in latitude 50° 15′ N and longitude 27° 19′ W is bound for a port situated in latitude 47° 30′ N and longitude 31° 14′ W. Find: (*a*) the course to steer; (*b*) the distance to be run.

Ans. $\begin{cases} (a) & \text{S } 43° \ 8' \text{ W} \\ (b) & 226.1 \text{ mi.} \end{cases}$

2. Find the distance and bearing between a ship in latitude 42° 3′ N and longitude 70° 4′ W and another ship in latitude 36° 59′ N and longitude 25° 10′ W.

Ans. $\begin{cases} \text{Dist.} = 2{,}097.5 \text{ mi.} \\ \text{Bearing} = \text{S } 81° \ 40' \text{ E} \end{cases}$

3. A ship in latitude 37° N and longitude 22° 56′ W steers N 33° 19′ E until her latitude in is 51° 18′ N. What is her longitude in?

Ans. 9° 44′ W

4. From a place in latitude 42° 30′ N and longitude 58° 51′ W, a ship sails S 3 points E 300 miles. Find the latitude and longitude in.

Ans. $\begin{cases} \text{Lat. in} = 38° \ 21' \text{ N} \\ \text{Long. in} = 55° \ 12' \text{ W} \end{cases}$

5. (*a*) What true course should a ship steer to get into latitude 47° 12′ N and longitude 30° 20′ W, her present latitude and longitude being 50° 25′ N and 27° 15′ W, respectively? (*b*) If her average speed is 8 knots, how many hours will be required to cover the distance?

Ans. $\begin{cases} (a) & \text{Course} = \text{S } 32° \ 16' \text{ W} \\ (b) & \text{Time required} = 28\frac{1}{2} \text{ hr. (nearly)} \end{cases}$

6. From latitude 47° 10′ N and longitude 32° 15′ W, a ship sails S 48° 58′ W 175.2 miles. Required, her present latitude and longitude in.

Ans. $\begin{cases} \text{Lat. in} = 45° \ 15' \text{ N} \\ \text{Long. in} = 35° \ 26' \text{ W} \end{cases}$

MERCATOR'S SAILING

41. In the foregoing it was shown that plane sailing serves every purpose for small distances when the comparatively small portion of the earth's surface sailed over is regarded as plane; and that middle-latitude sailing, the principal object of which is to render the results of plane sailing available for the purpose of determining the difference of longitude, is correct for short runs and when the difference of latitude is small in comparison with the difference of longitude. But, in cases where great accuracy is required, and especially in high latitudes, or on occasions when the differences of latitude and longitude are large, neither of these methods are exact enough. In such cases, the method about to be explained—**Mercator's sailing**—offers great advantages to the navigator.

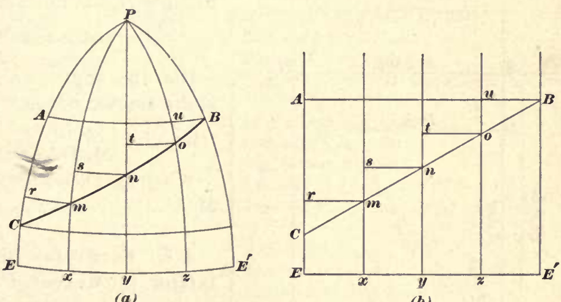

FIG. 19

42. Principles Involved.—Mercator's sailing is based on the same principles as those involved in the construction of a Mercatorial chart. On such charts, as shown in Fig. 19, the meridians are drawn parallel to one another, and, therefore, the arcs rm, sn, to, and uB, the sum of which equals the departure, are increased until they become equal to the corresponding parts Ex, xy, yz, and zE' of the equator EE'. Now, in order to preserve the similarity of the triangular

spaces on the chart, the sides Cr, ms, nt, and ou of the triangles Crm, msn, etc., Fig. 19 (a), must be increased in the same ratio, or proportion, as rm, sn, etc. have been increased. Hence, when $rm + sn + to + uB$, Fig. 19 (b), increases and becomes equal to the difference of longitude AB or EE', the arcs $Cr + ms + nt + ou$ are similarly increased and made equal to the *meridional difference of latitude* (M. D. Lat.) AC. Consequently, the difference of longitude AB, the meridional difference of latitude AC, and CB, representing the expanded distance, form a right plane triangle ABC, in which

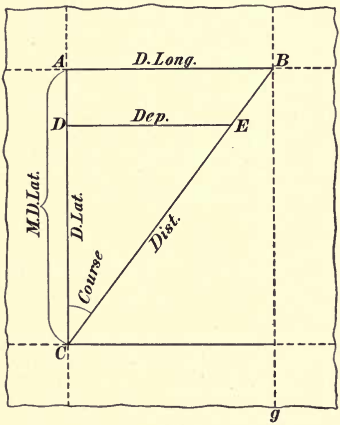

FIG. 20

$$\frac{AB}{AC} = \tan ACB$$

But the angle ACB is the course; hence,

$$\tan C = \frac{\text{D. Long.}}{\text{M. D. Lat.}}$$

Whence, D. Long. = M. D. Lat. $\times$ tan C.

43. Formulas Relating to Mercator's Sailing.—Let Fig. 20 represent a small portion of a Mercatorial chart, CDA the meridian of the place C, and Bg the meridian passing through another place B; let CD be the true difference of latitude, while AC represents the meridional difference of latitude. Now, if a line is drawn through D parallel to AB, then DE will be equal to the departure and CE equal to the distance between the places (on the globe), the angle ACB being, as shown before, the true course.

Referring to the triangles ABC and DEC, both being right-angled plane triangles, the principal formulas used in Mercator's sailing are readily deduced. Thus,

$$\tan C = \frac{AB}{AC} = \frac{\text{D. Long.}}{\text{M. D. Lat.}}$$

$$\text{Dist.} = \frac{DC}{\cos C} = \text{D. Lat.} \times \sec C$$

$$\text{D. Lat.} = CE \times \cos C = \text{Dist.} \times \cos C$$

$$\text{D. Long.} = AC \times \tan C = \text{M. D. Lat.} \times \tan C$$

TABLE IV

FORMULAS FOR MERCATOR'S SAILING

Given	Required	Formulas to be Used
Both latitudes and longitudes	Course Dist. Dep.	Tan C=D. Long.÷M. D. Lat. Dist.=sec C×D. Lat. Dep.=D. Lat.×tan C Dep.=(D. Lat.×D. Long.)÷M. D. Lat.
Both latitudes and departure	Course Dist. D.Long.	Tan C=Dep.÷D. Lat. Dist.=D. Lat.×sec C Dist.=Dep.×cosec C D. Long.=M. D. Lat.×tan C D. Long.=(Dep.×M. D. Lat.)÷D. Lat.
One latitude, course, and distance	Dep. D.Lat. D.Long.	Dep.=Dist.×sin C D. Lat.=Dist.×cos C D. Long.=M. D. Lat.×tan C
Both latitudes and course	Dist. Dep. D.Long.	Dist.=D. Lat.×sec C Dep.=D. Lat.×tan C D. Long.=M. D. Lat.×tan C
Both latitudes and distance	Course Dep. D.Long.	Cos C=D. Lat.÷Dist. Dep.=Dist.×sin C D. Long.=M. D. Lat.×tan C
One latitude, course, and departure	D.Lat. Dist. D.Long.	D. Lat.=Dep.×cot C Dist.=Dep.×cosec C D. Long.=M. D. Lat.×tan C D. Long.=(Dep.×M. D. Lat.)÷D. Lat.
One latitude, distance, and departure	Course D.Lat. D.Long.	Sin C=Dep.÷Dist. D. Lat=Dist.×cos C D. Long.=M. D.×tan C D. Long.=(Dep.×M. D. Lat)÷D. Lat.

44. Table IV gives a summary of the possible number of cases in Mercator's sailing, the given and required data, and the proper formula to use in each case. However, as has already been stated in regard to middle-latitude sailing, only the two cases that occur in actual practice will be considered here.

45. *To find the meridional difference of latitude*, or, what is the same thing, the difference of latitude expressed in meridional parts, proceed as follows:

1. *Should both places be north or both south of the equator, the M. D. Lat. between the two places is the difference between the meridional parts of the greater latitude and those of the smaller latitude as obtained from the table of meridional parts found in the collection of Nautical Tables* (pages 145 to 152).

2. *Should the places be on opposite sides of the equator, then the sum of the meridional parts, corresponding to the latitude of each place, is the M. D. Lat. of the two places.*

3. *If one place is on the equator, the M. D. Lat. between it and a place north or south of the equator is obtained directly from the table of meridional parts.*

Example 1.—Find the M. D. Lat. between a place in latitude 35° 10′ N and another in latitude 16° 58′ N.

Solution.—Enter table of meridional parts and find the values corresponding to each latitude. Thus,

	opposite 35° 10′	is	2,243.3 M. P.
and	opposite 16° 58′	is	1,026.5 M. P.

Subtracting, gives 18° 12′ as D. Lat. and 1,216.8 as M. D. Lat.

Hence, the D. Lat. of 18° 12′, or 1,092′, between 35° 10′ N and 16° 58′ N on a Mercator's chart is equivalent to 1,216.8′ measured on the equator; in other words, the true D. Lat. is expanded 1,216.8 − 1,092 = 124.8′. Ans.

Example 2.—Find the M. D. Lat. between 42° 9′ S and 18° 50′ N.

Solution.—

	Opposite 42° 9′	is	2,778.4 M. P.
and	opposite 18° 50′	is	1,143.5 M. P.

Adding, gives 60° 59′ as D. Lat. and 3,921.9 as M. D. Lat.

On a Mercatorial chart, therefore, 60° 59′ of a meridian between 42° 9′ S and 18° 50′ N is equivalent to 3,921.9′, or 65° 21.9′, on the equator. Ans.

Example 3.—Required, the M. D. Lat. between a place on the equator and another in latitude 36° 15′ S.

Solution.—Opposite 36° 15′ in the tables will be found 2,323.0 M. P. Hence, on a Mercator's chart, the length of 36° 15′ of a meridian from the equator N or S is equivalent to 2,323.0′ on the equator. Ans.

46. Solution of Problems by Traverse Tables.—It is evident that problems in Mercator's sailing can also be solved by means of Traverse Tables. For instance, when the course is sought, the meridional difference of latitude is entered in a latitude column and the difference of longitude in a departure column, when the number of degrees corresponding to where they stand opposite one another will be the required course. With this course, the distance is found opposite the proper difference of latitude. The following examples in Mercator's sailing will tend to show, better than any rules, how this is done in each case.

47. *To find the course and distance between any two places, the latitude and longitude of each being known.*

Example 1.—A ship in latitude 32° 15′ N and longitude 64° 52′ W is bound for a port in latitude 49° 57′ N and longitude 5° 12′ W. Required, the course to steer and the distance to be run.

Solution.—First find the D. Lat., the M. D. Lat., and the D. Long. as follows, and then calculate the course and distance according to proper formulas taken from Table IV.

1st Lat. = 32° 15′ N	M. P. = 2,033.9
2d Lat. = 49° 57′ N	M. P. = 3,452.2
D. Lat. = 17° 42′	M. D. Lat. = 1,418.3
Or = 1,062′ N	

1st Long. = 64° 52′ W
2d Long. = 5° 12′ W
D. Long. = 59° 40′
Or = 3,580′ E

tan C = D. Long. ÷ M. D. Lat.	Dist. = D. Lat. × sec C
log 3,580 (+10) = 13.55388	log 1,062 = 3.02612
log 1,418 = 3.15168	log sec 68° 23′ = 10.43369
log tan C = 10.40220	log Dist. = 3.45981
Course = N 68° 23′ E. Ans.	Dist. = 2,883 mi. Ans.

SOLUTION BY INSPECTION.—Enter the Traverse Tables with M. D. Lat. in a latitude column and the D. Long. in a departure column (see Fig. 21) and find the corresponding course. Then, with this course and the D. Lat., find the required Dist. In this case the numbers 1,418 and 3,580 are too large for the Traverse Tables; therefore, divide them by 100 and enter with 14.2 and 35.8 instead. Again, with the corresponding course 68° and the D. Lat. worked in similar manner (1,062 ÷ 10, or 106.2), the Dist. found is 2,835. This Dist., however, does not agree with that obtained by calculation, but it can be made much closer by a simple proportion, if deemed necessary. The correct course is 68° 23′, not 68°; therefore, an allowance should be made for the 23. Thus,

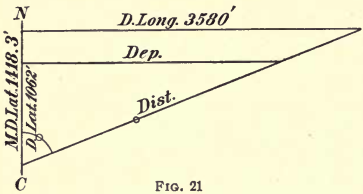

FIG. 21

with 68° as course and 1,062 D. Lat., the Dist. is 2,835 mi.
and with 69° as course and 1,062 D. Lat., the Dist. is 2,963 mi.

The difference, therefore, for 60′ of the course is 128 mi.

Whence, for 23′, it must be $\frac{23 \times 128}{60} = 49$ mi. This, when added to the Dist. corresponding to the smaller course, will produce a more correct value of the required Dist., or 49 + 2,835 = 2,884 mi., which very nearly agrees with that derived by computation. Ans.

EXAMPLE 2.—A ship at the Cape of Good Hope, latitude 34° 21′ S and longitude 18° 29′ E, is bound for a place in latitude 6° 47′ S and longitude 104° 37′ E. What true course and distance should the vessel sail in order to reach her destination?

SOLUTION.—

1st Lat. =	34° 21′ S	M. P. =	2,183.9
2d Lat. =	6° 47′ S	M. P. =	405.3
D. Lat. =	27° 34′	M. D. Lat. =	1,778.6
Or =	1,654′ N		

1st Long. =	18° 29′ E
2d Long. =	104° 37′ E
D. Long. =	86° 8′
Or =	5,168′ E

tan C = D. Long. ÷ M. D. Lat.		Dist. = D. Lat. × sec C	
log 5,168 (+ 10) =	13.71332	log 1,654 =	3.21854
log 1,778.6 =	3.25008	log sec 71° 1′ =	10.48773
log tan C =	10.46324	log Dist. =	3.70627
Course =	N 71° 1′ E. Ans.	Dist. =	5,084.8 mi. Ans.

Solution by Inspection.—In this case (Fig. 22), the M. D. Lat. and the D. Long. are too large to be found in the Traverse Tables. Therefore, each is divided by 100, which gives 17.8 and 51.7, respectively, and the required course corresponding is 71°. Now, with the course 71° and the D. Lat. divided by 100 (= 16.5), the corresponding Dist. is 51 mi. Hence, the required Dist. must be 51 × 100 = 5,100 mi.

It will be noticed that the Dist. found by inspection differs somewhat from that obtained by calculation. This goes to show that the

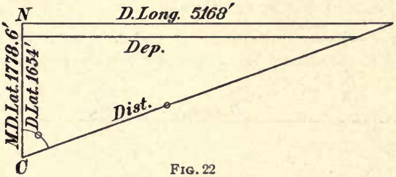

Fig. 22

Traverse Tables should not be relied on to give trustworthy results when the D. Lat. and D. Long. made good are very large.

48. *To find the latitude and longitude in, when the latitude and longitude from and the course and distance run are known.*

Example 1.—A ship from a place in latitude 52° 6′ N and longitude 35° 6′ W sails N W by W 229 miles. Find the latitude and longitude arrived at.

Solution.—First compute the D. Lat. and find the latitude in; then take out the meridional parts for each latitude, and with the M. D. Lat. thus found, compute the D. Long. according to the proper formula.

$$\begin{aligned}
\text{D. Lat.} &= \text{Dist.} \times \cos C \\
\log 229 &= 2.35984 \\
\log \cos 56° 15' &= 9.74474 \\
\log \text{D. Lat.} &= 2.10458 \\
\text{D. Lat.} = 127.2' &= 2° \ 7.2' \text{ N} \\
\text{Lat. from} &= 52° \ 6' \text{ N} \\
\text{Lat. in} &= 54° 13.2' \text{ N. Ans.}
\end{aligned}$$

$$\begin{aligned}
\text{M. P. 1st Lat.} &= 3{,}656.8 \\
\text{M. P. 2d Lat.} &= 3{,}868.2 \\
\text{M. D. Lat.} &= 211.4
\end{aligned}$$

$$\begin{aligned}
\text{D. Long.} &= \text{M. D. Lat.} \times \tan C \\
\log 211.4 &= 2.32510 \\
\log \tan 56° 15' &= 10.17511 \\
\log \text{D. Long.} &= 2.50021 \\
\text{D. Long.} &= 316.4' \text{ W}
\end{aligned}$$

$$\begin{aligned}
\text{Long. from} &= 35° \ 6' \text{ W} \\
\text{D. Long.} &= 5° 16.4' \text{ W} \\
\text{Long. in.} &= 40° 22.4' \text{ W. Ans.}
\end{aligned}$$

SOLUTION BY INSPECTION.—Entering the Traverse Tables with N W by W as course and 229 as Dist., the corresponding D. Lat. is 127.2′. Then again, with the same course (see Fig. 23) and M. D. Lat. (211.4) in a latitude column, enter the tables, when the corresponding D. Long. is obtained in a departure column. Thus,

corresponding to 166.7 is 249.4′
and corresponding to 44.7 is 66.9′
Whence, corresponding to 211.4′ is 316.3′ as D. Long.
or within $\frac{1}{10}$ mile of that by calculation. Ans.

EXAMPLE 2.—From latitude 27° S and longitude 123° W, a ship sails S S E $\frac{1}{2}$ E 150 miles. Required, her latitude and longitude in.

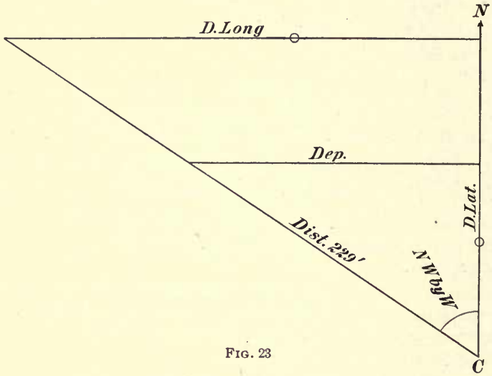

FIG. 23

SOLUTION.—Proceed exactly as in the preceding example. Thus,

$$
\begin{aligned}
\text{D. Lat.} &= \text{Dis.} \times \cos C \\
\log 150 &= 2.17609 \\
\log \cos 28° \, 7' &= 9.94546 \\
\log \text{D. Lat.} &= 2.12155 \\
\text{D. Lat.} = 132.3' &= 2° \, 12.3' \text{ S} \\
\text{Lat. from} &= 27° \, 0' \text{ S} \\
\text{Lat. in} &= 29° \, 12.3' \text{ S. Ans.}
\end{aligned}
$$

$$
\begin{aligned}
\text{M. P. 1st Lat.} &= 1{,}673.1 \\
\text{M. P. 2d Lat.} &= 1{,}822.0 \\
\text{M. D. Lat.} &= 148.9
\end{aligned}
$$

$$
\begin{aligned}
\text{D. Long.} &= \text{M. D. Lat.} \times \tan C \\
\log 148.9 &= 2.17289 \\
\log \tan 28° \, 7' &= 9.72780 \\
\log \text{D. Long.} &= 1.90069 \\
\text{D. Long.} &= 79.6' \text{ E}
\end{aligned}
$$

$$
\begin{aligned}
\text{Long. from} &= 123° \, 0' \text{ W} \\
\text{D. Long.} &= 1° \, 19.6' \text{ E} \\
\text{Long. in} &= 121° \, 40.4' \text{ W. Ans.}
\end{aligned}
$$

SOLUTION BY INSPECTION.—The result as obtained by Traverse Tables (see Fig. 24) agrees very nearly with that obtained by computation, or D. Lat. = 132.3′, and D. Long. = 79.7′. Ans.

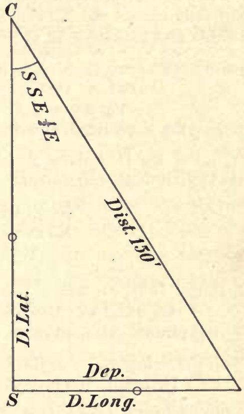

FIG. 24

EXAMPLE 3.—After leaving Cape Horn, latitude 55° 59′ S and longitude 67° 16′ W, a vessel runs 4,748 miles on a true N 76° 37′ W course. Required, the latitude and longitude of the position arrived at.

SOLUTION.—

$$\begin{aligned}
\text{D. Lat.} &= \text{Dist.} \times \cos C \\
\log 4{,}748 &= 3.67651 \\
\log \cos 76^\circ\ 37' &= 9.36449 \\
\log \text{D. Lat.} &= 3.04100 \\
\text{D. Lat.} &= 1{,}099'\ \text{N}
\end{aligned}$$

$$\begin{aligned}
\text{Lat. from} &= 55^\circ\ 59'\ \text{S} \\
\text{D. Lat. } 1{,}099' &= 18^\circ\ 19'\ \text{N} \\
\text{Lat. in} &= 37^\circ\ 40'\ \text{S.}\quad \text{Ans.}
\end{aligned}$$

$$\begin{aligned}
\text{M. P. Lat. } 55^\circ\ 59' &= 4{,}053.1 \\
\text{M. P. Lat. } 37^\circ\ 40' &= 2{,}428.9 \\
\text{M. D. Lat.} &= 1{,}624.2
\end{aligned}$$

$$\begin{aligned}
\text{D. Long.} &= \text{M. D. Lat.} \times \tan C \\
\log 1{,}624 &= 3.21059 \\
\log \tan 76^\circ\ 37' &= 10.62356 \\
\log \text{D. Long.} &= 3.83415 \\
\text{D. Long.} &= 6{,}826'\ \text{W}
\end{aligned}$$

$$\begin{aligned}
\text{Long. from} &= 67^\circ\ 16'\ \text{W} \\
\text{D. Long. } 6{,}826' &= 113^\circ\ 46'\ \text{W} \\
\text{Long. in} &= 181^\circ\ \ 2'\ \text{W} \\
\text{Or} &= 178^\circ\ 58'\ \text{E.}\quad \text{Ans.}
\end{aligned}$$

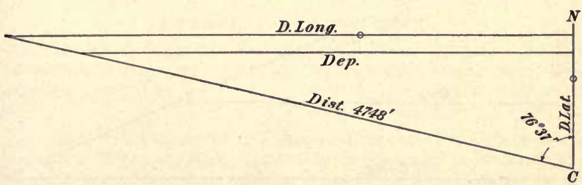

FIG. 25

SOLUTION BY INSPECTION.—Entering the Traverse Tables with 76° as course and $\frac{1}{44}$ of 4.748 mi., or 108, in the distance column (see Fig. 25), the corresponding D. Lat. = 26.1′; and, with the same Dist. and 77° as course, the corresponding D. Lat. = 24.3′. Now, the mean of these two values multiplied by 44 (or 25.2 × 44 = 1.109′) is a fair approximation of the required D. Lat. for 76° 37′. However, no attempt

should be made to use the tables for finding the D. Long. in a case of this kind, as the result obtained would be unreliable.

Example 4.—From latitude 56° 30′ N and longitude 28° 10′ W, a ship sails true S 66° 22′ E for a distance of 257 miles. What is her latitude and longitude in at the end of this run?

Solution.—

D. Lat. = Dist. × cos C

log 257 =	2.40993	Lat. from =	56° 30′ N
log cos 66° 22′ =	9.60302	D. Lat. 103′ =	1° 43′ S
log D. Lat. =	2.01295	Lat. in =	54° 47′ N Ans.
D. Lat. =	103′ S		

M. P. Lat. 56° 30′ =	4,108.7
M. P. Lat. 54° 47′ =	3,926.6
M. D. Lat. =	182.1

D. Long. = M. D. Lat. × tan C

log 182.1 =	2.26031	Long. from =	28° 10′ W
log tan 66° 22′ =	10.35894	D. Long. 416.1′ =	6° 56.1′ E
log D. Long. =	2.61925	Long. in =	21° 13.9′ W. Ans.
D. Long. =	416.1′ E		

Solution by Inspection.—The D. Lat. in the Traverse Tables for a course of 66° and a Dist. of 257 mi. (see Fig. 26) is 104.5, and for a

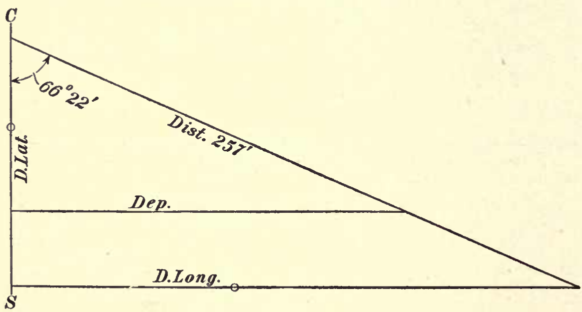

Fig. 26

course of 67° and the same Dist., it is 100.4. Taking the mean of these values gives 102.5 for the required D. Lat. Now, to find the D. Long., enter the tables with 66° as course and ½ M. D. Lat., or 91.1, in a latitude column, the corresponding value in the departure column is 204.6; and with 67° as course and the same value in the latitude column,

the number opposite in the departure column is 214.5. Hence, the required D. Long. is

$$\frac{204.6 + 214.5}{2} \times 2 = \frac{419.1}{2} \times 2 = 419.1' \text{ E}$$

It will be noted that, although the distance in this case is not very large, the course is not an exact number of degrees. For this reason, the results obtained by inspection differ somewhat from those found by calculation.

49. Remarks on the Middle-Latitude and the Mercator Methods.—If the distance run by a ship is less than 300 miles, the middle-latitude method may be used; if greater than 300 miles, Mercator's method should be employed, except in cases where the course is large or very near east or west, when it is preferable to use the former method. The reason it is preferable to use the middle-latitude method in finding the difference of longitude when the course is large is that tangents for angles between 80° and 90° change very rapidly, and, hence, when using the formula D. Long. = M. D. Lat. × tan C, if there is an error in the course, the resulting difference of longitude will be considerably in error. Therefore, when the course is large, or nearly 90°, it is better to find the difference of longitude by the middle-latitude formula, D. Long. = Dep. × sec M. Lat., in which the tangent is not used.

EXAMPLES FOR PRACTICE

1. Required, the course and distance between Ushant, in latitude 48° 28′ N and longitude 5° 3′ W, and St. Michael's, in latitude 37° 44′ N and longitude 25° 40′ W.
Ans. $\begin{cases}\text{Course} = \text{S } 54° \ 30' \text{ W}\\ \text{Dist.} = 1{,}109 \text{ mi.}\end{cases}$

2. Required, the course and distance from Cape Cod lighthouse, in latitude 42° 3′ N and longitude 70° 4′ W, to the island of St. Mary, one of the Western Islands, in latitude 36° 59′ N and longitude 25° 10′ W.
Ans. $\begin{cases}\text{Course} = \text{S } 81° \ 42' \text{ E}\\ \text{Dist.} = 2{,}106 \text{ mi.}\end{cases}$

3. Required, the course and distance from Cape Henry lighthouse, in latitude 36° 55′ N and longitude 76° 0′ W, to Point David, Bermuda Islands, in latitude 32° 21′ N and longitude 64° 36′ W.
Ans. $\begin{cases}\text{Course} = \text{S } 64° \ 8' \text{ E}\\ \text{Dist.} = 628 \text{ mi.}\end{cases}$

4. Required, the course and distance from Cape Palmos (African Coast), in latitude 4° 24′ N and longitude 7° 46′ W, to St. Paul de Loando, the latitude of which is 8° 48′ S and longitude 13° 8′ E.

Ans. {Course = S 57° 49′ E; Dist. = 1,487 mi.

5. From a place in latitude 70° 14′ S and longitude 25° 30′ E, a ship sails S 37° 7′ E 370 miles. Required, her latitude and longitude in.

Ans. {Lat. = 75° 9′ S; Long. = 38° 5′ E

6. From a place in latitude 42° 30′ N and longitude 58° 51′ W, a ship sails S E by S 300 miles. Required, her latitude and longitude in.

Ans. {Lat. = 38° 21′ N; Long. = 55° 13′ W

7. From a place in latitude 27° 20′ N and longitude 25° 12′ E, a ship sails N 37° 10′ E for a distance of 472.6 miles. Find the latitude and longitude of the place arrived at.

Ans. {Lat. = 33° 37′ N; Long. = 30° 42.2′ E

8. From a place in latitude 27° 15′ S and longitude 93° 21′ E, a ship sails S 61° W 325 miles. Required, her latitude and longitude in.

Ans. {Lat. = 29° 53′ S; Long. = 87° 58.1′ E

In the following examples, give the answers both by middle-latitude and by Mercator's sailing:

9. A ship at the Cape of Good Hope, in latitude 34° 22′ S and longitude 18° 24′ E, is bound for the Falkland Islands, in latitude 52° 21′ S and longitude 59° 18′ W. What course must be steered and how many miles must be run in order to reach the port of destination?

By Middle-Latitude Sailing

Using the ordinary method	*Using true middle latitude*
Ans. {Course = S 72° 20′ W; Dist. = 3,555 mi.	Ans. {Course = S 72° 9′ W; Dist. = 3,520 mi.

By Mercator's Sailing

Ans. {Course = S 72° 12′ W; Dist. = 3,530 mi.

10. From a place in latitude 51° 20′ S and longitude 60° 15′ W, a ship sails N 73° E for a distance of 3,400 miles. Required, her latitude and longitude in.

By Middle-Latitude Sailing

Using the ordinary method	*Using true middle latitude*
Ans. {Lat. in 34° 46′ S; Long. in = 13° 55′ E	Ans. {Lat. in =34° 46′ S; Long. in = 14° 37′ E

By Mercator's Sailing

Ans. {Lat. in = 34° 46′ S; Long. in = 14° 23′ E

DEAD RECKONING

(PART 2)

CURRENT SAILING

METHODS OF DETERMINING THE SET, DRIFT, AND EFFECT OF CURRENTS

1. A **current** may be defined as a large volume of water, or portion of the sea, that, occasioned by winds and other impulses, is put in motion.

2. The direction of a current, or the point of the compass toward which it flows, is called the **set** of the current; and its velocity, or rate, is called the **drift.**

The mode adopted in speaking of the direction of the wind, which is named according to the point *from* which it blows, is reversed in speaking of the direction of a current. Hence, a current setting toward N W is called a *northwest current*, and a current setting toward N by E is called a *north-by-east current.*

3. Effect of Currents.—When a current acts on a ship, her rate of sailing is necessarily affected by it, and, in general, both her speed and the direction in which she would otherwise move over ground are altered. By the expression "over ground" is meant the movement of a ship in reference to some fixed point on the bottom of the sea.

4. When a ship sails directly with or directly against the current, her speed over ground, only, will be affected. In

the former case she is assisted to the full extent of the current, while in the latter her speed is reduced with the amount of the drift.

5. When sailing across a current, its effect in assisting or in hindering will depend on the angle that the set of the current makes with the ship's track.

6. Current sailing is the method of determining the true course and distance sailed by a ship when its motion is either affected by or combined with that of a current.

In general, current sailing may be treated as a particular case of traverse sailing by regarding the set and drift of the current as a separate course and distance run.

7. Before entering into the solution of current problems, a clear understanding of Newton's first and second laws of motion, otherwise known as the laws relating to the connection of force with motion, will be very useful. In order that this knowledge may be attained, a few fundamental principles of mechanics will be explained.

8. Motion and Rest.—When a body changes its position with regard to some fixed point, it is said to be **in motion** relatively to that point; when no such change occurs, it is said to be **at rest** relatively to that point. Thus, a ship that sails by a lighthouse is in motion relatively to the lighthouse. On the other hand, if a person on board that ship walks from bow to stern, that is, in a direction opposite to that in which the ship sails and with the same speed, he will be in motion in relation to the ship, but at rest with respect to the lighthouse; since, until he gets to the stern, he will be directly over the spot at which he was when he started to walk.

9. Path, or Track.—When a point moves from one position to another, it describes a line, either straight or curved, connecting the two positions. This line is called the **path**, or **track**, of the point.

10. Velocity.—The rate at which a point or a body moves from one position to another is called its **velocity.**

Velocity is measured by the relation between the *distance passed over* and the *time occupied* in traversing that distance.

In regard to the motion of a ship, the term *velocity* is usually replaced by **speed**; and in regard to currents, by **drift.**

11. Representation of Motion.—Motion may take place along either a straight or a curved path; in the former case it is termed *rectilinear motion*, and in the latter case *curvilinear motion*. In current sailing, rectilinear motion only is considered.

12. It is evident that the rectilinear motion of a point or a body may be represented on a sheet of paper by a straight line; for the direction of the line may represent the direction of the motion, while the velocity may conveniently be indicated by its length. By thus representing the motion of points or bodies, certain relations are established by which problems connected with such motions may readily be solved by geometrical construction.

13. Laws of Motion.—Newton's first and second laws of motion already referred to are as follows:

I. *All bodies continue in a state of rest, or of uniform motion, in a straight line, unless acted on by some external or outside force that compels a change.*

II. *Every motion, or change of motion, is proportional to the acting force, and takes place in the direction of the straight line along which the force acts.*

14. Composition of Motions.—As a corollary of these laws, certain conclusions are established. Thus, if a body receives a single impulse in a given direction, it will move in that direction with a certain velocity; and, as just explained, its motion may be represented by a straight line having the same direction as the motion and a length proportional to the velocity. Again, if a body is acted on, at the same time, by two impulses, or forces, in different directions, it will obey both and move in an intermediate

direction with a velocity differing from that due to either impulse alone.

15. At the same instant, the body may also receive several impulses, each tending to impart to it a motion in a certain direction. It is evident, however, that the body can move only in one direction; this direction may properly be termed the *resultant*, and the separate directions, considered singly, the *components*.

16. Parallelogram of Motion.—If two components are represented in direction and velocity by the adjacent sides of a parallelogram, the resultant will be similarly represented by the diagonal passing through their point of intersection. Thus, in Fig. 1, let a body at A be acted on at the same instant by two forces represented in direc-

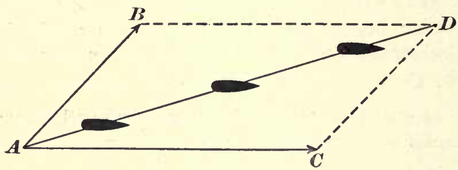

FIG. 1

tion and velocity by the straight lines AB and AC, respectively; then, the resultant, or combined effect of these forces, will be represented in direction and velocity by the diagonal AD of the parallelogram having the components AB and AC as adjacent sides.

17. Applying this explanation to a case of current sailing, it will be seen that if a ship at A steams or sails from A to C in an hour, and, if in the same interval of time, a current is flowing from A to B, it is evident that at the close of the hour the ship will be neither at B nor C, but at D, having been moved by her own propelling force and that of the current along the diagonal AD, as indicated in the figure. The result is precisely the same as though the ship had first sailed from A to C without being influenced by the current,

then stopped, and traveled from C to D with the current only. Hence, AD will represent the actual course and distance made good by the ship.

18. Solution of Problems in Current Sailing. From the foregoing, it is evident that when the set and drift of a current, also the course and distance steered by a ship, are known, the course and distance made good by the ship can be found either by construction, by the application of trigonometry, or by considering the set and drift of the current as a *separate course and distance*, in a case of traverse sailing. When using trigonometry, the required data are found by solving a triangle, one side of which represents the course and distance steered, while the other side represents the set and drift of the current. In practice, however, the method in which the set and drift are inserted in a traverse is the one most commonly used.

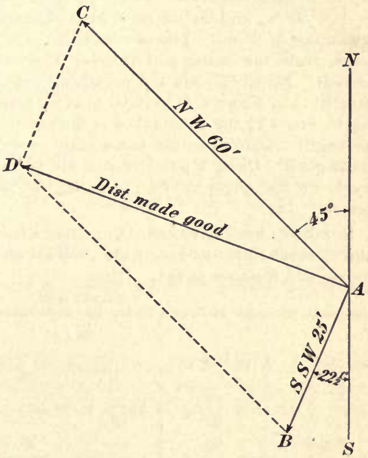

FIG. 2

19. When the set and drift of a current are known, together with the course and distance to be made good, the course to steer is found by entering in a traverse the course and distance to be made good and the set and drift of the current *reversed*.

20. When the course and distance run and the course and distance made good are known, the set and drift of the current are found by entering in a traverse the course and distance made good, also the course or courses sailed, *reversed*.

The examples that follow will illustrate the foregoing statements.

21. *To find the course and distance made good when the course and distance sailed and the set and drift of the current are known.*

Example 1.—A ship sails N W a distance of 60 miles; during the same interval of time she is influenced by a current that sets S S W 25 miles. Find the course and distance made good.

Solution by Construction.—Referring to Fig. 2, draw a line NS to represent the direction of a meridian; then, from any point A draw AC in a N W direction or N 45° W, and make this line 60 mi., according to an assumed scale. From the same point A draw AB = S S W = S 22° 30′ W, and lay off on it from A, according to the same scale, a distance of 25 mi. The two lines AC and AB will then represent, respectively, the course and distance sailed and the set and drift of the current. Now complete the parallelogram $ABDC$ by drawing BD parallel to AC and CD parallel to AB; then join A and D. According to Art. **17**, the diagonal AD is then the course made good, and its length measured on the same scale as AC and AB is the distance made good. Using a protractor, it will be found that the course made good, or the angle NAD, is N $69\frac{1}{2}$° W and the distance $55\frac{1}{2}$ mi., nearly. Ans.

Solution by Traverse.—Construct a traverse and enter the course and distance sailed as usual; also, the set and drift of the current as a course and distance sailed. Thus,

Traverse

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N W	60	42.4	——	——	42.4
S S W	25	——	23.1	——	9.6

D. Lat. = 19.3′ N Dep. = 52.0′ W

Then calculate the course and distance made good according to the proper formulas. Thus,

$\tan C$ = Dep. ÷ D. Lat.	Dist. = D. Lat. × $\sec C$
log 52 (+ 10) = 11.71600	log 19.3 = 1.28556
log 19.3 = 1.28556	log sec 69° 38′ = 10.45839
log tan C = 10.43044	log Dist. = 1.74395
Course made good = N 69° 38′ W. Ans.	Dist. = 55.46 mi. Ans.

The result thus found agrees very nearly with that obtained by construction.

Example 2.—From 12 o'clock noon until 6 p. m., a ship sails N E by N 31 miles and E N E 8 miles. During the same time a current is running N by W 8.5 miles. Required, the course and distance made good.

Solution by Traverse.—Form a traverse and enter the courses and the current as in the preceding example. Thus,

Traverse

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N E by N	31	25.8	——	17.2	——
E N E	8	3.1	——	7.4	——
N by W	$8\frac{1}{2}$	8.3	——	——	1.7

D. Lat. = 37.2′ N   24.6   1.7

1.7

Dep. = 22.9′ E

$\tan C = \text{Dep.} \div \text{D. Lat.}$

$\log 22.9\ (+10) = 11.35984$

$\log 37.2 = 1.57054$

$\log \tan C = 9.78930$

Course made good = N 31° 37′ E. Ans.

$\text{Dist.} = \text{D. Lat.} \times \sec C$

$\log 37.2 = 1.57054$

$\log \sec 31° 37' = 10.06978$

$\log \text{Dist.} = 1.64032$

Dist. = 43.68 mi. Ans.

22. *To find what course to steer when the course to be made good (or true bearing) and the set and drift of the current are known.*

Example 1.—The port of destination bears S 22° 30′ W, a distance of 25 miles; a current sets S by E, its drift being 4 miles per hour. Find what course to steer and what speed to make in order to reach the port in 5 hours.

Solution by Construction.—Referring to Fig. 3, draw the meridian NS; from any point A draw AD in a S S W direction, or S 22° 30′ W, and make it 25 mi., according to some scale; draw also the line AB = S by E = S 11° 15′ E, and lay off on it a distance of $4 \times 5 = 20$ mi. The line AD then represents the course and distance to be made good, and AB the set and drift of the current in 5 hr. Now, by completing the parallelogram, with AD as a diagonal, the side AC will represent the course and distance to steer in order to reach the destination D. Measuring the angle SAC, it will be found that the course to be steered is S $75\frac{1}{2}°$ W, and the distance through water equal to 14 mi., nearly. Ans.

SOLUTION BY TRAVERSE.—By considering the problem as a case of traverse sailing, the same result is obtained, as shown below. It should

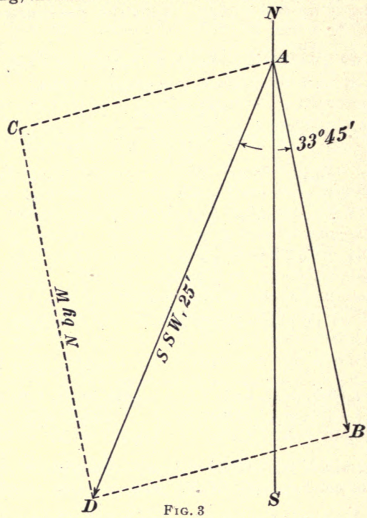

FIG. 3

be noticed that the set of the current is *reversed* before it is entered in the traverse, according to instructions given in Art. **19.**

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S S W	25	——	23.1	——	9.6
N by W	20	19.6	——	——	3.9
		19.6	23.1	Dep. =	13.5′ W
			19.6		

D. Lat. = 3.5′ S

$\tan C = \text{Dep.} \div \text{D. Lat.}$
$\log 13.5\ (+10) = 11.13033$
$\log 3.5 = 0.54407$
$\log \tan C = 10.58626$
Course = S 75° 28′ W. Ans.

$\text{Dist.} = \text{D. Lat.} \times \sec C$
$\log 3.5 = 0.54407$
$\log \sec 75° 28′ = 0.60042$
$\log \text{Dist.} = 1.14449$
Dist. = 13.9 mi. Ans.

In order to reach the port in 5 hr., the speed need not exceed $13.9 \div 5 = 2.8$ knots. Ans.

EXAMPLE 2.—A ship sailing at the rate of 8 knots an hour is bound to a port bearing N 45° W, but the passage is in a current that sets south 3 miles an hour. Find what course should be steered in order to counteract the effect of the current and keep the port on the same bearing.

SOLUTION BY CONSTRUCTION. From *A*, on the meridian *NB*, Fig. 4, draw *AD* = N 45° W, and make it equal in length to the number of miles the ship sails in 1 hr., or 8 mi. Then draw *AB* to represent the set and drift of the current, or south, 3 mi. With *AD* as a diagonal, construct the parallelogram *ABDC*; the line *AC* will then represent the direction in which to steer so as to counteract the effect of the current. Measuring the angle *NAC*, the course thus indicated is found to be N 33° W. Ans.

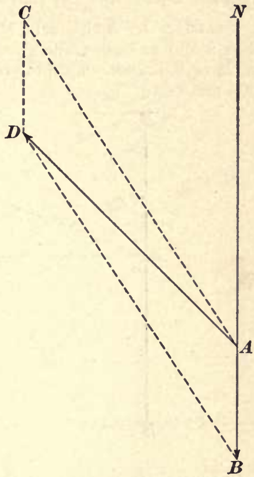

FIG. 4

SOLUTION BY TRAVERSE.—Using a traverse, the problem is solved by considering the ship to sail from *A* to *D*, and thence to *C* in a direction opposite to that of the current. Thus,

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N 45° W	8	5.7	——	——	5.7
North	3	3.0	——	——	——

D. Lat. = 8.7′ N   Dep. = 5.7′ W

By entering the Traverse Tables with D. Lat. 8.7′ and Dep. 5.7′ in their respective columns, the course corresponding nearest to these values is found on the page for 33°. Hence, the course to be steered is N 33° W. Ans.

The course may also be calculated by the formula tan *C* = Dep. ÷ D. Lat., in which case it is found to be N 33° 14′ W. Ans.

23. *To find the set and drift of a current by the course and distance sailed, and the course and distance made good.*

Example 1.—A ship sails N 69° E for a distance of 80 miles and is then found to have made good a course due east and covered a distance of 103 miles. Find the set and drift of the current in which the ship has sailed.

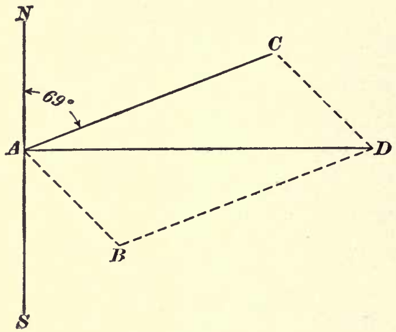

Fig. 5

Solution by Construction.—From any point A on the meridian NS, Fig. 5, draw AC equal to the course and distance steered, and also draw AD equal to the course and distance made good; complete the parallelogram with AD as the diagonal. CD or AB then represent the set and drift of the current, which in this case is found to be S 44½° E 40.3 mi. Ans.

Solution by Traverse.—When using a traverse in this case, reverse the course steered before entering, according to Art. **20**. Thus,

Traverse

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S 69° W	80	——	28.7	——	74.7
East	103	——	——	103	——

D. Lat. = 28.7′ S Dep. = 28.3′ E

Whence, according to plane sailing, the course or set of current is found to be S 44° 36′ E, and the distance or drift of the current to be about 40 mi. in the same interval of time as the ship sailed 80 mi. Ans.

EXAMPLE 2.—A ship steers S 62° W 70.5 miles for 8 hours; her course and distance made good is S 49° W 84 miles. Find the set and drift of the current in which she sails.

SOLUTION BY CONSTRUCTION.—From A, Fig. 6, draw AC and AD to represent, respectively, the course and distance steered and the course and distance made good. Complete the parallelogram as before: CD or AB is then the set and drift of the current. This, when measured, is found to be S 3° W 22 mi. Ans.

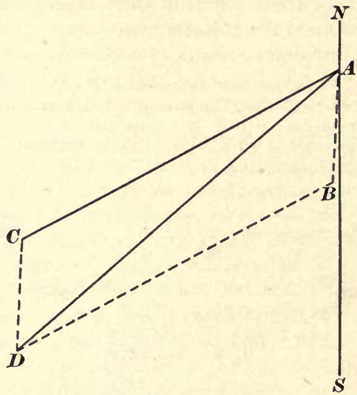

FIG. 6

SOLUTION BY TRAVERSE.—Using a traverse, the operation is the same as in the preceding examples, as follows:

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N 62° E	70.5	33.1	——	62.2	——
S 49° W	84	——	55.1	——	63.4

D. Lat. = 22.0′ S Dep. = 1.2′ W

$\tan C = \text{Dep.} \div \text{D. Lat.}$	$\text{Dist.} = \text{D. Lat.} \times \sec C$
$\log 1.2\ (+10) = 0.07918$	$\log 22 = 1.34242$
$\log 22 = 1.34242$	$\log \sec 3°\,7' = 0.00064$
$\log \tan C = 8.73676$	$\log \text{Dist.} = 1.34306$
C or set = S 3° 7′ W	Dist. or drift = 22 mi.

Therefore, the drift of the current per hr. is $22 \div 8 = 2\frac{3}{4}$ mi. Ans.

24. Determination of Current by Difference of Noon Positions.—In actual navigation, the simplest problem in current sailing occurs at sea every noon, when the positions of the ship both by dead reckoning and by astronomical

observations are obtained. To find, then, the set and drift of the current, which in most cases is assumed to be the main cause of the difference between the two results, the following example may serve as a general guide.

EXAMPLE.—By observation, a ship at noon is in latitude 36° 14.5′ S and longitude 2° 49′ E, and by dead reckoning, in latitude 36° 7′ S and longitude 3° 13.8′ E. Required, the set and drift of the current experienced during the 24 hours that have elapsed since the previous noon observations.

SOLUTION.—

Position	*Lat.*	*M. P.*	*Long.*
By observation.	36° 14.5′ S	2,322.4	2° 49′ E
By D. R. . . .	36° 7′ S	2,313.1	3° 13.8′ E
	D. Lat. = 7.5′ S	M. D. Lat. = 9.3	D. Long. = 24.8′ W

$\tan C$ = D. Long. ÷ M. D. Lat.
log 24.8 (+ 10) = 11.39445
log 9.3 = 0.96848
log $\tan C$ = 10.42597
Set of current, or C = S 69° 27′ W

Dist. = D. Lat. × sec C
log 7.5 = 0.87506
log sec 69° 27′ = 10.45466
log Dist. = 1.32972
Drift, or Dist. = 21.4 mi.

Hence, drift of current per hr. = 21.4 ÷ 24 = .9 mi., nearly. Ans.

A glance at Fig. 7 will explain the result arrived at in this example. Let EE' represent the equator, A the position of the ship according to dead reckoning, and B her position according to astronomical observations. Here it will be seen that, by observations, the ship is more south and more west than was expected from the courses steered and distances run by the log; hence, the current must run in a southwesterly direction. Now, the 24.8′ of difference of longitude, owing to the convergence of the meridians as the pole is approached, is less than 24.8 nautical miles; but, by considering the problem as a case of Mercator's sailing—where the place *from* is the position by

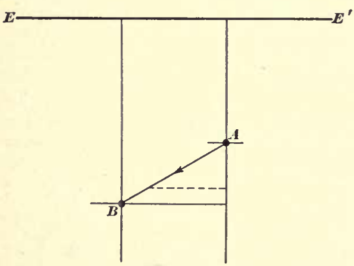

FIG. 7

dead reckoning and the place *in* the position determined astronomically—the true set and drift of the current is readily found, as shown in the example. In practice, however, the most convenient way is to plot the two positions on the chart, connect them by a straight line, and find, by means of a protractor and dividers, the bearing and distance between the two.

25. Cautionary Remarks.—The difference between the observed position of the ship and the position indicated by dead reckoning should not always be considered as the effect of a current. This difference may be due to the leeway made by the ship and insufficiently allowed for in the log, or it may be the result of careless steering, or from inaccuracy of the officer, whose duty it was to record the course and speed of the vessel. The principal ocean currents are now so well known and understood that no commander or officer should accept a current as the explanation until he has satisfied himself that the difference is not due to some other cause.

26. Allowance for Tidal Streams.—When navigating in localities where the direction and velocity of the tidal streams are known, the course to steer must be shaped so as to make allowance for the probable distance over which the vessel will be set by these streams. For instance, if the course across a channel or bay is east, and the distance 70 miles, a vessel sailing or steaming 10 knots will probably experience *both* tidal streams during the passage. Supposing that during 6 hours the stream sets north with an average drift of 4 knots per hour, and then south at the rate of 2 knots per hour, the whole drift of the vessel will therefore be about 22 miles to the northward.

Before reaching a coast line and when navigating along a coast line, careful attention should be paid to tides and currents of the locality. Much information on this subject is found on charts and in sailing directions, and these publications should accordingly be diligently studied.

27. To Find the Set and Drift of a Current by Bearings.—When sailing or steaming along and in sight of

land, the set and drift of a current can be determined as follows: Let *A*, Fig. 8, represent the position of a ship determined by cross-bearings, or other method, and *A B* the course steered and distance run from that point to another point *B*. At this latter point, the ship's position is again determined by reference to some known object or objects on shore. Should this indicate the vessel to be at *C*, it is evident that if *B* and *C* are connected by a straight line, the magnitude and direction of *B C* will represent the set and drift of the current that the ship experienced during her run from *A* to *B*.

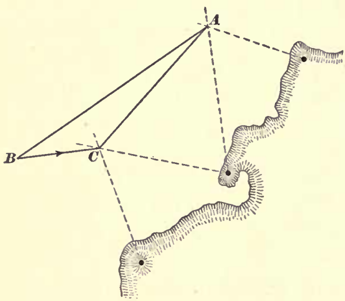

FIG. 8

28. When at anchor, the set and drift of the current is found by the log; the set is determined by the direction of the log line and the drift by the number of knots run out.

EXAMPLES FOR PRACTICE

1. A ship sails S E by E for a distance of 58 miles, and a current sets E N E 40 miles. Find: (*a*) the course and (*b*) the distance made good.

Ans. $\begin{cases} (a) & \text{S } 78° 47' \text{ E} \\ (b) & 86.9 \text{ mi.} \end{cases}$

2. During 24 hours a ship sails as follows: S W 40 miles, W S W 27 miles, and S by E 47 miles; throughout the run the ship is acted on by a current setting S E by S, the drift of which is $1\frac{1}{2}$ miles per hour. What is: (*a*) her course and (*b*) her distance made good?

Ans. $\begin{cases} (a) & \text{S } 11° 50' \text{ W} \\ (b) & 117.1 \text{ mi.} \end{cases}$

3. Suppose a port bears east and the set of a current is N N E with a drift of 2 miles per hour, the speed of the ship being 9 knots. What course should be steered so as to counteract the effect of the current?

Ans. S 77° 37′ E

4. At noon, the ship's position by astronomical observations is found to be latitude 50° 10′ N and longitude 19° 28′ W; but, by dead reckoning, its position is indicated to be in latitude 49° 47′ N and longitude 19° 59′ W. Assuming the vessel to have been accurately steered and the distances run exactly determined, find: (*a*) the set and (*b*) the drift of the current wherein she has sailed.

Ans. $\begin{cases} (a) \text{ N } 41^\circ\ 3' \text{ E} \\ (b)\ 1\tfrac{1}{4} \text{ mi. per hr.} \end{cases}$

5. A port bears N 75° E and a current is running N 40° W 2 miles per hour; the ship's rate of sailing is estimated to be 9 knots. Find what course to steer in order to counteract the current and keep the port on the same bearing. Ans. N 85° 26′ E

6. A ship sailing in a current has, by her dead reckoning, run S 57° E 48 miles; by observations, she is found to have made good 31.6′ southerly difference of latitude and 44.7′ easterly departure. Required, (*a*) the set and (*b*) the drift of the current.

Ans. $\begin{cases} (a) \text{ S } 38^\circ\ 40' \text{ E} \\ (b)\ 7 \text{ mi.} \end{cases}$

DAY'S WORK BY DEAD RECKONING

29. Data in Day's Work.—On board of every well-governed ship a careful record is kept of the several courses steered and distances run; also of the interval of time occupied on each course. These data are entered either on a slate, known as the *log slate*, or in a book generally termed the *rough log*, or *scrap log book*. From these data are computed, at noon of each day, the course and distance made good from the preceding noon, thus obtaining the ship's position by dead reckoning. This operation comprises what is called the **day's work**, and the data taken from the log book are usually termed *log-book account*.

The *data given* in the day's work are the latitude and longitude at the preceding noon (obtained by astronomical observations), the compass courses, the distance run on each course, the variation of the compass, the deviation due to each separate direction of the ship's head, the leeway, the wind, and the set and drift of currents (if known).

The *data required* in the day's work are the true course and distance made good, the latitude and longitude in, and the compass bearing and distance from the position

thus obtained to the port of destination, or that point toward which the ship, by reason of circumstances, is to be directed.

Strictly speaking, the day's work includes the determination of the ship's position *both* by dead reckoning and by astronomical observations. For the present, however, only that by dead reckoning will be considered.

30. Order of Procedure.—The order of procedure in carrying out the day's work by dead reckoning is as follows:

1. *Correct each course for leeway, deviation, and variation, thus finding the true courses, and arrange the same in tabular form, as will be shown. Add together the hourly distances for each course, and insert the same in the table opposite the corresponding true course.*

2. *Find in the Traverse Tables the difference of latitude and departure corresponding to each true course and distance, and place them in their respective columns, having previously marked (with a horizontal dash) the spaces not to be filled.*

3. *By adding the quantities in the four columns and taking the difference between north and south difference of latitude, and between east and west departure, the difference of latitude and departure due to the whole traverse is found, and thence the true course and distance made good according to the proper formulas. With this true course and distance, the difference of longitude made is found either by calculation or by inspection of the Traverse Tables. The position thus found is the position of the ship as determined by dead reckoning.*

31. The following example worked out in detail will illustrate the operations explained in the preceding article.

Illustration.—On July 10, at noon, a ship, by astronomical observations, was in the following position: Latitude 54° 3′ N and longitude 42° 18′ W; afterwards she sailed according to the following log-book account. Required, the latitude and longitude in on July 11, at noon.

NOON, JULY 10

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	7	2	N N W ½ W	N E ½ N	½	3° E	P. M.
2	6	8					
3	6	5					
4	6	8					
5	7	2					
6	7	5					Variation of compass = 27° W
7	5	8	N by W ¾ W	N E	¾	2° 30′ E	
8	5	5					
9	4	8					
10	4	5					
11	5	2					
12	4	2					Midnight
	JULY 11						
1	5	5					A. M.
2	5	5					
3	4	8	S E by E ¼ E	N E by E	1	1° W	
4	4	8					
5	4	2					
6	4	2					Variation of compass = 27° W
7	4	8	E S E	N E by N	½	2° W	
8	4	8					
9	5	2					
10	5	2					
11	7	8	E by N	N by W	0	2° 30′ W	
12	8	2					Noon, July 11

Proceed as follows: The courses recorded being compass courses, each must be reduced to true by applying the necessary corrections.

Correction of First Course

Comp. course = N N W $\frac{1}{2}$ W
Leeway (to left) = $\frac{1}{2}$ point

Course through water = N W by N
Or = N 33° 45′ W
Deviation = 3° 0′ E

Mag. course = N 30° 45′ W
Variation = 27° 0′ W

True course = N 57° 45′ W
Or = N 58° W

Correction of Second Course

Comp. course = N by W $\frac{3}{4}$ W
Leeway (to left) = $\frac{3}{4}$ point

Course through water = N N W $\frac{1}{2}$ W
Or = N 28° 7′ 30″ W
Deviation = 2° 30′ 0″ E

Mag. course = N 25° 37′ 30″ W
Variation = 27° 0′ 0″ W

True course = N 52° 37′ 30″ W
Or = N 53° W

In a similar manner, each of the remaining courses is corrected for leeway, deviation, and variation.

In practice, it is not necessary to work to seconds as shown here. If the minutes in the course are less than 30, they may be neglected; if over, the next higher whole degree may be used as the course.

The distance for the first course is found by adding up the hourly distances until the course is altered at 6 o'clock, which in this case gives a total distance of 42 mi. for the course N N W $\frac{1}{2}$ W; the distances for the second and remaining courses are found in exactly the same manner.

To find the difference of latitude and departure for each course and distance, construct a traverse as usual and take out from the Traverse Tables the D. Lat. and Dep. corresponding to each true course and distance. Thus,

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N 58° W	42	22.3	——	——	35.6
N 53° W	41	24.7	——	——	32.7
S 76° E	18	——	4.4	17.5	——
N 89° E	20	0.3	——	20.0	——
N 49° E	16	10.5	——	12.1	——
		57.8	4.4	49.6	68.3
		4.4			49.6
		D. Lat. = 53.4′ N			Dep. = 18.7′ W

To find the latitude in, apply the D. Lat. thus found to the latitude from in the usual manner. Thus,

Lat. from = 54° 3′ N
D. Lat. = 0° 53.4′ N
Lat. in = 54° 56.4′ N

To find the longitude in use the latitude from and latitude in, and find the M. Lat.; then compute the D. Long. according to the formula D. Long. = Dep. × sec M. Lat., and thence the longitude in, as follows:

Lat. from =	54° 3′ N	log 18.7 =	1.27184
Lat. in =	54° 56.4′ N	log sec 54° 30′ =	0.23605
Sum =	108° 59.4′	log D. Long. =	1.50789
M. Lat. =	54° 29.7′ N	D. Long. =	0° 32.2′ W
		Long. from =	42° 18′ W
		Long. in =	42° 50.2′ W

32. Taking the Departure.—When a vessel leaves one port for another she is usually conducted out of the harbor, bay, or river, by a pilot. The pilot lays his courses by ranges and landmarks with which long acquaintance has made him familiar. Arriving at a point where his local knowledge is of no further value, the pilot leaves the vessel and from there the navigating officer assumes the navigation of the ship. This officer's duty now is to fix what is called a *point of departure*, or a position accurately determined, from which to shape and plot courses to be run by the vessel. This is done according to some of the methods described in *Piloting;* cross-bearings, however, should be used for this purpose whenever available. Some navigators prefer to postpone establishing this position until the vessel is about to lose sight of land, but it is good policy to have the position fixed while the objects selected on shore are plainly visible instead of waiting until they are nearly lost on the horizon. If only a single object, such as a light, is in sight, the point of departure should be fixed not by a single bearing and estimated distance, but by two bearings and the distance run between them. At night, in addition to this, the time and bearing should be noted when the light disappears below the horizon; the distance of the light is then found from the chart or the list of lights giving the range of visibility of the light observed. This operation is usually known as **taking the departure** or "taking the farewell bearing."

33. Departure Course.—When working out the first day's work, this bearing is *reversed* and, together with the distance, is entered in the traverse and treated like the rest of the courses, the deviation applied being that due to the direction of the ship's head at the moment of taking the departure. Thus, if the bearing of an object used in establishing the point of departure is N E by E, and the distance is 10 miles, it is entered in the traverse as S W by W, distance 10 miles. The bearing thus reversed and entered in the traverse is usually known as the **departure course.** At the time of fixing the point of departure, the patent log should be read and noted.

34. In the following example, the departure course will be introduced.

EXAMPLE.—On September 11, at noon, a lighthouse in latitude 36° 30′ S and longitude 8° 42′ W bore, by compass, E N E $\frac{1}{2}$ E, distance 14 miles. The ship's head at the time of taking the departure was S by W, the deviation for that point being 7′ W. Afterwards, the ship sailed according to the following log-book account. Required, the latitude and longitude in on September 12 at noon.

NOON, SEPTEMBER 11

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	2	5	S W by W $\frac{1}{4}$ W	S S E	$2\frac{1}{4}$	14° W	P. M.
2	3	4					
3	2	3					
4	3	2	W $\frac{1}{2}$ S	South	$2\frac{1}{2}$	20° W	
5	4	4					Variation of compass 20° E
6	2	3					
7	2	3	N W by W	S W	2	23° 20′ W	
8	3	3					
9	4	0					
10	5	4					
11	4	2					
12	4	4					Midnight

(*Continued on next page.*)

SEPTEMBER 12

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	3	3	N W ¾ N	S W by W	2½	21° W	A. M.
2	3	3					
3	3	5					
4	4	2					
5	6	3	W ¼ N	S by W	1½	23° W	
6	3	7					Variation of compass 20° E
7	2	5					
8	5	0					
9	10	0	W ¼ S	S by E	0	20° 10′ W	
10	10	3					
11	8	0	E N E ½ E	S by E	0	21° 30′ E	
12	8	0					Noon, Sept. 12

SOLUTION.—

Bearing Reversed

W S W ½ W = S 73° 7′ W
Deviation = 7′ W
Mag. bearing = S 73° 0′ W
Variation = 20° 0′ E
True bearing rev. = S 93° 0′ W
Or = N 87° 0′ W

First Course

Comp. course = S W by W ¼ W
Leeway (to right) = 2¼ points
Course through water = W ½ S
Or = S 84° 22′ W
Deviation = 14° 0′ W
Mag. course = S 70° 22′ W
Variation = 20° 0′ E
True course = S 90° 22′ W
Or = west

Second Course

Comp. course = W ½ S
Leeway (to right) = 2½ points
Course through water = W N W
Or = N 67° 30° W
Deviation = 20° 0′ W
Mag. course = N 87° 30′ W
Variation = 20° 0′ E
True course = N 67° 30′ W
Or = W N W

Third Course

Comp. course = N W by W
Leeway (to right) = 2 points
Course through water = N W by N
Or = N 33° 45′ W
Deviation = 23° 20′ W
Mag. course = N 57° 5′ W
Variation = 20° 0′ E
True course = N 37° 5′ W
Or = N 37° 0′ W

The rest of the courses should then be corrected. This being done, the following traverse is formed:

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N 87° W	14.0	0.7	——	——	14.0
West	8.2	——	——	——	8.2
W N W	9.9	3.8	——	——	9.2
N 37° W	23.6	18.9	——	——	14.2
N 9° W	14.3	14.1	——	——	2.2
N 73° W	17.5	5.1	——	——	16.7
S 87° W	20.3	——	1.1	——	20.3
S 65° E	16.0	——	6.8	14.5	——
		42.6	7.9	14.5	84.8
		7.9			14.5
	D. Lat. =	34.7′ N		Dep. =	70.3′ W

For the latitude in,

Lat. from = 36° 30′ S
D. Lat. = 34.7′ N

Lat. in = 35° 55.3′ S. Ans.

For the longitude in,

Lat. from = 36° 30′ S
Lat. in. = 35° 55.3′ S

Sum = 72° 25.3′
M. Lat. = 36° 12.6′ S

D. Long. = Dep. × sec M. Lat.
log 70.3 = 1.84696
log sec 36° 13′ = 0.09324

log D. Long. = 1.94020
D. Long. = 87.1′ W

Long. from = 8° 42′ W
D. Long. = 1° 27.1′ W

Long. in = 10° 9.1′ W. Ans.

35. Introduction of Current in the Day's Work. In the following examples, it should be noted that the ship is supposed to have sailed in a current. The set and drift of the current should here be treated according to the instructions given in Art. **18.**

EXAMPLE 1.—At noon, May 2, the departure is taken from a point in latitude 38° 43′ S and longitude 86° 24′ E. The bearing by compass is

S S E $\frac{1}{2}$ E, and the distance 16 miles. The deviation for the direction of the ship's head (N W) at the time of taking the departure was 16° W. The set of the current, which affects the ship during her entire cruise, from noon to noon, is S E by E (magnetic) 39 miles. The courses and distances sailed are as follows. Find the latitude and longitude in at noon, May 3.

Noon, May 2

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	9	8	N W by W	N by E	1	16° W	P. M.
2	8	2					
3	8	8					
4	8	2					
5	9	8	S E by S	S W by S	$\frac{1}{4}$	11° E	
6	9	2					Variation of compass 25° W
7	10	8					
8	10	2					
9	10	2	S $\frac{1}{2}$ W	East	0	2° W	
10	10	2					
11	10	2					
12	10	4					Midnight
			May 3				
1	9	8	W $\frac{1}{4}$ N	North	$\frac{1}{2}$	26° W	A. M.
2	9	6					
3	8	8					
4	8	8					
5	7	8	N N E $\frac{1}{2}$ E	N W by W	$1\frac{3}{4}$	8° E	
6	8	6					Variation of compass 25° W
7	8	8					
8	8	8					
9	9	2	S by E	E by S	$\frac{3}{4}$	3° E	
10	9	2					
11	9	2					
12	9	4					Noon, May 3

SOLUTION.—Correction of courses:

Bearing Reversed

N N W $\frac{1}{2}$ W = N 28° W
Deviation = 16° W
Mag. bearing = N 44° W
Variation = 25° W
True bearing = N 69° W

First Course

Comp. course = N W by W
Leeway (to left) = 1 point
Course through water = W N W
Or = N 67° W
Deviation = 16° W
Mag. course = N 83° W
Variation = 25° W
True course = N 108° W
Or = S 72° W

Second Course

Comp. course = S E by S
Leeway (to left) = $\frac{1}{4}$ point
Course through water = S E $\frac{3}{4}$ S
Or = S 36° 34′ E
Deviation = 11° 0′ E
Mag. course = S 25° 34′ E
Variation = 25° 0′ W
True course = S 50° 34′ E
Or = S 51° 0′ E

Third Course

Comp. course = S $\frac{1}{2}$ W
Leeway = 0
Course through water = S $\frac{1}{2}$ W
Or = S 5° 37′ W
Deviation = 2° 0′ W
Mag. course = S 3° 37′ W
Variation = 25° 0′ W
True course = S 21° 23′ E
Or = S 21° 0′ E

Fourth Course

Comp. course = W $\frac{1}{4}$ N
Leeway (to left) = $\frac{1}{2}$ point
Course through water = W $\frac{1}{4}$ S
Or = S 87° W
Deviation = 26° W
Mag. course = S 61° W
Variation = 25° W
True course = S 36° W

Fifth Course

Comp. course = N N E $\frac{1}{2}$ E
Leeway (to right) = $1\frac{3}{4}$ points
Course through water = N E $\frac{1}{4}$ E
Or = N 47° 49′ E
Deviation = 8° 0′ E
Mag. course = N 55° 49′ E
Variation = 25° 0′ W
True course = N 30° 49′ E
Or = N 31° 0′ E

Sixth Course

Comp. course = S by E
Leeway (to left) = $\frac{3}{4}$ point
Course through water = S $\frac{1}{4}$ E
Or = S 3° E
Deviation = 3° E
Mag. course = south
Variation = 25° W
True course = S 25° E

Current

Mag. set = S E by E
Or = S 56° E
Variation = 25° W
True set = S 81° E

Adding together the distances for each course and placing the whole in form of a traverse, the result is as follows:

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N 69° W	16	5.7	——	——	14.9
S 72° W	35	——	10.8	——	33.3
S 51° E	40	——	25.2	31.1	——
S 21° E	41	——	38.3	14.7	——
S 36° W	37	——	29.9	——	21.7
N 31° E	34	29.1	——	17.5	——
S 25° E	37	——	33.5	15.6	——
S 81° E	39	——	6.1	38.5	——
		34.8	143.8	117.4	69.9
			34.8	69.9	
		D. Lat. =	109.0′ S	47.5′ E	= Dep.

For the latitude in,

$$\begin{aligned} \text{Lat. from} &= 38^\circ\, 43'\ \text{S} \\ \text{D. Lat.} &= 1^\circ\, 49'\ \text{S} \\ \hline \text{Lat. in} &= 40^\circ\, 32'\ \text{S.}\quad \text{Ans.} \end{aligned}$$

For the longitude in,

$$\text{D. Long.} = \text{Dep.} \times \sec \text{M. Lat.}$$

$$\begin{aligned} \text{Lat. from} &= 38^\circ\, 43'\ \text{S} & \log 47.5 &= 1.67669 \\ \text{Lat. in} &= 40^\circ\, 32'\ \text{S} & \log \sec 39^\circ\, 37' &= 0.11332 \\ \hline \text{Sum} &= 78^\circ\, 75' & \log \text{D. Long.} &= 1.79001 \\ \text{M. Lat.} &= 39^\circ\, 37'\ \text{S} & \text{D. Long.} &= 61.7'\ \text{E} \end{aligned}$$

$$\begin{aligned} \text{Long. from} &= 86^\circ\, 24'\ \ \text{E} \\ \text{D. Long.} &= 1^\circ\ \ 1.7'\ \text{E} \\ \hline \text{Long. in} &= 87^\circ\, 25.7'\ \text{E.}\quad \text{Ans.} \end{aligned}$$

EXAMPLE 2.—On August 24, at noon, a ship, by astronomical observations, was in latitude 23° 30′ S and longitude 15° 20′ E; and at noon on August 25, by similar observations, she was found to be in latitude 25° 30.4′ S and longitude 15° 36.4′ E. Between these periods she sailed according to the following log-book account. For the first 4 hours, a current set the ship 2 knots an hour E N E (magnetic); at 8 o'clock A. M. a tidal stream of unknown direction and velocity was

encountered. Find the latitude and longitude in by dead reckoning at noon, August 25, the course and distance made good, and the set and velocity of the tidal stream.

NOON, AUGUST 24

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	5	4	S W by S	W by N	1	6° 10′ W	P. M.
2	5	9					
3	4	7					
4	4	9					
5	6	1					
6	6	3					Variation of
7	6	0					compass 28° 7′ W
8	5	1					
9	5	0	E by S	N N E	2	7° 10′ E	
10	7	2					
11	8	7					
12	7	3					Midnight
	AUGUST 25						
1	5	0	S E	E N E	½	2° E	A. M.
2	5	7					
3	5	9					
4	6	4	W S W	East	0	8° W	
5	6	0					Variation of
6	5	1					compass 28° 7′ W
7	3	9					
8	4	7					
9	5	2					
10	5	9					
11	6	8					
12	8	4					Noon, August 25

SOLUTION.—Correct the courses in the usual manner and add together the distances for each course. Remember to correct the current for variation only, since its given set is magnetic. The traverse formed from these courses and distances is as follows:

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S 12° E	44.4	——	43.4	9.2	——
S 77° E	28.2	——	6.3	27.5	——
S 65° E	16.6	——	7.0	15.0	——
S 31° W	52.4	——	44.9	——	27.0
N 39° E	8.0	6.2	——	5.0	——
		6.2	101.6	56.7	27.0
			6.2	27.0	
			D. Lat. = 95.4′ S	29.7′ E = Dep.	

For the course and distance made good,

$$\tan C = \text{Dep.} \div \text{D. Lat.}$$
$$\log 29.7\ (+10) = 11.47276$$
$$\log 95.4 = \underline{\ 1.97955}$$
$$\log \tan C = 9.49321$$
$$\text{Course} = \text{S } 17° 18' \text{ E. Ans.}$$

$$\text{Dist.} = \text{D. Lat.} \times \sec C$$
$$\log 95.4 = 1.97955$$
$$\log \sec 17° 18' = \underline{0.02011}$$
$$\log \text{Dist.} = 1.99966$$
$$\text{Distance} = 99.9 \text{ mi. Ans.}$$

For the latitude in,

$$\text{Lat. from} = 23° 30' \text{ S}$$
$$\text{D. Lat.} = \underline{\ 1° 35.4' \text{ S}}$$
$$\text{Lat. in} = 25° 5.4' \text{ S}$$

For the longitude in, using Mercator's sailing, the result is as follows:

$$\text{1st Lat. } 23° 30' \text{ S} \ldots \text{M. P. } 1442.1$$
$$\text{2d Lat. } 25° 5.4' \text{ S} \ldots \text{M. P. } \underline{1546.2}$$
$$\text{M. D. Lat.} = 104.1$$

$$
\begin{aligned}
\text{D. Long.} &= \text{M. D. Lat.} \times \tan C \\
\log 104.1 &= 2.01745 \\
\log \tan 17^\circ 18' &= 9.49341 \\
\hline
\log \text{D. Long.} &= 1.51086 \\
\text{D. Long.} &= 32.4' \text{ E}
\end{aligned}
$$

$$
\begin{aligned}
\text{Long. from} &= 15^\circ 20' \text{ E} \\
\text{D. Long.} &= 32.4' \text{ E} \\
\hline
\text{Long. in} &= 15^\circ 52.4' \text{ E. Ans.}
\end{aligned}
$$

For the set and velocity of tidal stream proceed as in Art. **24**. Thus,

Position	*Lat.*	*M. P.*	*Long.*
By observation	25° 30.4′ S	1573.7	15° 36.4′ E
By D. R. . . .	25° 5.4′ S	1546.2	15° 52.4′ E
	D. Lat. = 25′ S	M. D. Lat. = 27.5	D. Long. = 16′ W

$$
\begin{aligned}
\tan C &= \text{D. Long.} \div \text{M. D. Lat.} \\
\log 16\,(+10) &= 11.20412 \\
\log 27.5 &= 1.43933 \\
\hline
\log \tan C &= 9.76479 \\
\text{Course} &= \text{S } 30^\circ 12' \text{ W}
\end{aligned}
$$

$$
\begin{aligned}
\text{Dist.} &= \text{D. Lat.} \times \sec C \\
\log 25 &= 1.39794 \\
\log \sec 30^\circ 12' &= 10.06335 \\
\hline
\log \text{Dist.} &= 1.46129 \\
\text{Dist.} &= 28.9 \text{ mi.}
\end{aligned}
$$

Hence, the set of the tidal stream met with at 8 A. M. was S 30° 12′ W, and, assuming it to have affected the ship with a uniform velocity up to 12 o'clock noon, the stream's drift, or velocity, per hour was 28.9 ÷ 4 = 7.2 mi. Ans.

EXAMPLE FOR PRACTICE

1. On November 6, at noon, a departure was taken from a point in latitude 54° 20′ S and longitude 6° 42′ E, which bore by compass S S E $\frac{1}{2}$ E a distance of 12 miles. The deviation for ship's head at time of taking departure was 15° W. Afterwards, the following courses and distances were sailed. During the entire period from noon to noon the ship was influenced by a current, the magnetic set of which was S E by E 38 miles. Required, the latitude and longitude in at noon, November 7.

NOON, NOVEMBER 6

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	9	8	N N W	N E by E	$\frac{1}{2}$	7° W	P. M.
2	9	8					
3	9	8					
4	9	6					
5	8	6	W S W	South	1	18° W	
6	7	2					Variation of compass 24° W
7	7	6					
8	8	6					
9	9	8	S S E	S W	$\frac{1}{4}$	7° E	
10	10	8					
11	10	8					
12	10	6					Midnight
	NOVEMBER 7						
1	8	0	E $\frac{1}{2}$ S	S by E	$1\frac{1}{4}$	21° E	A. M.
2	7	0					
3	7	0					
4	8	0					
5	8	2	S W by W	S S E	$\frac{3}{4}$	16° W	
6	9	6					Variation of compass 24° W
7	9	2					
8	9	0					
9	8	0	N W by W	N by E	$1\frac{1}{2}$	15° W	
10	7	0					
11	7	0					
12	6	0					Noon, Nov. 7

Ans. $\begin{cases} \text{Lat. in} = 55° 36' \text{ S} \\ \text{Long. in} = 6° 24.6' \text{ E} \end{cases}$

2. On November 29, at noon, a vessel, by astronomical observations, was found to be in latitude 60° 51′ N and longitude 20° 54′ W; afterwards she sailed according to the following log-book account. During the 24 hours from noon, November 29, to noon, November 30, the vessel was affected by a current, the magnetic set of which was S W by S and the drift $1\frac{1}{2}$ knots per hour. Between the hours of 8 and 12 P. M., a tidal stream was encountered, the magnetic set of which

was N E $\frac{1}{2}$ N and the estimated velocity 3 knots per hour. Required, the latitude and longitude in on November 30 at noon.

Noon, November 29

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	9	8	N E $\frac{1}{2}$ E	N W by W	1	14° E	P. M.
2	7	6					
3	7	8					
4	7	8					
5	8	8	N N E	N W by W	$\frac{1}{2}$	6° E	
6	9	2					Variation of compass 26° W
7	9	0					
8	9	0					
9	8	0	E S E	South	$1\frac{1}{4}$	17° E	
10	7	2					
11	7	8					
12	7	0					Midnight
	November 30						
1	9	8	S E $\frac{1}{4}$ E	S S W	$\frac{3}{4}$	13° E	A. M.
2	8	8					
3	9	8					
4	9	6					
5	8	8	E by N	N $\frac{1}{2}$ E	$1\frac{3}{4}$	21° E	
6	7	0					Variation of compass 26° W
7	6	0					
8	6	2					
9	6	8	N $\frac{1}{2}$ W	W by N	$2\frac{1}{2}$	2° W	
10	6	6					
11	5	6					
12	7	0					Noon, Nov. 30

Ans. $\begin{cases} \text{Lat. in} = 61° \ 35.2' \text{ N} \\ \text{Long. in} = 16° \ 47.3' \text{ W} \end{cases}$

3. On March 5, at noon, a ship was in the following position, determined by astronomical observations: Latitude 39° 55.9′ N and longitude 60° 20′ W. Afterwards, the courses and distances recorded in the accompanying log-book account were sailed. A current set S W by S

(magnetic) 38 miles during the 24 hours, and a tidal stream was met at 3 P. M., the magnetic set of which was S 7° W and the velocity 2 miles per hour, affecting the ship for 3 hours. Required, the latitude and longitude in on March 6 at noon; also find, by Mercator's sailing, the course and distance from the position thus found to Sandy Hook Lightship in latitude 40° 27′ N and longitude 73° 52′ W.

NOON, MARCH 5

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	6	2	S E by E	S by W½	2½	15° E	P. M.
2	5	0					
3	6	0					
4	5	8					
5	5	0	S by W	S E by E	3	3° W	
6	4	8					
7	4	2					Variation of compass 26° W
8	3	0					
9	3	0	W ½ S	S by W	4	24° W	
10	2	6					
11	2	4					
12	1	0	W N W	North	5	10° W	Midnight
			MARCH 6				
1	1	0					A. M.
2	1	0					
3	1	0					
4	2	8	N W	N N E	3	14° W	
5	3	4					Variation of compass 26° W
6	4	8					
7	4	8	N N E	East	2¼	7° E	
8	5	0					
9	5	0					
10	5	2					
11	6	8	N by W	N E by E	1½	2° W	
12	7	2					Noon, March 6

Ans. $\begin{cases} \text{Lat. in} = 39° 14' \text{ N} \\ \text{Long. in} = 60° 50' \text{ W} \\ \text{True course to Sandy Hook Lightship} = \text{N } 83° \text{ W, nearly} \\ \text{Distance} = 607.6 \text{ mi.} \end{cases}$

4. Referring to the preceding example, suppose that at 6 o'clock P. M. a ship near by displays signals to the effect that she has lost her reckoning on account of having been tossed about by severe weather and asks, as a favor, to be given the latitude and longitude in at that hour. What would be the answer? Ans. $\begin{cases} \text{Lat. in} = 39° 33.4' \text{ N} \\ \text{Long. in} = 59° 52.8' \text{ W} \end{cases}$

GREAT-CIRCLE SAILING

INTRODUCTION

36. Geometry teaches that the shortest distance between two places, or points, is the straight line that joins them; consequently, the shortest path, or distance, between two places on the earth's surface is the straight line connecting one with the other. This path, however, would necessarily pass through some portion of the earth, while the methods of traveling or communicating from one place to another are confined to its surface only. Hence, the question is limited to the shortest distance between two places on and over the surface of the earth, and this distance on a globe, or representation of the earth, is found by stretching a thread from one place to the other on its surface. Such an arc extended around the entire globe would be a **great circle,** the plane of which would pass through the center of the globe. The art of sailing along such an arc is termed by navigators **great-circle sailing,** and the arc itself is called a **great-circle track.**

37. The properties of great-circle sailing were known as far back as the 14th century, and in ancient works on navigation it was described as *globular sailing.* The general applications of great-circle sailing for the purpose of practical navigation, were not effected, however, until the introduction of steam power had rendered steamships practically

independent of the wind, and at the present day of long voyages and great competition, considerable time and distance may be saved in reaching the port of destination by following the great-circle track leading to it.

38. Advantages of Great-Circle Sailing.—In the main, there are two advantages in favor of the great-circle track over the rhumb track; namely, the saving in distance and the saving in time. Reduction in distance is greatest in high latitudes between places not differing much in latitude but differing greatly in longitude; it is least between places that are situated nearly on the same meridian, either in north or south latitude, or one in north latitude and the other in south latitude. For instance, the distance on the great-circle track between Cape Agulhas, the southerly extremity of Africa, and Perth, Australia, is 4,595 miles, while the distance that has to be traversed by a ship following the rhumb track between the same places is 4,799 miles. A saving of 204 miles is thus effected by sailing along the great circle passing through the two places.

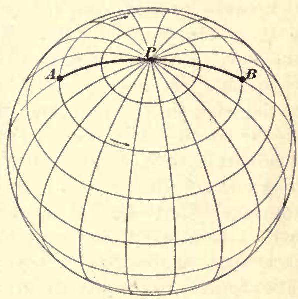

Fig. 9

The saving in time can only be estimated after having laid out the great-circle track on the chart and considered whether the regions through which it passes are favorable in regard to winds, currents, etc. The contrast between the great-circle track and the rhumb track would be more noticeable if it were possible to navigate the Arctic Ocean between two places in the same north latitude, but differing 180° in longitude. Thus, in Fig. 9, let A and B represent two places situated on the same latitude parallel, the difference in longitude being 180°, and let P represent the north pole. It is

evident, then, that by following the rhumb course a vessel at A would have to steer either east or west in order to reach B; but, by following the great-circle track APB, her first course would be north from A to P and then south from P to B. A glance at the figure will show that the difference in distance, and, consequently, in time, between the two routes would be considerable.

39. Advantage of Great-Circle Track to Sailing Ships.—The advantages just mentioned apply in particular to steamers, which, by virtue of their motive power, can readily adopt the constant changes of courses necessary in great-circle sailing. But sailing ships may also profit in certain cases by following the great-circle track, even with an unfavorable wind blowing. Thus, if the rhumb course to the port of destination is E N E, the great-circle course N E, and the wind E N E, it is far more advantageous to lay the ship on the starboard tack, in which case she would be only 4 points off the direct course leading to the destination; whereas, if laid on the port tack, she would be 8 points off the course. In general, when starting on a transoceanic voyage, or at any period during such voyage, with a head-wind, the ship should be close-hauled on the tack that approaches most nearly the great-circle track. The term "windward great-circle sailing" is used with special reference to these facts.

The rule for sailing ships, as laid down by Towson, a prominent advocate of great-circle sailing, is as follows: *Ascertain the great-circle course and put the ship on that tack which is the nearer to the great-circle course.*

The object of great-circle sailing, therefore, is to determine what courses should be run in order that the ship may be kept on the great-circle track and thus traverse the shortest route to the port of destination.

40. The Great-Circle Track.—If a ship could be kept exactly on the great-circle track when sailing from one port to another, her head would be always pointing toward the port of destination; this would, however, necessitate a continual

change of the course, unless the track ran along a meridian or along the equator.

In order to avoid this constant alteration of courses, it is customary in practice to make use of the principle that a small arc of a large circle is practically a straight line. Points are therefore taken on the great-circle track at convenient distances apart and the bearing of each point to the next is noted on the chart; the ship is then kept on these courses from point to point. Thus, in Fig. 10, if a ship

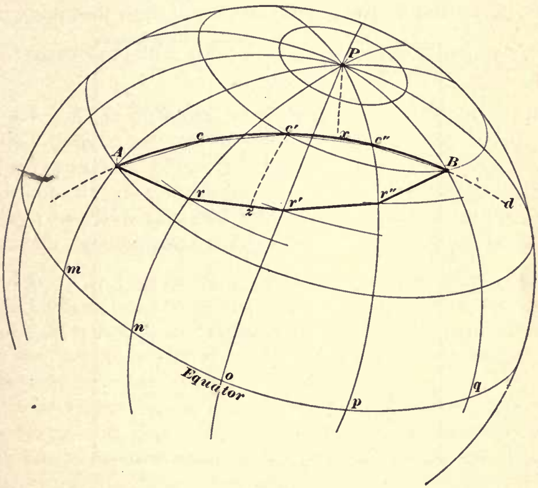

FIG. 10

were to sail from A to B along the great-circle track $A c c' c'' B$, her first course would be from A to c, her second from c to c', and so on, until B was reached. In this manner, the advantages to be gained by following the great-circle track are obtained without frequent change of courses.

41. From the foregoing it is evident that a ship sailing on a great-circle track makes straight for the port of destination, and her path through the water intersects the meridians

Pm, *Pn*, *Po*, etc. at an angle that is always varying; whereas, by following the rhumb track $A\,r\,r'\,r''\,B$, the ship crosses all meridians at the same angle. In other words, on the rhumb track the ship's head is kept on the same point of the compass, and she never steers directly for the port until it is in sight.

42. Representation of Track on a Mercator's Chart. From the preceding explanation, it follows that on a Mercator's chart the great-circle track must necessarily be represented by a *curve*, and a little consideration will show that this curve

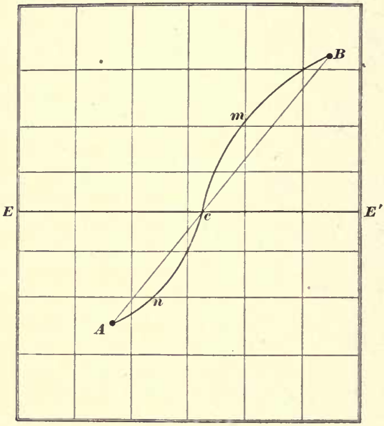

Fig. 11

must lie in a higher latitude than the rhumb track, as shown in Figs. 10 and 19. Thus, if the great-circle track is in the northern hemisphere, it lies nearer to the north pole than the rhumb track; if in the southern hemisphere, it lies nearer the south pole. This explains how the curved track *A V B*, Fig. 19, between *A* and *B* on a Mercator's chart, represents a shorter distance between the two places than does a straight line (rhumb track). The difference of longitude is the same for both tracks, but the great-circle track has the

advantages of *shorter longitude degrees*, measured on higher latitude parallels. The higher the latitude, the more the tracks differ; this is particularly so when the two places are nearly on the same parallel.

When the two places are situated in different hemispheres and not on the same meridian, as A and B, Fig. 11, the great-circle track $A\,n\,c\,m\,B$ between them will be represented on a Mercator's chart, as shown in the figure, by a curve of double flexure intersecting the equator EE' at the point c.

DEFINITIONS RELATING TO GREAT-CIRCLE SAILING

43. When following a great-circle track, the first course sailed on is called the **initial course,** and the last one the **final course.** Thus, in Fig. 10, the angle PAc is the initial course, and the angle PBd or qBd is the final course. When the place from and the place arrived at lie on the same latitude parallel, the initial and the final course are equal.

44. The **vertex** of a great circle is the point at which the highest latitude is reached, or the point at which the track most nearly approaches the pole and runs perpendicular to the meridian. Thus, in Fig. 10, x is the vertex of the great-circle track between A and B.

It is evident that on every great circle there must be two vertexes, and that they must be situated midway between the points of intersection of the great circle and the equator, one in north and the other in south latitude. However, it is not necessary to consider more than one vertex; namely, the one relating to the great-circle track. The vertex may or may not be situated *between* the two places, but, if they are in the same latitude, it lies midway between them.

If both of the angles PAc and PBc'', Fig. 10, are less than 90°, the vertex will lie between the two places; but if one of the angles is greater than 90°, the vertex will lie on the arc produced on that side on which the greater angle is situated.

45. Point of Maximum Separation.—When the two places are situated in the same hemisphere, the ship following the great-circle track is always in a higher latitude than if following the rhumb track. Hence, since both tracks coincide at A and B (see Fig. 10), there must be some point on the great circle at which its distance from the rhumb track is greater than any other point. This point c' is called the **point of maximum separation**, and is evidently situated between the two places. When the vertex is also situated between the places, the point of maximum separation falls between the vertex and the place most distant in longitude from the vertex. If the two places are situated on the same latitude parallel, the vertex and the point of maximum separation coincide.

46. When the two places are situated in different hemispheres, as in Fig. 11, there must be two points of maximum separation—one to the northward of the rhumb track in north latitude, and one to the southward in south latitude. In the figure, m is the former and n the latter.

The point of maximum separation may also be defined as the point on a great circle where two ships sailing, one on the great circle and the other on the rhumb track, but changing their longitude at a uniform rate, are most widely separated in latitude. At these places c' and z, Fig. 10, it is evident that the true course steered on one track must be the same as that steered on the other track, because at these points the tracks run parallel.

TO DETERMINE A GREAT-CIRCLE TRACK

47. Problems in great-circle sailing may be solved either by spherical trigonometry, according to the data in each case, or by graphical methods.

The method of determining by calculation the different courses and distances to be run along a great-circle track is very seldom used in practice. In nautical examinations, however, candidates are frequently required to use formulas and to derive formulas employed in great-circle sailing. For

this reason this subject will be treated here in the simplest form possible.

48. To Find the Initial and the Final Course.—In Fig. 12, let P represent the pole nearest the great-circle track AVB, A the place having *lowest* latitude, and B the other place. Also, in this case, let A denote the initial course PAV, B the supplement of the final course PBd, and V the vertex.

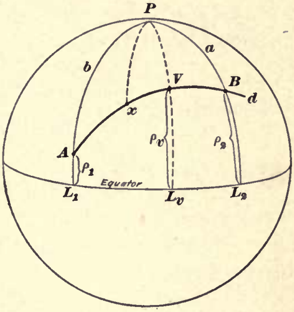

Fig. 12

If the latitude of A is denoted by ρ_1 and its longitude by L_1, the latitude of B by ρ_2 and its longitude by L_2, and the latitude of V by ρ_v and its longitude L_v, then, the side $b = 90° - \rho_1$, and the side $a = 90° - \rho_2$.

Since an angle at the pole between two meridians is measured by the arc of the equator intercepted by the meridians, it is evident that the angle APB is equal to $L_1 - L_2$, or equal to the difference of longitude between A and B. By Napier's analogies,

$$\tan \tfrac{1}{2}(A + B) = \frac{\cos \dfrac{a - b}{2}}{\cos \dfrac{a + b}{2}} \times \cot \frac{APB}{2}$$

and
$$\tan \tfrac{1}{2}(A - B) = \frac{\sin \dfrac{a - b}{2}}{\sin \dfrac{a + b}{2}} \times \cot \frac{APB}{2}$$

Substituting in these equations the values of a and b just given,

$$\tan \tfrac{1}{2}(A + B) = \frac{\cos \dfrac{90° - \rho_2 - 90° + \rho_1}{2}}{\cos \dfrac{90° - \rho_2 + 90° - \rho_1}{2}} \times \cot \tfrac{1}{2} \text{ D. Long.},$$

$$\text{or, } \tan \tfrac{1}{2}(A+B) = \frac{\cos \dfrac{\rho_1 - \rho_2}{2}}{\cos\left(90° - \dfrac{\rho_1 + \rho_2}{2}\right)} \times \cot \tfrac{1}{2} \text{ D. Long.}$$

Whence,

$$\tan \tfrac{1}{2}(A+B) = \frac{\cos \dfrac{\rho_1 - \rho_2}{2}}{\sin \dfrac{\rho_1 + \rho_2}{2}} \times \cot \tfrac{1}{2} \text{ D. Long.} \qquad (1)$$

Similarly,

$$\tan \tfrac{1}{2}(A-B) = \frac{\sin \dfrac{90° - \rho_2 - 90° + \rho_1}{2}}{\sin \dfrac{90° - \rho_2 + 90° - \rho_1}{2}} \times \cot \tfrac{1}{2} \text{ D. Long.,}$$

$$\text{or, } \tan \tfrac{1}{2}(A-B) = \frac{\sin \dfrac{\rho_1 - \rho_2}{2}}{\sin\left(90° - \dfrac{\rho_1 + \rho_2}{2}\right)} \times \cot \tfrac{1}{2} \text{ D. Long.}$$

Whence,

$$\tan \tfrac{1}{2}(A-B) = \frac{\sin \dfrac{\rho_1 - \rho_2}{2}}{\cos \dfrac{\rho_1 + \rho_2}{2}} \times \cot \tfrac{1}{2} \text{ D. Long.} \qquad (2)$$

Hence, the initial course, or $A = \frac{1}{2}(A+B) - \frac{1}{2}(A-B)$, and the final course, or $B = \frac{1}{2}(A+B) + \frac{1}{2}(A-B)$.

49. To Find the Distance.—In the triangle APB, Fig. 12, the distance AB is found as follows:

$$\sin A : \sin APB = \sin a : \sin AB$$

Whence, $$\sin AB = \frac{\sin a \times \sin APB}{\sin A}$$

But a is equal to $90° - \rho_2$ and the angle APB represents the difference of longitude; hence,

$$\sin AB = \frac{\sin(90° - \rho_2) \times \sin \text{ D. Long.}}{\sin A}$$

Whence, denoting the distance AB by D,

$$\sin D = \frac{\cos \rho_2 \times \sin \text{ D. Long.}}{\sin A}$$

NOTE.—When the difference of longitude is near 90°, the distance D is more accurately and more conveniently determined by the formula

$$\tan \tfrac{1}{2} D = \frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} \times \cot \frac{\rho_1 + \rho_2}{2}$$

No doubt should then be entertained as to whether D is less or greater than 90°.

50. To Find the Vertex.—In the right triangles AVP and BVP, Fig. 12, when the angles A and B are known,

$$\sin PV = \sin A \times \sin b$$

and

$$\sin PV = \sin B \times \sin a$$

Substituting in these equations the value of a, b, and PV, the latter being equal to $90° - \rho_v$,

$$\sin(90° - \rho_v) = \sin A \times \sin(90° - \rho_1)$$

and

$$\sin(90° - \rho_v) = \sin B \times \sin(90° - \rho_2)$$

But, $\sin(90° - \rho_v) = \cos \rho_v$, the *latitude* of the vertex V; and, likewise, $\sin(90° - \rho_1) = \cos \rho_1$ and $\sin(90° - o_2) = \cos \rho_2$. Hence,

$$\cos \rho_v = \sin A \times \cos \rho_1 = \sin B \times \cos \rho_2 \qquad (1)$$

The *longitude* of the vertex (L_v) is obtained from the same right triangles by calculating the angles APV and BPV, each being equivalent to $(L_1 - L_v)$ and $(L_v - L_2)$, respectively. Thus,

$$\cos b = \cot A \times \cot APV$$

and

$$\cos a = \cot B \times \cot BPV$$

Substituting and transposing,

$$\left.\begin{aligned} \cot(L_1 - L_v) &= \frac{\cos(90 - \rho_1)}{\cot A} = \sin \rho_1 \times \tan A \\ \text{and} \quad \cot(L_v - L_2) &= \frac{\cos(90 - \rho_2)}{\cot B} = \sin \rho_2 \times \tan B \end{aligned}\right\} \qquad (2)$$

Or, by using one or the other of these formulas the difference of longitude between the vertex and either of the places is obtained.

51. To Find the Point of Maximum Separation. The latitude and longitude of the point of maximum separation can be calculated by remembering that at this point the great-circle track and the rhumb track are *parallel;* that is, the courses on both are the same. If x, Fig. 12, represents the position of this point, then, in the right triangle PVx,

the side PV and the angle PxV are known, the latter being equal to the rhumb course as laid out between the two places on a Mercator's chart. From these known data, Px, the complement of the latitude of x, and VPx, the difference of longitude between x and V (the vertex), are readily determined. Denoting the latitude of x by ρ_x, then, in the triangle PVx,

$$\sin PV = \sin PxV \times \sin Px$$

Substituting and transposing,

$$\sin(90° - \rho_x) = \frac{\sin(90° - \rho_v)}{\sin PxV}$$

Whence,
$$\cos \rho_x = \frac{\cos \rho_v}{\sin \text{ rhumb course}} \qquad (1)$$

The longitude of $x(L_x)$ is found by calculating the angle $xPV(= L_x - L_v)$ in the same triangle. Thus,

$$\cos Px = \cot PxV \times \cot xPV$$

Substituting and transposing,

$$\cot(L_x - L_v) = \frac{\cos(90° - \rho_x)}{\cot PxV}$$

Whence,

$$\cot(L_x - L_v) = \frac{\sin \rho_x}{\cot \text{ rhumb course}} \qquad (2)$$

Or the cotangent of the difference of longitude between x and V is obtained by dividing the sine of the latitude of x by the cotangent of the number of degrees in the rhumb course between A and B.

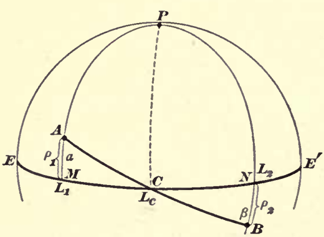

FIG. 13

52. To Find the Point of Intersection of the Great-Circle Track With the Equator, and the Course to be Steered There.—Let EE', Fig. 13, represent the equator; ACB, the great-circle track between A and B; α, the initial, and β, the final course, both being reckoned from that portion of the meridian nearest the equator; and L_c

the longitude of the point of intersection C. L_c is then found from either of the triangles AMC or BNC by calculating the difference of longitude $L_1 - L_c$ or $L_c - L_2$. Thus,

$$\sin \rho_1 = \cot \alpha \times \tan (L_1 - L_c)$$

and $$\sin \rho_2 = \cot \beta \times \tan (L_c - L_2)$$

Whence,

$$\left.\begin{aligned} \tan (L_1 - L_c) &= \sin \rho_1 \times \tan \alpha \\ \text{and} \quad \tan (L_c - L_2) &= \sin \rho_2 \times \tan \beta \end{aligned}\right\} \quad (1)$$

From the same triangles, the complement of the course, $MCA = NCB = S$ is obtained as follows:

$$\left.\begin{aligned} \cos S &= \sin \alpha \times \cos \rho_1 \\ \text{and} \quad \cos S &= \sin \beta \times \cos \rho_2 \end{aligned}\right\} \quad (2)$$

53. Order of Procedure.—To determine, by calculation, the great-circle track between two places situated on the same side of the equator, the following order of procedure will be found convenient.

1. Find the initial and final course by formulas **1** and **2**, Art. **48**.
2. Find the distance by the formula of Art. **49**.
3. Find the position of the vertex by formulas **1** and **2**, Art. **50**.
4. Find the position of the point of maximum separation by formulas **1** and **2**, Art. **51**.
5. Then, on a Mercator's chart, draw the rhumb course between the two places; lay off the initial and final course, mark the position of the vertex and the point of maximum separation, and through the latter draw a line parallel to the rhumb course. The curve traced through these four points will then represent the great-circle track.

If one place is situated in south and the other in north latitude, or vice versa, proceed as follows:

1. Calculate the initial and final course, and the distance as just directed.
2. Find the point of intersection of the great-circle track with the equator by formula **1**, Art. **52**, and the course to be steered when crossing the equator by formula **2**, Art. **52**.
3. Find the rhumb course between the point of intersection and each place, according to Mercator's sailing

4. Find the position of the point of maximum separation on both sides of the equator by formulas **1** and **2**, Art. **51**.

5. Having determined these five points, the two places, the point of intersection with the equator, and the two points of maximum separation, the great-circle track is traced on a Mercator's chart.

54. Determination of Courses to be Steered.—To find the different courses to be steered when following the great-circle track, get the difference between the initial course and the course at the point of maximum separation (equal to the rhumb course) and find how many quarter points are contained in it. Divide the distance between the first place and the point of maximum separation by this number of quarter points; the result will be the number of miles to be sailed on each quarter-point course.

For instance, assume the initial course to be N W, the course at the point of maximum separation W N W, and the distance between these points 800 miles. Now, the difference between N W and W N W is 2 points, or 8 quarter points; hence, dividing 800 miles by 8 will give 100 miles for each quarter-point course. In other words, the course will have to be changed one-quarter point to the west for every 100 miles run.

Proceed similarly to find the course and distance from the point of maximum separation to the point of destination.

55. It is evident that the difference between the courses can be divided into still smaller divisions if required. For instance, in the case just mentioned, the difference could be divided into eighths of a point; the result would then be to change the course one-eighth point for each 50 miles run. For ordinary practice, however, quarter points will suffice.

If the great-circle track crosses the equator, find the courses and distance from the first place to the nearest point of maximum separation; thence to the point of intersection with the equator; thence to the other point of maximum separation; and thence to the port of destination.

56. Courses Computed Are True.—It must be borne in mind that the courses computed are all *true* courses and subject to corrections for variation, deviation, and leeway, if necessary.

ILLUSTRATIVE EXAMPLES

Example 1.—A ship having left New York bound for a port in the English Channel is in latitude 40° N and longitude 70° W. Calculate

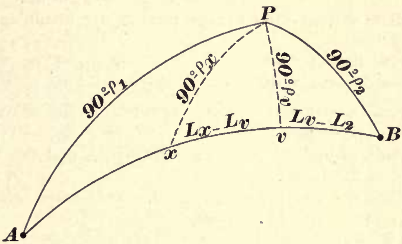

Fig. 14

the great-circle track between that point and another situated near the entrance of the channel, in latitude 50° N and longitude 10° W. The rhumb course between the two places is N 76° 44′ E.

Solution.—See Fig. 14.

Lat. of A = 40° N	Long. of A = 70° W
Lat. of B = 50° N	Long. of B = 10° W
$\rho_1 - \rho_2 = 10°$	D. Long. = 60°
$\rho_1 + \rho_2 = 90°$	$\frac{1}{2}$ D. Long. = 30°
$\frac{\rho_1 - \rho_2}{2} = 5°$	
$\frac{\rho_1 + \rho_2}{2} = 45°$	

These data being found, calculate the *initial* and *final* course according to formulas **1** and **2**, Art. **48**. Thus,

log cos 5° = 9.99834	log sin 5° = 8.94030
log cot 30° = 10.23856	log cot 30° = 10.23856
log cosec 45° = 10.15051	log sec 45° = 10.15051
log tan $\frac{1}{2}(A+B)$ = 10.38741	log tan $\frac{1}{2}(A-B)$ = 9.32937
$\frac{1}{2}(A+B)$ = 67° 43′	$\frac{1}{2}(A-B)$ = 12° 3′

$$A = 67° \, 43' - 12° \, 3' = 55° \, 40'$$
$$B = 67° \, 43' + 12° \, 3' = 79° \, 46'$$

The initial course is therefore N 55° 40′ E and the final course 180° − 79° 46′ = N 100° 14′ E = S 79° 46′ E.

To calculate the *distance*, use the formula of Art. **49.** Thus,

$$\begin{aligned} \log\cos 50° &= 9.80807 \\ \log\sin 60° &= 9.93753 \\ \log\operatorname{cosec} 55° \, 40' &= 10.08314 \\ \hline \log\sin D &= 9.82874 \\ \text{Dist.} &= 42° \, 23' = 2{,}543 \text{ mi.} \end{aligned}$$

To calculate the position of the *vertex*, use formulas **1** and **2**, Art. **50.** Thus,

$$\begin{aligned} \log\sin 79° \, 46' &= 9.99304 \\ \log\cos 50° &= 9.80807 \\ \hline \log\cos \rho_v &= 9.80111 \\ \rho_v &= 50° \, 46' \end{aligned} \qquad \begin{aligned} \log\sin 50° &= 9.88425 \\ \log\tan 79° \, 46' &= 10.74345 \\ \hline \log\cot (L_v - L_2) &= 10.62770 \\ L_v - L_2 &= 13° \, 16' \end{aligned}$$

Hence, the latitude of the vertex is 50° 46′ N, and the longitude is 13° 16′ + 10° = 23° 16′ W.

To calculate the position of the *point of maximum separation*, use formulas **1** and **2**, Art. **51.** Thus,

$$\begin{aligned} \log\cos 50° \, 46' &= 9.80105 \\ \log\operatorname{cosec} 76° \, 44' &= 10.01175 \\ \hline \log\cos \rho_x &= 9.81280 \\ \rho_x &= 49° \, 28' \text{ N} \end{aligned} \qquad \begin{aligned} \log\sin 49° \, 28' &= 9.88083 \\ \log\tan 76° \, 44' &= 10.62750 \\ \hline \log\cot (L_x - L_v) &= 10.50833 \\ L_x - L_v &= 17° \, 14' \end{aligned}$$

Hence, the latitude of the point of maximum separation is 49° 28′ N

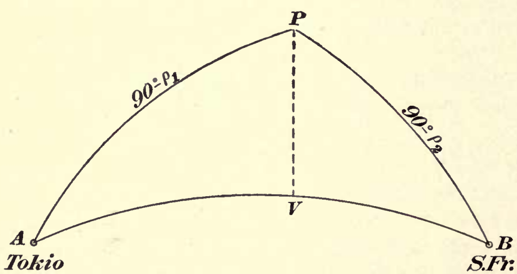

FIG. 15

and the longitude, according to Art. **51**, is 17° 14′ west of vertex, or 17° 14′ + 23° 16′ = 40° 30′ W. Ans.

EXAMPLE 2.—Calculate the initial and the final course; the distance, and the position of the vertex on the great-circle track between San

Francisco and Tokio, Japan. The latitude of San Francisco is 37° 49′ N and the longitude 122° 30′ W, while the latitude of Tokio is 35° 40′ N and the longitude 140° E.

SOLUTION.—See Fig. 15.

$$\begin{aligned} \text{Lat. of } A &= 35^\circ\ 40'\ \text{N} \\ \text{Lat. of } B &= 37^\circ\ 49'\ \text{N} \\ \hline \rho_1 - \rho_2 &= 2^\circ\ 9' \\ \rho_1 + \rho_2 &= 73^\circ\ 29' \\ \frac{\rho_1 - \rho_2}{2} &= 1^\circ\ 4.5' \\ \frac{\rho_1 + \rho_2}{2} &= 36^\circ\ 44.5' \end{aligned} \qquad \begin{aligned} \text{Long. of } A &= 140^\circ\ 0'\ \text{E} \\ \text{Long. of } B &= 122^\circ\ 30'\ \text{W} \\ \hline \text{D. Long.} &= 97^\circ\ 30' \\ \tfrac{1}{2}\ \text{D. Long.} &= 48^\circ\ 45' \end{aligned}$$

To calculate the *courses:*

$$\begin{aligned} \log\cos 1^\circ\ 4.5' &= 9.99992 \\ \log\cot 48^\circ\ 45' &= 9.94299 \\ \log\operatorname{cosec} 36^\circ\ 44.5' &= 10.22306 \\ \hline \log\tan \tfrac{1}{2}(A+B) &= 10.16597 \\ \tfrac{1}{2}(A+B) &= 55^\circ\ 41' \end{aligned} \qquad \begin{aligned} \log\sin 1^\circ\ 4.5' &= 8.27661 \\ \log\cot 48^\circ\ 45' &= 9.94299 \\ \log\sec 36^\circ\ 44.5' &= 10.09623 \\ \hline \log\tan \tfrac{1}{2}(A-B) &= 8.31583 \\ \tfrac{1}{2}(A-B) &= 1^\circ\ 11' \end{aligned}$$

$$A = 55^\circ\ 41' - 1^\circ\ 11' = 54^\circ\ 30'$$
$$B = 55^\circ\ 41' + 1^\circ\ 11' = 56^\circ\ 52'$$

Hence, the initial course, when leaving San Francisco, is N 56° 52′ W and the final course S 54° 30′ W. Ans.

To calculate the *distance:*

$$\begin{aligned} \log\cos 35^\circ\ 40' &= 9.90978 \\ \log\sin 97^\circ\ 30'\ (= \sin 82^\circ\ 30') &= 9.99627 \\ \log\operatorname{cosec} 56^\circ\ 52' &= 10.07707 \\ \hline \log\sin D &= 9.98312 \end{aligned}$$

Dist. = 74° 8′ = 4,448 mi. Ans.

To calculate the position of the *vertex:*

$$\begin{aligned} \log\sin 56^\circ\ 52' &= 9.92293 \\ \log\cos 37^\circ\ 49' &= 9.89761 \\ \hline \log\cos \rho_v &= 9.82054 \\ \rho_v &= 48^\circ\ 35' \end{aligned} \qquad \begin{aligned} \log\sin 37^\circ\ 49' &= 9.78756 \\ \log\tan 56^\circ\ 52' &= 10.18527 \\ \hline \log\cot (L_v - L_2) &= 9.97283 \\ L_v - L_2 &= 46^\circ\ 47' \end{aligned}$$

Hence, the latitude of the vertex is 48° 35′ N, and its longitude, 46° 47′ + 122° 30′ = 169° 17′ W. Ans.

NOTE.—The rhumb course from San Francisco to Tokio is S 88° W; the difference between this course and the initial course is therefore more than 3 points. By following the great-circle track between the two places, about 270 miles is saved.

GRAPHICAL METHODS RELATING TO GREAT-CIRCLE SAILING

57. Great-Circle Charts.—Charts constructed on the gnomonic projection are called **great-circle charts,** and on such charts the great-circle track between any two places is represented as a *straight line* connecting them. The principles of this projection are illustrated in Fig. 16. A globe

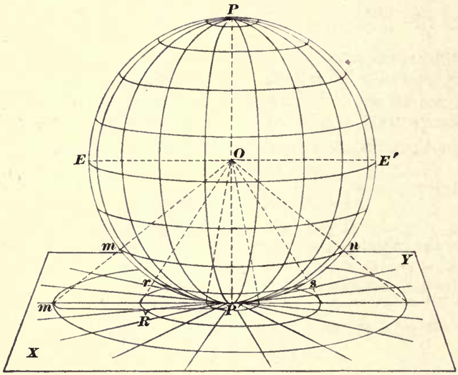

Fig. 16

representing the earth is placed on a flat sheet of paper XY in such a position that one of its poles P' touches the central part of the paper. To an observer situated at the center O of the globe, the latitude parallels mn and rs will now appear on the paper as concentric circles described around the center P', and the meridians will appear as straight lines radiating from the same center.

Suppose the meridians on the globe to be 10° apart, and to be projected from the center O on the plane XY; the result will, on the removal of the globe, appear as in Fig. 17, representing a chart on the gnomonic projection.

The difference of longitude between any two places on such a chart, for instance that between A and B, is measured

by the angle at the pole P included between the meridians passing through the two places. The radius R for each latitude parallel is obtained from the formula $R = r \times \cot$ Lat., in which r is a constant value and equal to the radius of the 45th parallel. By considering r equal to unity, a still simpler formula is obtained; namely, $R = \cot.$ Lat.

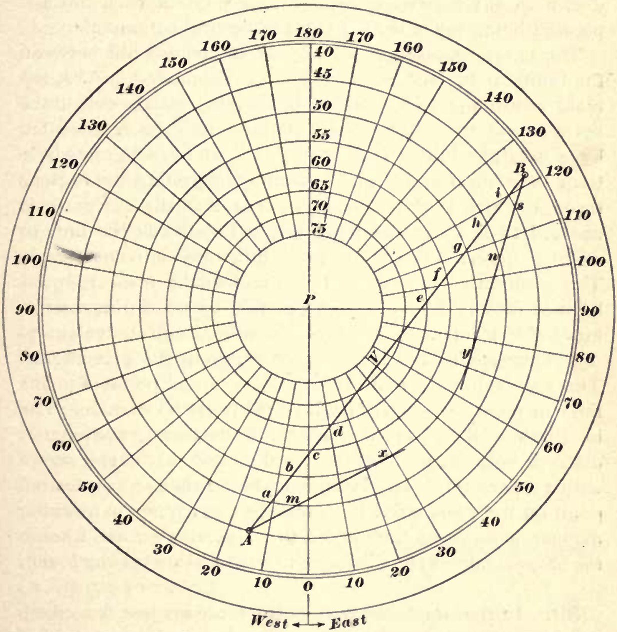

FIG. 17

58. Relation Between Latitude and Radius of Parallel.—The relations existing between the latitude and the radius of any parallel, when projected on a chart according to the gnomonic projection, is readily seen by inspecting Fig. 16. Take, for instance, the parallel $m\,n$, the latitude of

which is the arc $E m$; the radius $m P'$, or R, of that parallel when projected on the chart is equal to the tangent for the angle $m O P'$ or equal to the cotangent for the latitude of the parallel, which is measured by the angle $E O m$, or the arc $E m$, the radius of the globe being considered as unity in both cases. A gnomonic chart may therefore be readily drawn to any desired scale, the cotangent of each latitude parallel being taken from a table of natural cotangents.

The chart represented in Fig. 17 is constructed between the limits of 40° and 75° of latitude. As stated before, the main advantage of this chart is that the great-circle track between any two places within its limit may be represented by a straight line. Thus, the line $A B$ is the great-circle track between A and B, and by simply inspecting the regions through which it passes, it is evident that the navigator is enabled to see whether the track is practicable or not, or whether it passes through regions unfavorable to navigation. The position of the vertex V is obtained by inspection; its latitude may be found quite accurately by measuring its distance $P V$ from the pole, this distance being equivalent to the cotangent of the latitude according to the given scale. The corresponding numbers of degrees and minutes in the table of natural cotangents will be the required latitude. The longitude of the vertex is obtained by measuring the angular distance between the nearest meridian and that of the vertex with a protractor. The latitude and longitude of any desired point on the great-circle track may be determined in a similar manner, and can be transferred to a Mercator's chart, whence the proper courses and distances to be sailed are readily found.

59. In the simple form of gnomonic chart just described, the courses and distances along a great-circle track cannot be obtained directly; but on great-circle charts, devised by G. Herrle, and published by the United States Hydrographic Office, courses and distances are conveniently found by means of diagrams attached to the chart. Full information regarding the use of these diagrams is printed conspicuously on each chart under the heading "Explanation."

The gnomonic chart, although accurate in high latitudes, gives a very distorted representation of low latitudes. In such parts of the world, however, the advantage of great-circle sailing over that of Mercator's sailing is insignificant.

60. Airy's Method.—When gnomonic charts are not available, the following method for laying out a great-circle track on a Mercator's chart will be found satisfactory. This method was originally communicated to the Royal Astronomical Society by Sir G. B. Airy, Astronomer Royal of Great Britain, in 1858.

The sweep of the track should be accomplished as follows:

1. Join the two places on a Mercator's chart by a straight line. Find its middle point. From this point draw a perpendicular toward the equator and, if necessary, continue it beyond the equator.

2. With the middle latitude between the two places, enter Table I and take out the corresponding parallel.

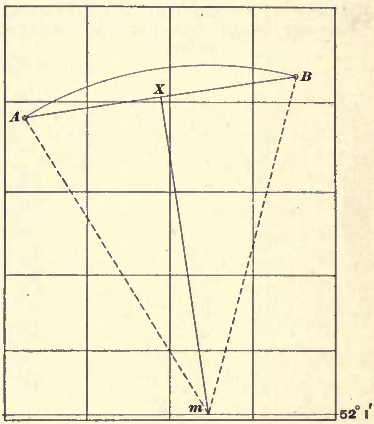

FIG. 18

3. The center of the required arc (which is drawn either with beam compasses or with a pencil attached to a thread) will be the intersection of this parallel with the perpendicular, the radius being the distance from the center to either place.

ILLUSTRATION.—To illustrate the foregoing, let A and B, Fig. 18, represent two places on a Mercatorial chart between which it is desired to lay out a great-circle track. Assuming that the M. Lat. of these places is 76°, Table I is entered with this value, and the corresponding latitude parallel is found to be 52° 1′ N. Now connect the

two places with a straight line *A B* representing the rhumb track, and from its middle point *x* drop a perpendicular, producing it until it intersects the latitude parallel 52° 1′ N at *m*. Then, with *m* as a center and a radius equal to *m A* or *m B*, draw by means of a pair of compasses an arc, connecting *A* and *B*. This arc is the required great-circle track according to Airy's method.

Since the perpendicular referred to may be quite long, it is evident that a practical illustration of this method requires considerable chart space; hence, Fig. 18 may not be proportioned accurately, although it serves well for the purpose of explanation.

TABLE I

DATA FOR FINDING THE "CORRESPONDING PARALLEL" ACCORDING TO AIRY'S METHOD

Middle Latitude	Corresponding Parallel (Opposite Name to the Latitude of Places)	Middle Latitude	Corresponding Parallel (Same Name as the Latitude of Places)
20°	81° 13′	—	—
22°	78° 16′	—	—
24°	74° 59′	—	—
26°	71° 26′	58°	4° 0′
28°	67° 38′	60°	9° 15′
30°	63° 37′	62°	14° 32′
32°	59° 25′	64°	19° 50′
34°	55° 5′	66°	25° 9′
36°	50° 36′	68°	30° 30′
38°	46° 0′	70°	35° 52′
40°	41° 18′	72°	41° 14′
42°	36° 31′	74°	46° 37′
44°	31° 38′	76°	52° 1′
46°	26° 42′	78°	57° 25′
48°	21° 42′	80°	62° 51′
50°	16° 39′	—	—
52°	11° 33′	—	—
54°	6° 24′	—	—
56°	1° 13′	—	—

NOTE.—The blank spaces arise from the fact that in relations otherwise than those contained in the table, great-circle sailing is of no particular advantage.

61. Sigsbee's Method.—Another method of determining graphically the data connected with great-circle sailing was devised by Admiral C. D. Sigsbee, U. S. N., and published by the United States Hydrographic Office under the title "Graphical Method for Navigators." This publication contains a large stereographic projection of the sphere, exhibiting meridians and parallels at intervals of 1°. Full explanations and numerous examples accompany each diagram. No drawing is necessary except to plot points and occasionally to trace parts of a curve from the diagram. The materials required are a piece of tracing paper about the size of the diagram, a lead pencil, and an eraser. Besides being useful for solving problems in great-circle sailing, the diagram can be advantageously used for the solution of spherical problems in general.

62. Comment on Graphical Methods.—Other graphical methods have been devised by which the great-circle track can be drawn on a Mercator's chart without any calculations whatever. Among these, the one devised by the late Richard A. Proctor, the astronomer, is most satisfactory in its results. His charts of the northern and southern hemispheres are constructed on the principles of the stereographic projection. Godfray's great-circle charts and course and distance diagrams also give very accurate results. Explanations and directions usually accompany these charts.

In simplicity, however, Airy's method is foremost; its disadvantages, however, are that the center of the arc to be drawn frequently falls beyond the limit of the chart to be used, and that the track may occupy more than one chart, thus rendering it difficult to draw the required arc. On the other hand, Herrle's method is a most excellent one for all practical purposes; it combines simplicity and accuracy. By Herrle's method, the course and distance run by a vessel can be measured independently of any great-circle track that may have been laid down, just as the rhumb course and distance to a desired place are measured on the Mercator's chart from one position to another.

COMPOSITE SAILING

63. Explanation.—Whenever the great-circle track between two places passes through land or brings the ship into too high a latitude, or into a region frequented by ice and bad weather, or is met by other obstacles that would tend to either lengthen the passage or make it dangerous, a method of sailing called **composite sailing** is resorted to. This method is essentially a combination of great-circle sailing and parallel sailing, as shown in the following: Assume a ship in latitude 45° N and longitude 15° W, or at *A*, Fig. 17, to be bound for a place *B* in latitude 42° N and longitude

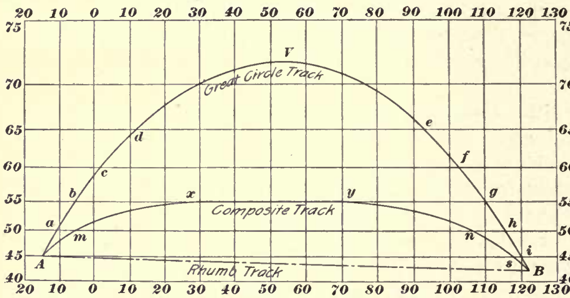

Fig. 19

123° E. Joining the two places with a straight line, the master of the ship at once discovers that by following the great-circle track he will come as far north as 72°. Now, on account of the ice, fog, and cold weather that is likely to be encountered in this latitude, and which would seriously interfere with the successful navigation of his vessel, he decides to go no farther north than 55°; in other words, 55° N will be his maximum latitude. A tangent from *A* to the 55th parallel is then drawn on the gnomonic chart, and likewise a tangent from *B* to the same parallel. The ship will now proceed first along the great-circle track *A x*, then along the 55th parallel to *y*, and finally proceed along the great-circle track *y B* until her destination *B* is reached.

This, in brief, comprises what is known as *composite sailing*, and the track $AxyB$, composed of two separate great-circle tracks and an arc of a parallel, is called a **composite track.** Measuring roughly, it will be found that the first great-circle track Ax, Fig. 17, crosses the 50th parallel at about 3° W or at m, and is tangent to the 55th parallel at about 28° E, or at x; the second great-circle track commences at about 73° E, or at y, and crosses the 50th and 45th parallels at n and s, or at 105° E and 117° E, respectively.

These positions, together with those of A and B, can then be plotted on a Mercator's chart, as shown in Fig. 19, and through the points A, m, x and y, n, s, B thus found curves are drawn that will coincide with the 55th parallel at x and y.

64. In a similar manner, the great-circle track AVB, Fig. 17, can be transferred to the Mercatorial chart by plotting on it the positions of $a, b, c, d, V, e, f, g, h, i$, and then connecting all places by a curve, as shown in Fig. 19. In general, to transfer a great-circle track to a sailing chart, select a few points along the track, the closer the better, and find the latitude and longitude of each point. Plot these points carefully on the sailing chart, and draw a uniform curve through all points established. This curve will be the great-circle track, and the compass courses and distances to be run on each course may then be readily determined along the entire length of the track.

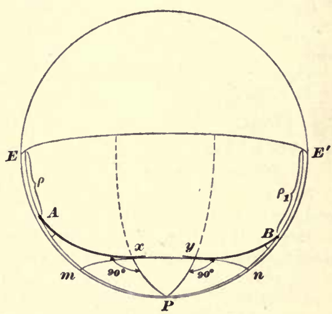

FIG. 20

65. Solution by Trigonometry.—Courses and distances may also be found by calculation, as shown in the following example. The formulas here used may serve in any case of composite sailing.

EXAMPLE.—A ship in latitude 30° S and longitude 18° W is to sail for a place in latitude 38° 49′ S and longitude 145° E. It is decided to utilize the great-circle route, but not to go farther south than the 55th parallel. Required, the initial and final course of the composite track, the total distance to be run, and the number of miles to be run on the parallel.

SOLUTION.—Let P, Fig. 20, represent the south pole, A the place of departure, B the port of destination, and $m\,x\,y\,n$ the 55th parallel; also, let the arcs $A\,x$ and $B\,y$ represent, respectively, the great-circle tracks drawn from A and B to the parallel. Now, in the triangles $P\,A\,x$ and $P\,B\,y$, the sides have the following values:

$$A\,P = 90° - \rho \;\;= 90° - 30° \;\;\;= 60°$$
$$B\,P = 90° - \rho_1 = 90° - 38°\,49' = 51°\,11'$$

and $$P\,x = \;\; P\,y \;\;= 90° - 55° \;\;\;= 35°$$

The angles $A\,x\,P$ and $B\,y\,P$ are each 90°, since the great circles are tangent to the parallel at x and y, and are therefore at right angles to the meridians passing through these places. Hence, the triangles are right-angled, and to find the initial course $P\,A\,x$ and the final course $P\,B\,y$, the following formulas are used:

$$\sin P\,A\,x = \sin P\,x \operatorname{cosec} A\,P$$

and $$\sin P\,B\,y = \sin P\,y \operatorname{cosec} B\,P$$

Thus,

$$\begin{array}{rl} \log \sin 35° = & 9.75859 \\ \log \operatorname{cosec} 60° = & 10.06247 \\ \hline \log \sin P\,A\,x = & 9.82106 \\ P\,A\,x = & 41°\,29' \end{array} \qquad \begin{array}{rl} \log \sin 35° = & 9.75859 \\ \log \operatorname{cosec} 51°\,11' = & 10.10838 \\ \hline \log \sin P\,B\,y = & 9.86697 \\ P\,B\,y = & 47°\,24' \end{array}$$

Hence, initial course = S 41° 29′ E

and final course = N 47° 24′ E. Ans.

To find the distance $A\,x$ and $B\,y$ on the respective great-circle tracks, the following formulas are used:

$$\cos A\,x = \cos A\,p \sec P\,x$$

and $$\cos B\,y = \cos B\,P \sec P\,y$$

$$\begin{array}{rl} \log \cos 60° = & 9.69897 \\ \log \sec 35° = & 10.08664 \\ \hline \log \cos A\,x = & 9.78561 \end{array} \qquad \begin{array}{rl} \log \cos 51°\,11' = & 9.79715 \\ \log \sec 35° = & 10.08664 \\ \hline \log \cos B\,y = & 9.88379 \end{array}$$

Whence, $A\,x = 52°\,23' = 3{,}143'$ and $B\,y = 40°\,4' = 2{,}404'$.

According to parallel sailing, the distance on the parallel, or $x\,y$ = D. Long. × cos Lat. Now, the D. Long. is measured by the angle at the pole $x\,P\,y$, which is equivalent to $A\,P\,B - (A\,P\,x + B\,F\,y)$; or,

$$x\,P\,y = A\,P\,B - (A\,P\,x + B\,P\,y)$$

Hence, the value of these angles must be found. Thus,

$$\sin A\,P\,x = \sin A\,x \operatorname{cosec} A\,P$$

and $$\sin B\,P\,y = \sin B\,y \operatorname{cosec} B\,P$$

$$\begin{array}{rl} \log \sin 52^\circ\ 23' = & 9.89879 \\ \log \text{cosec}\ 60^\circ = & 10.06247 \\ \hline \log \sin APx = & 9.96126 \\ APx = & 66^\circ\ 10' \end{array} \qquad \begin{array}{rl} \log \sin 40^\circ\ 4' = & 9.80867 \\ \log \text{cosec}\ 51^\circ\ 11' = & 10.10838 \\ \hline \log \sin BPy = & 9.91705 \\ BPy = & 55^\circ\ 42' \end{array}$$

Inserting these values in the preceding formulas, it will be found that xPy, or the D. Long., $= 163^\circ - (66^\circ\ 10' + 55^\circ\ 42') = 41^\circ\ 8' = 2{,}468'$. By parallel sailing, xy is then computed as follows:

$$\begin{array}{rl} \log 2{,}468 = & 3.39235 \\ \log \cos 55^\circ = & 9.75859 \\ \hline \log xy = & 3.15094 \\ xy = & 1{,}416 \text{ mi. Ans.} \end{array}$$

The total distance on the composite track is therefore 3,143 + 1,416 + 2,404 = 6,963 mi. Ans.

66. When, from any cause, a ship has deviated from the original great-circle track, it is not necessary to resume this track. A new track should be laid out from the actual position of the ship to her port of destination, or any other desired point.

EXAMPLES FOR PRACTICE

1. A ship in latitude 40° S and longitude 16° E is ordered to proceed to a place in latitude 48° S and longitude 147° E. She is to follow the great-circle track. Required, the initial and final courses and the highest latitude reached.

$$\text{Ans.} \begin{cases} \text{Initial course} = \text{S } 30^\circ\ 40' \text{ E} \\ \text{Final course} = \text{N } 35^\circ\ 44' \text{ E} \\ \text{Highest Lat.} = 67^\circ \text{ S} \end{cases}$$

2. Find the initial and final courses, also the distance and position of vertex, of the great-circle track between a place in latitude 15° 55′ S and longitude 5° 44′ W and another in latitude 55° 59′ S and longitude 67° 16′ W.

$$\text{Ans.} \begin{cases} \text{Initial course} = \text{S } 34^\circ\ 12' \text{ W} \\ \text{Final course} = \text{S } 75^\circ \text{ W} \\ \text{Dist.} = 3{,}662 \text{ mi.} \\ \text{Lat. vertex} = 57^\circ\ 17' \text{ S} \\ \text{Long. vertex} = 85^\circ\ 11' \text{ W} \end{cases}$$

3. A ship at Otago Harbor, New Zealand, in latitude 45° 47′ S and longitude 170° 45′ E, is about to leave for Callao, Peru, in latitude 12° 4′ S and longitude 77° 15′ W. Calculate the initial and final courses, also the distance, using the formula given in the note of Art. **49.**

$$\text{Ans.} \begin{cases} \text{Initial course} = \text{S } 65^\circ\ 45' \text{ E} \\ \text{Final course} = \text{N } 40^\circ\ 35' \text{ E} \\ \text{Dist.} = 5{,}764 \text{ mi.} \end{cases}$$

A SERIES OF QUESTIONS

Relating to the Subjects
Treated of in This Volume.

It will be noticed that the questions contained in the following pages are divided into sections corresponding to the sections of the text of the preceding pages, so that each section has a headline that is the same as the headline of the section to which the questions refer. No attempt should be made to answer any of the questions until the corresponding part of the text has been carefully studied.

ELEMENTS OF NAVIGATION

EXAMINATION QUESTIONS

(1) Explain, briefly, the aim and purposes of navigation.

(2) What do you understand: (*a*) by meridians? (*b*) by latitude parallels?

(3) Define latitude and longitude and state how they are measured.

(4) (*a*) Suppose that it is 10 A. M. in longitude 15° W; what is the time in longitude 90° W? (*b*) If it is 9 A. M. in longitude 60° W, what is the corresponding time at Greenwich?

(5) What is: (*a*) difference of latitude? (*b*) difference of longitude?

(6) Convert into time the following angular measurements: (*a*) 100° 17′ 45″; (*b*) 82° 52′ 40″; (*c*) 49° 31′ 45″; (*d*) 17° 44′ 45″.

(7) How long is: (*a*) the nautical mile? (*b*) the statute mile?

(8) Find the difference of latitude and the difference of longitude between San Francisco, California, and San Diego, California, the latitude and longitude of each being, respectively, 37° 48′ N, 122° 28′ W, and 32° 43′ N, 117° 10′ W.

$$\text{Ans.}\begin{cases}\text{Diff. lat.} = 5° \ 5' \text{ S}\\ \text{Diff. long.} = 5° \ 18' \text{ E}\end{cases}$$

(9) Define course, distance, and departure. Illustrate your answer by a sketch and mark the different parts of the triangle with the proper names.

(10) Convert into angular measurements the following times: (*a*) $10^h\ 47^m\ 25^s$; (*b*) $0^h\ 14^m\ 30^s$; (*c*) $9^h\ 13^m\ 31^s$; (*d*) $5^h\ 58^m\ 17^s$.

(11) (*a*) How is the circumference of the compass card divided? (*b*) Name the quadrantal points. (*c*) What is the lubber's point and what is its purpose?

(12) A vessel having run a certain distance finds that she is 2° 38′ to the northward and 5° 14′ to the westward of her original position, which was in latitude 27° 36′ N and longitude 82° 46′ W; find the latitude and longitude of the place arrived at.

$$\text{Ans.}\begin{cases}\text{Lat. in} = 30°\ 14'\ \text{N}\\ \text{Long. in} = 88°\ 0'\ \text{W}\end{cases}$$

(13) (*a*) Name in regular order, the points, half points, and quarter points contained between N 45° E and S 11° 15′ W. (*b*) How many degrees are contained between the points N by W $\frac{1}{4}$ W and S W $\frac{3}{4}$ W?

(14) (*a*) By what compass should the ship's course be set and all bearings taken? (*b*) Name the requirements of a good compass.

(15) A vessel leaves Port-au-Prince, Haiti, in latitude 18° 34′ N and longitude 72° 22′ W, and is bound for Galveston, Texas, in latitude 29° 22′ N and longitude 94° 46′ W; what is the difference of latitude made, also the difference of longitude in time?

$$\text{Ans.}\begin{cases}\text{Diff. lat.} = 10°\ 48'\ \text{N}\\ \text{Diff. long. in time} = 1^h\ 29^m\ 36^s.\end{cases}$$

(16) State the law of magnetic attraction and repulsion.

(17) What are the advantages and disadvantages of pole compasses?

(18) What do you understand: (*a*) by a vessel being close hauled? (*b*) by a vessel being under way?

(19) Name the relative bearings from dead ahead to two points abaft the port beam.

(20) The latitude and longitude of two places *A* and *B* are as follows: *A* is in latitude 4° 32′ N and longitude

176° 53′ W; *B* is in latitude 4° 58′ S and longitude 173° 5′ E. What is: (*a*) the difference of latitude and (*b*) the difference of longitude between these two places? Ans. $\begin{cases} (a)\ 9° 30' \\ (b)\ 10° 2' \end{cases}$

(21) What is meant: (*a*) by the fore-and-aft line and (*b*) by the athwartship line of a vessel?

(22) What is magnetic polarity?

(23) Give a brief description of the Ritchie liquid compass.

(24) Find the latitude in in the following examples: (*a*) lat. from = 4° 48′ N, diff. lat. = 228′ S; (*b*) lat. from = 3° 4′ S, diff. lat. = 192′ N; (*c*) lat. from = 0° 15′ N; diff. lat. = 223′ N.

(25) Find the difference of latitude in each of the following cases: (*a*) lat. from = 3° 4′ S, lat. in = 14° 46′ S; (*b*) lat. from = 40° 30′ N, lat. in = 33° 32′ N; (*c*) lat. from = 3° 58′ S, lat. in = 2° 42′ N.

(26) (*a*) What is meant by the rhumb track? (*b*) Describe its relation to geographical meridians.

(27) Referring to Fig. 15, name all square sails attached to the mainmast and mizzenmast.

(28) Name all fore-and-aft sails carried by the ship shown in Fig. 15.

(29) In the following examples find the longitude in: (*a*) long. from = 43° 40′ W, diff. long. = 251′ E; (*b*) long. from = 3° 22′ W, diff. long. = 202′ E; (*c*) long. from = 179° 41′ E, diff. long = 122′ W.

(30) Find the difference of longitude in each of the following cases: (*a*) long. from = 23° 40′ W, long. in = 19° 58′ W; (*b*) long. from = 178° 10′ E, long. in = 178° 10′ W; (*c*) long. from = 2° 15′ W, long. in = 0° 23′ E.

CHART, LEAD, AND LOG

EXAMINATION QUESTIONS

(1) Explain, briefly, the characteristic features: (*a*) of a polyconic chart; (*b*) of a Mercator chart.

(2) (*a*) When measuring distances between any two places on a Mercator chart, what scale should be used? (*b*) From what part of such scale should measurements be taken?

(3) What scale is used for measuring distances on a polyconic chart?

(4) Describe, in your own words, how the latitude and longitude of a place or position are found from the chart.

(5) What are the superior qualities possessed by the sounding machine in comparison with the common lead?

(6) State the general principles on which: (*a*) the several patent logs are constructed; (*b*) on which the common log is founded.

(7) Explain the relation between the log-line knot and the nautical mile.

(8) What should be the length of the knot: (*a*) when using the nautical mile as a unit and a glass of 30 seconds? (*b*) when using a glass that runs out in 28 seconds?

Ans. $\begin{cases} (a) \ 50.67 \text{ ft.} \\ (b) \ 47.29 \text{ ft.} \end{cases}$

(9) What should be the length of a knot on the log line: (*a*) when using the nautical mile as a unit and a 32-second

log glass? (*b*) when using the statute mile as a unit and a 26-second glass?

Ans. $\begin{cases} (a) & 54 \text{ ft.} \\ (b) & 38.1 \text{ ft.} \end{cases}$

(10) When two places are situated on, or very nearly on, the same parallel, how will you find the distance between them on a Mercator chart?

(11) (*a*) Explain how you find the course or bearing between any two places on a chart. (*b*) In what direction do the meridians run on a chart?

(12) What do you understand: (*a*) by meridional parts? (*b*) by meridional difference of latitude?

(13) The distance run according to the common log is 280 miles; a comparison between the log glass and an accurate watch shows that the former indicates only 26 instead of 28 seconds. What is the correct distance? Ans. 301.5 mi.

(14) State, clearly, how to find the position of a vessel on the chart by means of cross-bearings.

(15) How are soundings indicated on charts, and at what condition of the tide are they expressed?

(16) (*a*) Describe the usual method of marking a lead line. (*b*) What is the purpose of the arming of the lead?

(17) Construct the frame of a Mercator chart on the scale of 1 inch to 1° of longitude, the chart to extend from latitude 45° N to 50° N, and from longitude 25° W to 31° W. When the chart is completed, plot on it three positions *A*, *B*, and *C*, as follows: *A*, latitude 45° 40′ N, longitude 29° 50′ W; *B*, latitude 47° 30′, longitude 26° 15′ W; *C*, latitude 49° 20′ N, longitude 28° 35′ W. Then find: (*a*) the true course from *A* to *B* and from *B* to *C*; (*b*) calculate the time it will require a steamer making 15 knots to cover the distance *A B C*.

(18) Suppose that soundings at a certain place 4 hours after high water show a depth of 42 feet; what is the actual depth, assuming the range of the tide to be 16 feet?

Ans. 38 ft.

(19) Describe how sounding is done with the hand lead, and state what precaution should be taken to insure a true sounding.

(20) How is the lead line read at night?

(21) (*a*) What is a depth recorder? (*b*) On what principle is this instrument constructed?

(22) What precautions should be taken when using the chemical tubes in place of the depth recorder, in sounding with Lord Kelvin's sounding machine?

(23) (*a*) Can the depth shown by a tube or a depth recorder be checked by the amount of wire run out? (*b*) What is the principal thing to be guarded against in using the sounding machine?

(24) What do you understand: (*a*) by general charts? (*b*) by sailing charts? (*c*) by plans? (*d*) What charts are generally used in coast navigation?

(25) (*a*) Are pilot charts intended for use as sailing charts? (*b*) Before using a chart, what important precautions should be taken?

DEVIATION AND COMPASS COMPENSATION

EXAMINATION QUESTIONS

(1) What do you understand: (*a*) by a magnetic meridian? (*b*) by magnetic dip, or inclination? (*c*) by the magnetic equator?

(2) What causes the compass needle to be deflected from the magnetic north and south, and why is this deflection different for different positions of the ship's head?

(3) What is magnetic variation, and where is the amount usually recorded?

(4) Referring to the chart, Fig. 3 of the text, state the amount and name of variation: (*a*) in the neighborhood of Cape Horn; (*b*) near the Cape of Good Hope; (*c*) in the vicinity of latitude 20° N and longitude 60° W.

(5) In reference to magnetism, what do you understand by hard iron and soft iron?

(6) What is: (*a*) subpermanent magnetism? (*b*) retentive magnetism? State how acquired.

(7) Describe the general principles on which to compensate a compass. Illustrate your answer by a sketch of your own, clearly indicating the red and blue pole of each magnet shown.

(8) What causes semicircular deviation, and how does it acquire its name?

(9) (*a*) What name is given to the deviation caused by the magnetism of horizontal soft iron of the ship? (*b*) In what direction of the ship's head is this deviation greatest?

(10) Suppose that an iron or steel vessel is built with her bow toward N W. What will be the logical position of her magnetic poles?

(11) Name the principal errors of the compass that should be compensated, and state how these errors, in reference to compensation, are divided and denoted.

(12) How would you compensate that part of the semicircular error denoted by *C*, assuming that the ship is not equipped with a compensating binnacle?

(13) Describe how that part of the semicircular error denoted by *B* is compensated without a compensating binnacle.

(14) (*a*) Is it necessary that the magnets used for compensating *B* and *C* should be fastened on deck? (*b*) If not, state where and how they should be placed.

(15) What means are used to compensate the quadrantal deviation?

(16) Describe the nature of the Flinders bar, and state for what purpose it is used.

(17) State the difference between variation, deviation, and local attraction.

(18) Describe briefly the method used to determine the deviation in port: (*a*) when the correct magnetic bearing of some object is known; (*b*) by means of reciprocal bearings.

(19) State the rule for determining whether deviation is easterly or westerly.

(20) (*a*) Is the deviation table that has been established for one compass applicable to other compasses on the same vessel? (*b*) How often is it advisable to test the accuracy of deviation tables?

(21) A compass is brought on shore for the purpose of determining the deviation of the ship's standard compass, whereupon the ship is slowly swung around. The result of the simultaneous bearings taken are as follows:

Ship's Head	Simultaneous Bearings	
	By Standard Compass	By Shore Compass
South	S 18° 40′ W	N 21° 20′ E
S E	S 29° 50′ W	N 31° 0′ E
East	S 47° 10′ W	N 44° 50′ E
N E	S 75° 0′ W	N 72° 30′ E
North	S 81° 30′ W	N 78° 0′ E

Required the deviation of the standard compass for each of these points.

(22) The correct magnetic bearing of a distant object is N 17° E. The ship is swung and the results are as follows:

Ship's Head	Bearing of Distant Object	Ship's Head	Bearing of Distant Object
North	N 18° E	South	N 14° E
N N E	N 16° E	S S W	N 17° E
N E	N 15° E	S W	N 19° E
E N E	N 13° E	W S W	N 21° E
East	N 11° E	West	N 23° E
E S E	N 10° E	W N W	N 24° E
S E	N $10\frac{1}{2}$° E	N W	N 23° E
S S E	N 12° E	N N W	N $20\frac{1}{2}$° E

Form a deviation table and find the deviation for the headings given.

(23) (*a*) What is heeling error? (*b*) Describe briefly how it is detected and the methods used to compensate it.

(24) In swinging a ship for deviation, if the magnetic bearing of the object selected is unknown, how can it be found?

(25) Describe the use to which a Napier's deviation diagram can be applied and state its special advantages.

(26) A ship was swung and the following compass bearings taken of the object on shore:

Ship's Head by Compass	Bearing of Object on Shore by Compass
North	N 44° W
N E	N 47° W
East	N 80° W
S E	S 76° W
South	S 79° W
S W	S 89° W
West	N 69° W
N W	N 52° W

Find the correct magnetic bearing and the deviation for the different headings.

(27) Suppose that when a ship is heading correct magnetic north, the steering compass shows N by E, and that when heading magnetic east, it indicates E by N; explain and show by sketch how the compensating magnet should be placed in each case in order to correct the errors.

(28) What is a compensating binnacle? Give a brief description of its interior arrangement.

(29) (*a*) What is the main object of compensating a compass? (*b*) How would a compass act if left uncompensated?

(30) State briefly the precautions necessary when selecting a position for a compass on board an iron vessel.

PILOTING

EXAMINATION QUESTIONS

(1) Explain the method of fixing a ship's position: (*a*) by bow bearings; (*b*) by range lines.

(2) The bearing of a certain lighthouse at 4 P. M. was N N W; 2 hours later it bore N E by N. The distance run in the interval between the bearings was 12 miles true west. Find, by construction, and also by the use of proper tables, the distance of the ship from the lighthouse at the second bearing. Ans. 13.3 mi.

(3) The angle subtended by a lighthouse known to be 150 feet high is measured by a sextant and found to be 10′; find its distance, in nautical miles, from the observer. Ans. 8.4 naut. mi.

(4) A vessel steers by compass N N E $\frac{1}{4}$ E, variation 14° E, deviation $7\frac{1}{2}$° W, leeway $1\frac{3}{4}$ points, wind E by S; find true course made good. Ans. N 12° 7′ E

(5) Name the different kinds of courses. Explain each clearly, and illustrate the answer by a sketch.

(6) (*a*) What is leeway? (*b*) How is it applied when converting a compass course to true, and vice versa?

(7) In correcting courses and bearings, to what point should the correction for deviation correspond?

(8) The officer on the bridge can just see the light of the Charleston lightship on the horizon, the height of this light, according to the List, being 40 feet. If the height of the

officer's eye above the water-line is 20 feet, find the ship's distance from the lightship, expressed in nautical miles.

Ans. 12.4 naut. mi.

(9) Explain how sounding curves can be used as guidance when running in a fog.

(10) Find the course to steer by compass if the true course is S by E $\frac{1}{2}$ E, wind S W by W, leeway 2 points, deviation 9° W, and variation 20° W. Ans. S 34° 38′ W

(11) The true course between two positions is S 29° E, variation from chart 17° E, deviation according to Table I; show, by diagram and calculations, how the course to steer is found. Ans. S 56° E

(12) The lookout, whose height above the water level is 40 feet, reports the Absecon light on the horizon; what is the distance, in nautical miles, of the ship from the light, the height of which is given as 167 feet? Ans. 22.1 mi.

(13) In fixing a ship's position by cross-bearings, how should bearings be taken in order to obtain satisfactory results?

(14) (*a*) How is a ship's position fixed by means of a station pointer? (*b*) Do you know of any other instrument by which it can be fixed, without a compass, when in sight of known objects on shore?

(15) The compass course is S W by W, wind N E, leeway 0°, variation of locality 17° E, deviation according to Table I; show, by figure and calculations, how the true course is found. Ans. S 78° 45′ W, or W by S

(16) Find the true course made by a vessel if the course indicated by the standard compass is N 30° E, wind S by E, leeway $1\frac{1}{4}$ points, deviation 10° W, variation 12° E.

Ans. N 17° 56′ E

(17) The compass course of a ship is E $\frac{1}{2}$ S; from the chart, the variation of locality is found to be 21° E, deviation 4° W, leeway $2\frac{1}{2}$ points, wind N N E $\frac{1}{2}$ E; find the ship's true course. Ans. S 39° 15′ E

(18) The true course between two places, as found from the chart, is S 79° W, deviation 19° W, variation 10° 30′ E, leeway ¾ point, wind S by W; what is the course to steer in order to arrive at the port of destination?

Ans. S 79° W, or W by S, nearly

(19) The navigating officer of a vessel wishes to reach a point the true bearing of which is N 44° W. He finds that the corrections to be applied for variation and deviation are, respectively, 6° E and 20° W; on what point of the compass must he set the ship's head?

Ans. $\begin{cases} \text{N 30° W, or} \\ \text{N N W ¾ W, nearly} \end{cases}$

(20) The bearing, by standard compass, of Cape Henlopen lighthouse was W N W ¼ W; ship was heading in a S by E direction. Assuming the variation to be 18.3° W and the deviation as given in Table I, what is the true bearing of this lighthouse from the ship?

Ans. N 75° 7′ W

(21) Describe briefly the present system of submarine signaling, and name the principal apparatus used in sending and receiving signals.

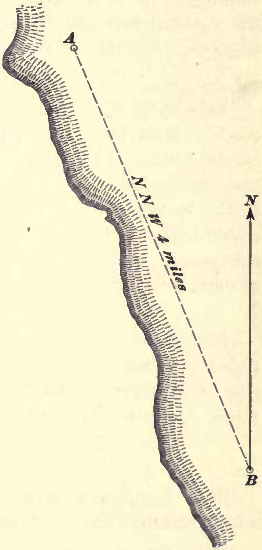

Fig. I

(22) Suppose that a vessel is sailing along the coast where two known objects A and B, Fig. I, are in sight; of these, A is the northernmost. According to the chart, the bearing from B to A is N N W (true), and the distance between the two is 4 miles. Outside and nearly midway between A and B lie a number of shoals and rocks just awash; the outermost point C of these shoals (not shown in

the figure but which should be located by the student) bears S 51° W from *A* and N 68° W from *B*. It is desired to pass 1 mile outside of that point. Draw a figure and plot the positions *A*, *B*, and *C*, using a scale of 1 inch to a mile. Then find, by construction, the horizontal danger angle.

Ans. Danger angle = 48°, very nearly

(23) The Diamond Shoal lightship bore W S W; after running S by W a distance of 11 miles, it bore N W. Find, by construction and by use of tables, the distance of ship from the lightship at the time of taking the second bearing.

Ans. 10 mi., nearly

(24) At 9:30 A. M. on May 17, 1906, the bearing, by compass, of a certain church spire was N by E. At 11 o'clock on the same day, its compass bearing was N W. The vessel was proceeding due east by compass at the rate of 10 knots. If the variation of the locality is 7° W and the deviation as given in Table I, find, by construction and also by the use of the proper tables, the distance of the ship from the church at each observation.

Ans. $\begin{cases} \text{First bearing, 12.75 mi.} \\ \text{Second bearing, 17.7 mi.} \end{cases}$

(25) The angular height of the St. Augustine, Florida, lighthouse was measured by a sextant and found to be 11′; find the observer's distance: (*a*) in nautical and (*b*) in statute miles, the recorded height of lighthouse being 161 feet.

Ans. $\begin{cases} (a) \text{ 8.2 naut. mi.} \\ (b) \text{ 9.5 stat. mi.} \end{cases}$

(26) Explain how the approximate bearing of a distant submarine bell can be determined.

(27) What is understood by danger bearings? Explain briefly and clearly the use of such bearings.

(28) In approaching a coast in foggy weather, state how you may fix, approximately, the position of your vessel by soundings. Illustrate, if possible, your answer by a sketch.

(29) (*a*) In misty weather and when near a coast, what should be the navigator's main reliance? (*b*) Explain how

a vertical danger angle may be used in clearing outlying hidden rocks and shoals.

(30) Correct the following courses: (*a*) The compass course of a vessel is S S E ½ E, wind east, leeway 1¼ points, deviation 1½ points W, variation 2¼ points W. (*b*) True course is N W by W ½ W, deviation 1½ points W, variation 19° E, leeway ¾ point, wind S W. (*c*) True course is E S E ½ E, variation 23° E, deviation according to Table I, leeway, none. (*d*) Compass course is east, deviation for this point as in Table I, wind S E by S, leeway 2 points, variation 15.5° E.

Ans. (*a*) S E by E
(*b*) N 72° 26′ W
(*c*) N 78° 23′ E
(*d*) East

DEAD RECKONING

(PART 1)

EXAMINATION QUESTIONS

(1) A ship in latitude 3° 52′ S is bound for a port in latitude 4° 30′ N, bearing N W by W $\frac{1}{2}$ W. Find, by plane sailing, what distance on that course must be covered in order to reach the intended port and how many miles of departure the ship will make during the trip. Illustrate the answer.

$$\text{Ans.}\begin{cases}\text{Dist.} = 1{,}065 \text{ mi.}\\ \text{Dep.} = 939 \text{ mi. W}\end{cases}$$

(2) From a place in latitude 54° 10′ N, a ship sails E S E $\frac{1}{2}$ E until a distance of 273 miles is covered. Find, by inspection of the Traverse Tables, the latitude of the place arrived at and the number of miles of departure made.

$$\text{Ans.}\begin{cases}\text{Lat. in} = 52° \ 50.8' \text{ N}\\ \text{Dep.} = 261.2 \text{ mi. E}\end{cases}$$

(3) (*a*) What do you understand by the expression "working a traverse"? (*b*) Of what elements are the Traverse Tables composed? (*c*) How are the Traverse Tables entered when the course is greater than 4 points, or 45°?

(4) From a point in latitude 49° 58′ N a ship sails the following courses: S by W 42 miles, W S W 36 miles, west 18 miles, E S E 22 miles, south 34 miles, and N E 21 miles. Draw a compass diagram and from its center lay out,

according to an adopted scale, the given courses and distances; find, also, by plane sailing, the latitude in and the course and distance made good.

Ans. $\begin{cases} \text{Lat. in} = 48° \ 35.4' \text{ N} \\ \text{Course} = \text{S } 16° \ 27' \text{ W, or S by W } \frac{1}{2} \text{ W} \\ \text{Dist.} = 86.2 \text{ mi.} \end{cases}$

NOTE.—All courses should be considered as true unless otherwise stated.

(5) Two ships A and B start from a point in latitude 37° 15′ N and longitude 73° 30′ W. Ship A steers S W 41 miles, and ship B steers S 69° E 81 miles. Find, by the Traverse Tables, the latitude and longitude of each ship after making the run, also their distance apart.

Ans. $\begin{cases} \text{Lat. A and B} = 36° \ 46' \text{ N} \\ \text{Long. A} = 74° \ 6' \text{ W} \\ \text{Long. B} = 71° \ 55' \text{ W} \\ \text{Dist.} = 104.6 \text{ mi.} \end{cases}$

(6) (*a*) Find the course and distance made good by a ship after she has sailed as follows: S S W 69 miles, east 29.5 miles, N by E 79 miles, west 42 miles, S by W 26 miles, and E N E 31 miles. (*b*) Assuming a favorable wind to be blowing and the ship capable of making 12 knots, how many hours will she require to run back to her starting point?

(7) (*a*) A ship sails as follows: S 67° 30′ W 51 miles, N 78° 45′ W 35 miles, S 1 point E 45 miles, S W by W 55 miles, and S 2 points E 41 miles. Reduce the courses to points, and find, by plane sailing, the course and distance made good. (*b*) Supposing her latitude from was 50° 13′ N, what is the latitude of the point arrived at?

Ans. $\begin{cases} (a) \begin{cases} \text{Course} = \text{S } 39° \ 19' \text{ W} \\ \text{Dist.} = 161.9 \text{ mi.} \end{cases} \\ (b) \ \text{Lat. in} = 48° \ 7.7' \text{ N} \end{cases}$

(8) (*a*) Under what conditions may the distance run by a vessel be considered equal to the difference of longitude? (*b*) When may the departure be considered equal to the difference of longitude?

(9) Name the methods used in navigation by dead reckoning by which the difference of longitude may be

determined when sailing in any other direction than the four cardinal points.

(10) From Fenwick Island Shoal Lightship, which is moored in latitude 38° 26′ N and longitude 74° 50′ W, a vessel runs true S 65° E for a distance of 54.5 miles. Find, by Traverse Tables, her latitude and longitude in.

Ans. $\begin{cases}\text{Lat. in} = 38° \ 3' \text{ N} \\ \text{Long. in} = 73° \ 47' \text{ W}\end{cases}$

(11) A ship in latitude 50° 25′ N and longitude 27° 15′ W is bound for a port in latitude 47° 12′ N and longitude 30° 20′ W. Find, by middle-latitude sailing, the true course and the number of miles to be run.

Ans. $\begin{cases}\text{Course} = \text{S } 32° \ 16' \text{ W} \\ \text{Dist.} = 228.2 \text{ mi.}\end{cases}$

(12) What precaution should be taken when using the method of middle-latitude sailing for a distance of 400 miles or more?

(13) From a point in latitude 32° 42′ N and longitude 57° 30′ W, a cruiser on patrol duty steams the following courses and distances: N E 240 miles, W S W ½ W 120 miles, N by E 250 miles, S E by S 120 miles, W N W 300 miles, S S W ½ W 210 miles, and west 120 miles. Find the course and distance made good.

Ans. $\begin{cases}\text{Course} = \text{N } 57° \ 12' \text{ W} \\ \text{Dist.} = 387.5 \text{ mi.}\end{cases}$

(14) When the course is large, that is, very nearly east or west, what method should be used in determining the difference of longitude? Explain the reason for your answer.

(15) How is the meridional difference of latitude between two places situated on opposite sides of the equator obtained?

(16) From a place in latitude 47° 10′ N and longitude 32° 15′ W, a ship steers a true course S 48° 58′ W. Find, by ordinary middle-latitude sailing, her latitude and longitude in after having covered a distance of 175.2 miles.

Ans. $\begin{cases}\text{Lat. in} = 45° \ 15' \text{ N} \\ \text{Long. in} = 35° \ 26' \text{ W}\end{cases}$

(17) A ship in latitude 65° S and longitude 25° 42′ W steers, by compass, E N E 112 miles. Deviation = $4\frac{1}{2}$° E and variation = 18° E. Required, her longitude in.

Ans. Long. in = 21° 17′ W

(18) A ship at Cape Palmas on the coast of Liberia, in latitude 4° 24′ N and longitude 7° 46′ W, is bound for a place in latitude 8° 48′ S and longitude 13° 8′ E. Assuming the mean value of the variation to be 20° W, find, by Mercator's sailing, the magnetic course to be steered; also, the distance between the two places.

Ans. { Mag. course = S 37° 49 E
Dist. = 1,487 mi.

(19) From a place in latitude 51° 37′ N and longitude 20° 24′ W, a ship sails S 48° W for a distance of 226 miles. Find, by middle-latitude sailing, her latitude and longitude in.

Ans. { Lat. in = 49° 6′ N
Long. in = 24° 47′ W

(20) In what latitude does a ship change her longitude 1° when sailing 28 miles true east or west?

Ans. In Lat. 62° 11′ N or S

(21) From a place situated in latitude 49° 52′ S and longitude 17° 22′ W, a ship steams 513.5 miles, steering a course N 26° 36′ E. Find, by Mercator's sailing, the latitude and longitude in.

Ans. { Lat. in = 42° 13′ S
Long. in = 11° 51′ W

(22) Find the course and distance from a place in latitude 0° 43′ N and longitude 105° 17.2′ E to another in latitude 0° 27.5′ S and longitude 101° 38′ E. In answering this question, use your own judgment in regard to method to be used.

Ans. { Course = S 72° 10′ W
Dist. = 230.2 mi.

(23) Calculate the course and distance from Cape of Good Hope, in latitude 33° 56′ S and longitude 18° 29′ E, to Cape Hatteras, in latitude 35° 15′ N and longitude 75° 30′ W. Use your own judgment in regard to the method to be used.

Ans. { Course = N 52° 1′ W
Dist. = 6,745 mi.

(24) Find the number of nautical miles a ship has to cover when sailing along a parallel in latitude 32° 14′ N in order to change her longitude 4° 15′. Ans. 215.7 mi.

(25) Find the course to steer by compass from a place in latitude 46° 40′ N and longitude 53° 7′ W to another in latitude 32° 43′ N and longitude 16° 40′ W, the mean value of the variation being 29° W and deviation 12° E. What is the distance in miles between the two places?

Ans. $\begin{cases}\text{Comp. course} = \text{S } 46^\circ\ 31'\ \text{E}\\ \text{Dist.} = 1{,}877\ \text{mi.}\end{cases}$

DEAD RECKONING

(PART 2)

EXAMINATION QUESTIONS

(1) (*a*) Define a current. (*b*) Explain what is meant by the *set* and *drift* of a current. (*c*) When the set and drift of a current and the course and distance run, are known, show, by construction, how you would find the course and distance made good.

(2) (*a*) How would you find the set and drift of a current when the ship's position by astronomical observations differs from that obtained by dead reckoning? (*b*) In your opinion, is this difference always due to a current, and, as a rule, should it be considered so?

(3) Explain in your own words, using a diagram if necessary, how the set and drift of a current may be determined when sailing along a coast in sight of known objects.

(4) A steamer capable of making a speed of 9 miles an hour is bound for a port, the bearing of which is N W by N and the distance 54 miles. A current is running in a direction N E $\frac{1}{2}$ N 3 miles per hour. Required, the course to steer so as to counteract the effect of the current.

Ans. Course = N 53° W, or N W $\frac{3}{4}$ W, nearly

(5) (*a*) What do you understand by the day's work? (*b*) Describe the data given and required in this operation.

(6) The bearing of a ship from her port of destination is W S W, and the distance is 102 miles. After proceeding on

DEAD RECKONING

(PART 2)

EXAMINATION QUESTIONS

(1) (*a*) Define a current. (*b*) Explain what is meant by the *set* and *drift* of a current. (*c*) When the set and drift of a current and the course and distance run, are known, show, by construction, how you would find the course and distance made good.

(2) (*a*) How would you find the set and drift of a current when the ship's position by astronomical observations differs from that obtained by dead reckoning? (*b*) In your opinion, is this difference always due to a current, and, as a rule, should it be considered so?

(3) Explain in your own words, using a diagram if necessary, how the set and drift of a current may be determined when sailing along a coast in sight of known objects.

(4) A steamer capable of making a speed of 9 miles an hour is bound for a port, the bearing of which is N W by N and the distance 54 miles. A current is running in a direction N E $\frac{1}{2}$ N 3 miles per hour. Required, the course to steer so as to counteract the effect of the current.

Ans. Course = N 53° W, or N W $\frac{3}{4}$ W, nearly

(5) (*a*) What do you understand by the day's work? (*b*) Describe the data given and required in this operation.

(6) The bearing of a ship from her port of destination is W S W, and the distance is 102 miles. After proceeding on

a course opposite to the bearing for 8 hours at a speed of $10\frac{1}{2}$ miles per hour, the port is sighted, and is then found to bear N N E, distance 24 miles. Evidently the ship has been affected by a current. Find the set and drift of that current.

$$\text{Ans.}\begin{cases}\text{Set} = \text{S } 26^\circ \text{ E, or S S E } \frac{1}{4} \text{ E, nearly}\\ \text{Drift} = 17 \text{ mi., or nearly 2 mi. per hr.}\end{cases}$$

(7) (*a*) What do you understand by the operation known as *taking the departure?* (*b*) How is this departure treated in connection with the determination of the ship's position by dead reckoning?

(8) (*a*) Find the Mercatorial course and distance between Sandy Hook Lighthouse, in latitude 40° 28′ N and longitude 74° W, and the Morro Lighthouse, San Juan, Porto Rico, in latitude 18° 29′ N and longitude 66° 7′ W. (*b*) How long would it require a steamer making 12 knots an hour to cover the distance between the two places?

$$\text{Ans.}\begin{cases}(a)\ \begin{cases}\text{Course} = \text{S } 17^\circ\ 15' \text{ E}\\ \text{Dist.} = 1{,}381 \text{ mi.}\end{cases}\\ (b)\ \text{4 da. 19 hr., nearly}\end{cases}$$

(9) (*a*) What is the shortest distance between two places on the surface of the earth? (*b*) What is meant by great-circle sailing? (*c*) What do you understand by a great-circle track?

(10) (*a*) Name the advantages of great-circle sailing. (*b*) In what region or portion of the earth do you consider the application of the principles of great-circle sailing most advantageous?

(11) On September 14, at noon, a point near Cape Romanzoff, Alaska, in latitude 62° 11′ N and longitude 166° 10′ W, bore S S W $\frac{1}{2}$ W, estimated distance 10 miles. The ship's head at the time of taking the departure was N N W, the deviation for that point being 7° W. Afterwards, the following courses and distances were sailed. A current set S W by W (magnetic) $1\frac{1}{2}$ miles per hour, affecting the ship's course during her entire cruise from noon to noon. Find the latitude and longitude in at noon, September 15.

NOON, SEPTEMBER 14, 1900

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	9	5	N N W	N E by E	½	7° W	P. M.
2	9	5					
3	10	4					
4	9	6					
5	9	6	N 67½° W	North	¾	18° W	
6	8	5					Variation of compass 20° W
7	9	5					
8	8	4					
9	9	0	S S W ½ W	S E	¼	9° W	
10	9	4					
11	10	0					
12	10	6					Midnight
	SEPTEMBER 15, 1900						
1	10	6	S by E	East	0	3° E	A. M.
2	10	4					
3	9	5					
4	9	5					
5	9	5	S ½ E	W S W	1	2° E	
6	8	7					Variation of compass 20° W
7	8	4					
8	8	4					
9	8	4	N N E	N W	1¼	6° E	
10	7	6					
11	7	0					
12	8	0					Noon, Sept. 15

Ans. $\begin{cases} \text{Lat. in} = 60° \ 45' \text{ N} \\ \text{Long. in} = 167° \ 26' \text{ W} \end{cases}$

(12) Does the advantage of great-circle sailing apply to steamers only? Explain how sailing vessels are benefited by this method.

(13) Suppose you are in command of a ship loading in New York harbor for some West Indian port, as, for

instance, San Juan, Porto Rico. By what method would you shape your course when starting on the voyage? Consult question (8), and give reason for your answer. State the advantages, if any, of great-circle sailing in this case.

(14) What is meant: (*a*) by the initial and final course? (*b*) by the vertex of a great-circle track?

(15) Find the longitude of the vertex of the great-circle track between a place in latitude 43° N and longitude 52° 30′ W and another in latitude 43° N and longitude 12° 10′ W. Ans. Long. of vertex = 32° 20′ W

(16) Define the point of maximum separation. Are there any cases in great-circle sailing where two points of maximum separation are possible? If so, state under what circumstances.

(17) Give formulas by which the latitude and the longitude of the point of maximum separation are determined.

(18) Required, the initial and final courses on the great-circle track between a place in latitude 36° N and longitude 70° W and another in latitude 50° N and longitude 10° W.

$$\text{Ans.}\begin{cases}\text{Initial course} = \text{N } 52° \ 16' \text{ E}\\ \text{Final course} = \text{S } 84° \ 28' \text{ E}\end{cases}$$

(19) How is the great-circle track between two places represented on a chart constructed on the gnomonic projection?

(20) (*a*) Can the courses and distances along the great-circle track be determined directly from a great-circle chart? (*b*) Describe how you would proceed to transfer a great-circle track from such a chart to another on Mercator's projection.

(21) On September 19, at noon, a vessel by astronomical observations was found to be in latitude 48° 10′ N and longitude 14° 6′ W. Afterwards, the following courses and distances were sailed. Find the latitude and longitude in at noon, September 20, and also the course made good from noon to noon.

Noon, September 19, 1900

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	2	0	N 28° W	E N E	0	14° 50′ W	P. M.
2	3	0					
3	4	0					
4	3	3					
5	2	4	E S E $\frac{1}{4}$ E	N E	0	18° E	
6	2	6					Variation of compass 23° W
7	3	2					
8	3	2					
9	3	4					
10	4	2	E S E	North	0	18½° E	
11	4	2					
12	4	3					Midnight
	September 20, 1900						
1	4	8	N N E $\frac{1}{4}$ E	W by N	0	11° E	A. M.
2	6	3					
3	7	2					
4	7	6					
5	8	2					
6	6	0	W S W	N W	0	16½° W	Variation of compass 23° W
7	5	4					
8	5	2					
9	6	2					
10	6	6					
11	6	3					
12	7	0					Noon, Sept. 20

Ans. Lat. in = 48° 2.8′ N
Long. in = 14° 1.7′ W
Course made good = S 22° E

(22) (*a*) Calculate the initial and final courses of the great-circle track between Manila, Philippine Islands, latitude 14° 29′ N and longitude 120° 55′ E, and San Francisco, Cal., latitude 37° 49′ N and longitude 122° 30′ W, assuming the entire track from port to port to be navigable. (*b*) What is the highest latitude reached by a ship sailing on this track?

Ans. (*a*) Initial course = N 46° E; Final course = S 61° 52′ E
(*b*) Highest Lat. = 45° 51′ N

(23) Supposing that two ships A and B start on a voyage from Manila to San Francisco, A following the great-circle track, and B the Mercatorial, or rhumb, course. Assuming the entire track to be navigable, what would be the saving in distance by A, and what course should be steered by B?

Ans. B's course = N 77° 22′ E
Saving = 350 mi.

(24) Do you know of any method or methods by which the process of laying out a great-circle track between two places on a Mercatorial chart can be accomplished without any computations whatever? If so, give a brief description of same.

(25) Explain the meaning of composite sailing, and point out the advantages offered by this method.

(26) (*a*) After having laid out the desired great-circle, or composite, track on a Mercator's chart, describe how you would determine the several courses and distances to be sailed. (*b*) What kind of courses are they: true, magnetic, or compass courses?

(27) Find the initial and final courses, also the distance on the great-circle track from Cape Hatteras, in latitude 35° 15′ N and longitude 75° 30′ W, to the Lizard, entrance of English Channel, in latitude 49° 58′ N and longitude 5° 12′ W.

Ans. Initial course = N 50° 28′ E
Final course = S 78° 12′ E
Dist. = 3,104 mi.

(28) When about to leave one port for another, for instance, on a transatlantic voyage, what method should be employed in shaping the course? If a sailing vessel, what tack would be the most advantageous in case of a head-wind?

(29) A ship sails from a place in latitude 34° 29′ S and longitude 18° 23′ E for another situated to the westward. She is to follow a composite track but not to go to a higher latitude than 45° S. Find her initial course, the longitude of the point where she will reach the parallel of maximum latitude, and the distance of that point from the place she left.

Ans. { Initial course = S 59° 5′ W
Long. of point = 28° 14′ W
Dist. = 2,208 mi. }

(30) On January 15, at noon, the magnetic bearing of the southernmost point of Barbadoes, latitude 13° 4′ N and longitude 59° 32′ W, was N 49° W, the distance being 12 miles. Afterwards the ship sailed according to the following log-book account. Find the latitude and longitude in on January 16 at noon.

Noon, January 15, 1900

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	4	5	S E by S	N E	1	7° 50′ E	P. M.
2	5	2					
3	6	3					
4	7	6					
5	7	4					
6	8	0					Variation of compass 2° 49′ E
7	4	0	E by S	N N E	1	7° 15′ E	
8	5	4					
9	5	6					
10	7	4					
11	8	3					
12	8	3					Midnight

(Continued on next page.)

JANUARY 16, 1900

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	3	0	E $\frac{3}{4}$ S	N by E	$\frac{3}{4}$	6° 10′ E	A. M.
2	3	2					
3	4	4					
4	5	4					
5	5	0					
6	5	5	N W by N	E N E	$\frac{1}{2}$	9° 15′ W	Variation of compass 2° 49′ E
7	6	2					
8	7	3					
9	9	0					
10	8	3	N by W	East	0	2° 57′ E	
11	6	3					
12	5	4					Noon, January 16

Ans. { Lat. in = 12° 26.6′ N; Long. in = 58° 44.3′ W }

(31) On July 7, at noon, the ship's position, as determined by astronomical observations, was latitude 35° 3′ S, and longitude 26° 52′ E. Twenty-four hours later, after having made good a course N 81° W, a distance of 39 miles, according to dead reckoning, the ship, by astronomical observations, was found to be in latitude 35° 13′ S and longitude 25° 5′ E. Find the set and drift of the current.

Ans. { Set of current = S 72° W; Drift of current = $2\frac{1}{6}$ mi. per hr. }

(32) On December 31, 1899, at noon, the lighthouse at Cape Race, Newfoundland, latitude 46° 40′ N and longitude 53° 4′ W, bore N $\frac{1}{2}$ E, the distance being 14 miles. Deviation for N $\frac{1}{2}$ E = 15° W. The log-book account for the next 24 hours was as follows. The magnetic set of a current experienced during these hours was N E $\frac{1}{2}$ E and its drift 36 miles. Find the latitude and longitude in at noon January 1, 1900; also the Mercatorial bearing and distance from this point to the City of Cardiff, England, in latitude 51° 28′ N and longitude 3° 10′ W.

Noon, December 31, 1899

Hours	Knots	Tenths	Courses	Wind	Leeway, Points	Dev.	Remarks
1	10	5	S W by W	S E	0	15° W	P. M.
2	9	5					
3	9	8					
4	9	2					
5	9	5	S E $\frac{1}{4}$ S	S W	0	10° W	
6	9	2					Variation of
7	9	3					compass
8	9	0					30° 30′ W
9	10	0	W N W $\frac{3}{4}$ W	S W by S	$1\frac{1}{4}$	7° 30′ W	
10	9	8					
11	9	4					
12	8	8					Midnight
	January 1, 1900						
1	8	8	E S E $\frac{1}{2}$ E	South	$\frac{3}{4}$	3° 30′ E	A. M.
2	8	5					
3	8	5					
4	9	2					
5	9	2	N N W $\frac{3}{4}$ W	W S W	$\frac{1}{2}$	5° 30′ E	
6	9	2					Variation of
7	9	2					compass
8	8	4					30° 30′ W
9	8	2	W S W $\frac{3}{4}$ W	S $\frac{1}{2}$ E	$\frac{1}{4}$	12° W	
10	8	4					
11	8	2					
12	7	2					Noon, January 1

Ans. $\begin{cases}\text{Lat. in} = 46° \ 22.3' \text{ N} & \text{Bearing} = \text{N } 81° \ 10' \text{ E} \\ \text{Long. in} = 53° \ 4' \text{ W} & \text{Dist.} = 1{,}993 \text{ mi.}\end{cases}$

A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

INCLUDED IN THE

EXAMINATION QUESTIONS IN THIS VOLUME

It will be noticed that the Keys have been given the same section numbers as the Examination Questions to which they refer. All article references refer to the Instruction Paper bearing the same section number as the Key in which they occur, unless the title of some other Instruction Paper is given in connection with the references.

ELEMENTS OF NAVIGATION

(1) Consult Arts. **1** and **28.**

(2) (*a*) Consult Art. **8.**
(*b*) Consult Art. **9.**

(3) Consult Arts. **12** and **20.**

(4) (*a*) By Art. **27** and also by reference to Fig. 7, it is evident that the time in longitude 90° W must be 90° – 15° = 75° or 5 hours earlier than the corresponding time in longitude 15° W; therefore, the time in longitude 90° W must be 10 – 5 = 5 A. M. Ans.

(*b*) If it is 9 A. M. in longitude 60° W, the corresponding time at Greenwich must be 60° or 4 hours later than 9 A. M., or 1 P. M. Ans.

(5) (*a*) Consult Art. **13.**
(*b*) Consult Art. **25.**

(6) (*a*) $100° \, 17' \, 45'' = 6^h \, 41^m \, 11^s$ Ans.
(*b*) $82° \, 52' \, 40'' = 5^h \, 31^m \, 30.7^s$. Ans.
(*c*) $49° \, 31' \, 45'' = 3^h \, 18^m \, 7^s$. Ans.
(*d*) $17° \, 44' \, 45'' = 1^h \, 10^m \, 59^s$. Ans. Arts. **22** and **23.**

(7) (*a*) and (*b*) Consult Art. **19.**

(8) By following the directions given in Arts. **15** and **25,** respectively, the difference of latitude between the two places is found to be 5° 5′ S, and the difference of longitude is 5° 18′ E. Ans.

(9) Consult Arts. **31, 34,** and **35.**

(10) (*a*) $10^h \, 47^m \, 25^s = 161° \, 51' \, 15''$. Ans.
(*b*) $0^h \, 14^m \, 30^s = 3° \, 37' \, 30''$. Ans.
(*c*) $9^h \, 13^m \, 31^s = 138° \, 22' \, 45''$. Ans.
(*d*) $5^h \, 58^m \, 17^s = 89° \, 34' \, 15''$. Ans. Arts. **22** and **23.**

(11) (*a*) Consult Arts. **73** and **75.**
(*b*) Consult Art. **73.**
(*c*) Consult Art. **77.**

(12) By following the directions given in Arts. **16** and **26,** the latitude in is found to be 30° 14′ N, and the longitude in, 88° W. Ans.

(13) (*a*) Consult representation of compass card, Fig. 19. Also see Table II.

(*b*) By an inspection of Fig. 19 and Table II, the answer is found to be 112° 30′ or $112\frac{1}{2}$°.

(14) (*a*) Consult Art. **84.**
(*b*) Consult Art. **79.**

(15) By following the directions given in Arts. **15** and **25,** respectively, the difference of latitude is found to be 10° 48′ N, and the difference of longitude, 22° 24′ W, and converting 22° 24′ into time according to Arts. **22** and **23,** gives $1^h\ 29^m\ 36^s$. Ans.

(16) Consult Art. **69.**

(17) Consult Art. **88.**

(18) (*a*) Consult Art. **45.**
(*b*) Consult Art. **57.**

(19) According to Art. **43,** they are as follows: Dead ahead; one point on port bow; two points on port bow; three points on port bow; broad on port bow; three points forward of port beam; two points forward of port beam; one point forward of port beam; port beam; one point abaft port beam; two points abaft port beam.

(20) (*a*) According to Art. **15,**

$$\left.\begin{array}{l}\text{Lat. } A = 4°\ 32'\ \text{N}\\ \text{Lat. } B = 4°\ 58'\ \text{S}\end{array}\right\}\text{Adding}$$

$$\text{Diff. of lat.} = 9°\ 30' \quad \text{Ans.}$$

(*b*) According to Art. **25,**

$$\left.\begin{array}{l}\text{Long. } A = 176°\ 53'\ \text{W}\\ \text{Long. } B = 173°\ \ 5'\ \text{E}\end{array}\right\}\text{Adding}$$

$$349°\ 58'$$

$$\text{Diff. of long.} = 360° - 349°\ 58' = 10°\ 2'. \quad \text{Ans.}$$

(21) (*a*) Consult Art. **38.**
(*b*) Consult Art. **39.**

(22) Consult Art. **68.**
(23) Consult Art. **86.**

(24) (*a*) In this case, the ship has decreased her latitude and therefore her latitude in is 4° 48′ − 228′ = 60′ or 1° N. Ans.

(*b*) Since the ship has changed her latitude 192′ to the northward of 3° 4′ S, she has crossed the equator and is in latitude 0° 8′ N. Ans.

(*c*) In this case, the ship has increased her latitude and therefore her latitude in is 3° 58′ N. Ans. Art. **16.**

(25) The difference of latitude in each case are as follows:
(*a*) 11° 42′ S. Ans.

(*b*) 6° 58′ S. Ans.

(*c*) 6° 40′ N. Ans. Arts. **15** and **17**.

(26) (*a*) and (*b*) Consult Art. **32**.

(27) According to Art. **63**, the square sails on the maintop, beginning with the uppermost, are: mainroyal, upper maintopgallantsail, lower maintopgallantsail, upper maintopsail, lower maintopsail, mainsail; while those on the mizzenmast are: mizzenroyal, mizzentopgallantsail, upper mizzentopsail, lower mizzentopsail.

(28) According to Art. **63**, the fore-and-aft sails beginning at the bow are: outer jib, inner jib, forestaysail, mainroyal-staysail, maintopgallant-staysail, mainstaysail, mizzentopgallant-staysail, maintrysail, spanker.

(29) (*a*) In this case, the ship has decreased her longitude and therefore her longitude in is 39° 29′ W. Ans.

(*b* Since the ship has changed her longitude 202′ or 3° 22′ to the eastward of 3° 22′ W, she is evidently on the meridian of Greenwich, or in longitude 0°. Ans.

(*c*) It is evident that 122′ to the westward of 179° 41′ E gives for longitude in, 177° 39′ E. Ans. Art. **26**.

(30) (*a*) Difference of longitude is 23° 40′ − 19° 58′ or 3° 42′ E. It is named east, since the vessel has changed her longitude in that direction.

(*b*) Proceed as follows:

Long. from	= 178° 10′ E
Long. in	= 178° 10′ W
	356° 20′
Subtract from	360° 0′
Diff. Long.	= 3° 40′ E Ans.

(*c*) In this case, the ship has crossed the meridian of Greenwich, sailing eastward, and the difference of longitude is therefore 2° 15′ + 0° 23′, or 2° 38′ E. Ans. Arts. **25** and **26**.

CHART, LEAD, AND LOG

(1) (*a*) Consult Arts. **4**, **5**, and **6**.
(*b*) Consult Arts. **8** and **10**.

(2) (*a*) The graduated meridian or latitude scale should be used. (*b*) A suitable unit of measurement should be taken from that part of the vertical or latitude scale that lies opposite a point midway between the two places, or near the middle latitude of the two places.

(3) The scale of miles printed on the polyconic chart is used for this purpose.

(4) Consult Arts. **28** and **29**.

(5) Consult Art. **52**.

(6) (*a*) Consult Art. **65**.
(*b*) Consult Art. **56**.

(7) As explained in Art. **61**, the relation between the log-line knot and the nautical mile may be expressed by the following proportion: The length of each knot is to the nautical mile as the time of the log glass is to the hour.

(8) By following the directions given in Art. **61**, the length of the knot in each case is found to be as follows:

(*a*) $$\frac{6{,}080 \times 30}{3{,}600} = 50.67 \text{ ft.} \quad \text{Ans.}$$

(*b*) $$\frac{6{,}080 \times 28}{3{,}600} = 47.29 \text{ ft.} \quad \text{Ans.}$$

(9) (*a*) Using the nautical mile and a 32-sec. glass, the length of the knot should be $\frac{6{,}080 \times 32}{3{,}600} = 54$ ft. Ans.

(*b*) Using the statute mile and a 26-sec. glass, the length of the knot should be $\frac{5{,}280 \times 26}{3{,}600} = 38.1$ ft. Ans. See Art. **61**.

(10) Consult Art. **31**.

(11) (*a*) Consult Art. **33.**

(*b*) The meridians on any chart run in a true north-and-south direction.

(12) (*a*) Consult Art. **11.**

(*b*) Consult Art. **13.**

(13) In this case, the time of the log glass is incorrect; therefore use formula **1,** Art. **64,** in order to find the correct distance. Inserting the values of d, or 280 mi., and g, or 26 sec., in the formula, $x = \frac{28 \times 280}{26} = 301.5$ mi. Ans.

(14) Consult Arts. **36** and **37.**

(15) Consult Art. **16.**

(16) (*a*) Consult Art. **44.**

(*b*) Consult Art. **42.**

(17) (*a*) The meridional parts and the corresponding meridional differences of latitude are as follows:

Lat.	M. P.	M. D. L.
45°	3,013.7	
		85.3 = 1° 25.3′
46°	3,099.0	
		86.9 = 1° 26.9′
47°	3,185.9	
		88.6 = 1° 28.6′
48°	3,274.5	
		90.2 = 1° 30.2′
49°	3,364.7	
		92.2 = 1° 32.2′
50°	3,456.9	

The required chart in reduced form is shown in Fig. 1. The true course from A to B as found by protractor is N 53° E and the distance is 183 mi. The true course from B to C is N 40° W and the distance is 144 mi. Ans. See Arts. **38** to **41.**

(*b*) The total distance $A\,B\,C$ is $183 + 144 = 327$ mi., and hence the steamer will require $\frac{327}{15} = 21.8$ hr. to cover the distance. Ans.

(18) According to Art. **54,** deduct one-fourth of the range of the tide from the sounding at the fourth hour after high water. Therefore, in this case, the actual depth is $42 - \frac{16}{4} = 42 - 4 = 38$ ft. Ans.

(19) Consult Art. **46.**

(20) If soundings are taken after dark, the leadsman should know the height of his position above the water-line, and at each cast this distance should be deducted from the amount of line out. If too dark to see the marks, the leadsman should be able to distinguish them by the sense of touch.

(21) (*a*) and (*b*) Consult Art. **49.**

(22) Consult Art. **51** (*j*).

(23) (*a*) Consult Art. **51** (*i*).
(*b*) Consult Art. **51** (*k* and *l*).

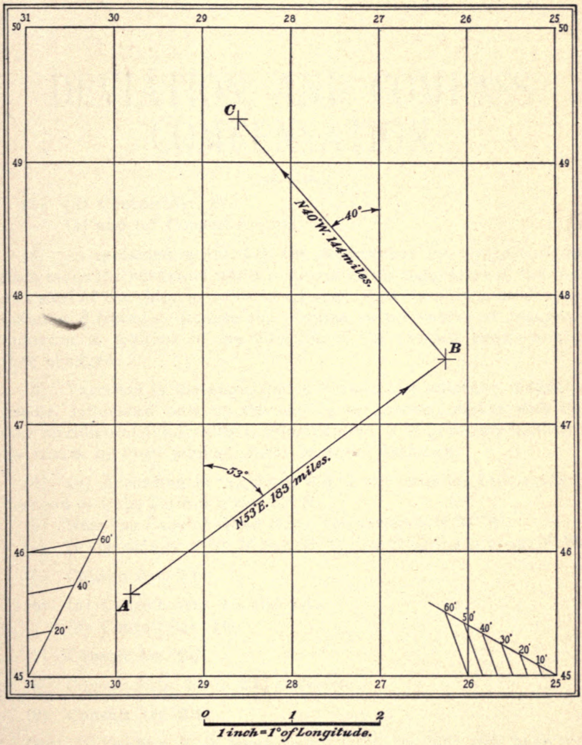

FIG. 1

(24) (*a* and *b*) Consult Art. **15.**
(*c*) Consult Art. **17.**
(*d*) Consult Art. **16.**

(25) (*a*) Consult Art. **20.**
(*b*) Consult Art. **19.**

DEVIATION AND COMPASS COMPENSATION

(1) (*a*) Consult Art. **2.**
(*b*) and (*c*) Consult Art. **6.**

(2) As explained in Art. **10,** the deflection of the compass needle from magnetic north and south is caused by the magnetism of the iron and steel of the ship; it varies in amount when the ship is heading in different directions, because the position of the source of magnetic influence in relation to the direction of the compass needle is also then changed.

(3) Variation is the angle that a horizontally balanced magnetic needle, influenced only by the earth's magnetism, makes with the true meridian of the locality considered, and it is generally indicated on charts by lines joining places of equal variation.

(4) (*a*) According to the chart, Fig. 3, the variation in the neighborhood of Cape Horn is about 20° E.
(*b*) Near the Cape of Good Hope, the variation is 30° W.
(*c*) In the vicinity of 20° N and 60° W, the variation is about 5° W.

(5) Consult Art. **11.**

(6) (*a*) Consult Arts. **14** and **15.**
(*b*) Consult Art. **16.**

(7) Consult Art. **26.**

8) Consult Arts. **18** to **21,** inclusive.

(9) Consult Art. **22.**

(10) If heading N W when being built, the ship will have red polarity on the starboard bow, and blue polarity on the port quarter. See Art. **14.**

(11) Consult Art. **28.**

(12) Consult Art. **30.**

(13) Consult Art. **31.**

(14) No; they need not be secured to the deck, but may be placed in any convenient position near the compass, either inside or outside the binnacle. However, the following rules should always be carefully regarded: The center of the fore-and-aft magnets must be exactly on the vertical athwartship plane passing through the center of the compass; the center of the athwartship magnets must be exactly on the vertical fore-and-aft plane passing through the center of the compass; all magnets must be parallel to the plane of the compass card when the ship is in an upright position.

(15) Consult Art. **29.**

(16) Consult Art. **34.**

(17) Consult Arts. **1, 3, 10, 24,** and **25.**

(18) (*a*) As explained in Arts. **46** to **49,** inclusive, the ship is first made ready for sea; a suitable object on shore is then selected, the correct magnetic bearing of which is known. The ship is now swung around, and as her head is steadied on each point of the compass, the bearing of the selected object by the standard compass is noted. The compass bearings thus obtained are then compared with the correct magnetic bearing of the object, which gives the deviation for each heading, respectively. The correct magnetic bearing of the object may be found either before "swinging" by one of the methods explained in Arts. **47** and **48,** or after "swinging" by taking the mean of all the compass bearings.

(*b*) As explained in Arts. **55** and **56,** a compass is placed on shore, or in a ship's boat located at a suitable distance from the ship where no magnetic influences exist, and an observer stationed by it. The ship is now swung as in the previous case, and as her head is steadied on each point, the observer at each compass takes a bearing of the other simultaneously, according to a prearranged signal. The bearings taken by the shore compass are then *reversed*, and are considered as the correct magnetic bearings with which to compare the bearings observed by compass on board; the difference between each pair of these bearings, named according to the rule of Art. **51,** is the deviation of the compass for each point, respectively.

(19) Consult Art. **51.**

(20) (*a*) Consult Art. **58.**
(*b*) Consult Art. **63.**

(21) By following the directions given in Arts. **51** and **56,** the resulting deviations are as follows:

$$\left.\begin{array}{ll}\text{For south} & \text{the deviation is } 2^\circ\ 40'\ \text{E}\\ \text{For S E} & \text{the deviation is } 1^\circ\ 10'\ \text{E}\\ \text{For east} & \text{the deviation is } 2^\circ\ 20'\ \text{W}\\ \text{For N E} & \text{the deviation is } 2^\circ\ 30'\ \text{W}\\ \text{For north} & \text{the deviation is } 3^\circ\ 30'\ \text{W}\end{array}\right\}\text{Ans.}$$

(22) Proceeding according to instructions given in Arts. **49** and **51**, the following deviation table is formed:

DEVIATION TABLE

Ship's Head	Deviation	Ship's Head	Deviation
North	1° W	South	3° E
N N E	1° E	S S W	0°, or none
N E	2° E	S W	2° W
E N E	4° E	W S W	4° W
East	6° E	West	6° W
E S E	7° E	W N W	7° W
S E	$6\frac{1}{2}$° E	N W	6° W
S S E	5° E	N N W	$3\frac{1}{2}$° W

(23) Consult Art. **35**.

(24) Consult Art. **50**.

(25) Besides indicating the deviation for any point, or subdivision, of the compass, Napier's diagram is especially useful for finding, by inspection, the magnetic course, or bearing, corresponding to any compass course, or bearing, and vice versa. See Art. **68**.

(26) By proceeding as explained in Arts. **50, 51, 54,** all the bearings are referred to the north point and the following deviation table is obtained:

DEVIATION TABLE

Ship's Head by Compass	Bearing of Object on Shore by Compass	Bearings Referred to North Point	Deviation of Compass
North	N 44° W	N 44° W	29° 30′ W
N E	N 47° W	N 47° W	26° 30′ W
East	N 80° W	N 80° W	6° 30′ E
S E	S 76° W	N 104° W	30° 30′ E
South	S 79° W	N 101° W	27° 30′ E
S W	S 89° W	N 91° W	17° 30′ E
West	N 69° W	N 69° W	4° 30′ W
N W	N 52° W	N 52° W	21° 30′ W

Sum of bearings = 588°

Mean = $\frac{588}{8}$ = 73.5° = 73° 30′

Correct magnetic bearing = N 73° 30′ W

(27) In the first case, the deviation is westerly, and by following instructions given in Art. **26,** this error is compensated by placing the magnet as shown in Fig. 1 (*a*). In the second case, the deviation is easterly, and is therefore compensated as shown in Fig. 1 (*b*).

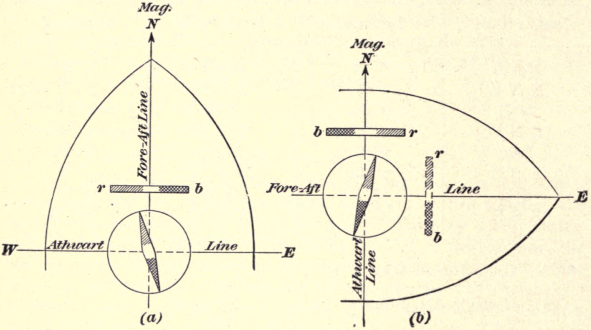

FIG. 1

(28) Consult Arts. **38** and **39.**

(29) Consult Art. **37.**

(30) Consult Arts. **69** and **70.**

PILOTING

(1) (*a*) Consult Arts. **4** and **5**.
(*b*) Consult Art. **31**.

(2) Follow the instructions in Arts. **8** to **12**, inclusive. The number of points between the first bearing BA and the ship's head, or course, BC, Fig. 1, is 6; between the second bearing DA and the ship's head there are 11 points. Entering the proper table on page 177 of Nautical Tables, with 6 points in the top row and with 11 points in

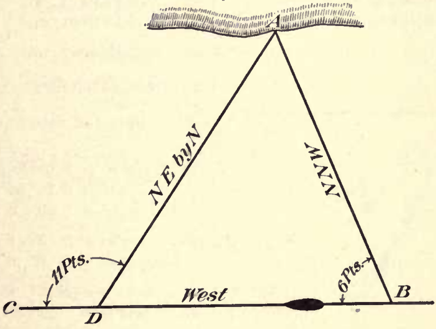

FIG. 1

the side column, below the former and opposite the latter, the number 1.11 is found; multiplying this by 12, the number of miles run in the interval between the bearings, will give the distance of the ship from A at the time the second bearing was taken. Thus, $1.11 \times 12 = 13.3$ mi. Ans.

(3) From the proper formula of Art. **13**, the distance

$$d = \frac{.56\,h}{v} = \frac{.56 \times 150}{10} = 8.4 \text{ naut. mi.} \quad \text{Ans.}$$

(4) Apply the rule of Art. **51.** Thus,

Compass course =	N N E $\frac{1}{4}$ E
Leeway =	$1\frac{3}{4}$ points
	N $\frac{1}{2}$ E
Or =	N 5° 37′ E
Deviation =	7° 30′ W
Magnetic course =	N 1° 53′ W
Variation =	14° 0′ E
True course =	N 12° 7′ E. Ans.

(5) Consult Arts. **43** to **46,** inclusive.

(6) Consult Arts. **48, 51,** and **53.**

(7) Consult Arts. **52** and **56.**

(8) According to Art. **16,** enter the Table of Distances of Objects at Sea in Nautical Miles, and find the distance corresponding to the height of the light and that corresponding to the observer, respectively. Thus,

the distance corresponding to 40 ft. = 7.27 naut. mi.
the distance corresponding to 20 ft. = 5.15 naut. mi.

Hence, the required distance = 12.42 naut. mi. Ans.

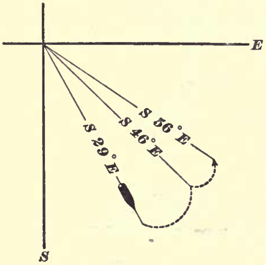

FIG. 2

(9) Consult Art. **22.**

(10) Apply the rule of Art. **53.** Thus,

True course =	S by E $\frac{1}{2}$ E
Or =	S 16° 52′ E
Variation =	20° 0′ W
Magnetic course =	S 3° 8′ W
Deviation =	9° 0′ W
	S 12° 8′ W
Leeway 2 points =	22° 30′
Compass course =	S 34° 38′ W. Ans.

(11) See Fig. 2. According to the rule of Art. **53** and instructions given in Art **54,**

True course =	S 29° E
Variation =	17° E
Magnetic course =	S 46° E
Deviation (for S 46° E mag.) =	10° E
Compass course =	S 56° E. Ans.

(12) According to Art. **16,** the solution is as follows:

The distance corresponding to 167 ft. = 14.86 naut. mi.
and the distance corresponding to 40 ft. = 7.27 naut. mi.
Hence, the distance of ship from light = 22.13 naut. mi. Ans.

(13) Consult Art. **3.**

(14) (*a*) Consult Art. **41.**
(*b*) Consult Art. **42.**

(15) See Fig. 3. Apply the rule of Art. **51** and follow instructions given in Art. **52.** Thus,

Compass course = S W by W
Or = S 56° 15′ W
Deviation (for S W by W) = 5° 30′ E
Magnetic course = S 61° 45′ W
Variation = 17° 0′ E
True course = S 78° 45′ W, or W by S. Ans.

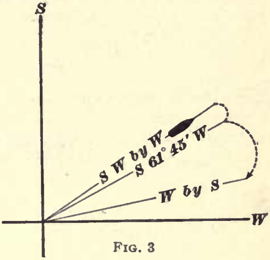

FIG. 3

(16) Apply the rule of Art. **51.** Thus,

Compass course = N 30° 0′ E
Leeway $1\frac{1}{4}$ points = 14° 4′
N 15° 56′ E
Deviation = 10° 0′ W
Magnetic course = N 5° 56′ E
Variation = 12° 0′ E
True course = N 17° 56′ E. Ans.

(17) Apply the rule of Art. **51.** Thus,

Compass course = E $\frac{1}{2}$ S
Leeway = $2\frac{1}{2}$ points
S E by E
Or = S 56° 15′ E
Deviation = 4° 0′ W
Magnetic course = S 60° 15′ E
Variation = 21° 0′ E
True course = S 39° 15′ E. Ans.

(18) Apply the rule of Art. **53.** Thus,

True course = S 79° 0′ W
Variation = 10° 30′ E
Magnetic course = S 68° 30′ W
Deviation = 19° 0′ W
S 87° 30′ W
Leeway $\frac{3}{4}$ point = 8° 26′
Compass course = S 79° 4′ W

Hence, in order to reach the desired port, the ship should steer by compass S 79° W, or W by S, nearly. Ans.

(19) Apply the rule of Art. **53.** Thus,

True bearing =	N 44° W
Variation =	6° E
Magnetic course =	N 50° W
Deviation =	20° W
Compass bearing =	N 30° W

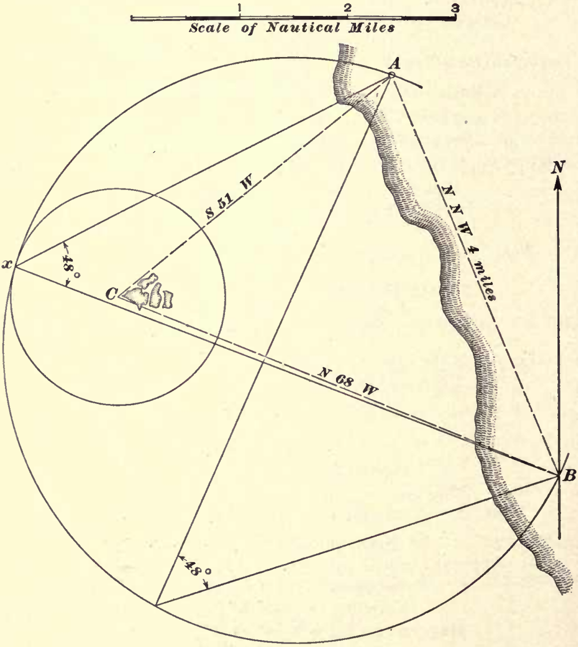

FIG. 4

Therefore, the ship's head should be set by compass N 30° W, or N N W $\frac{3}{4}$ W, nearly. Ans.

(20) Follow instructions given in Art. **56.** Thus,

Compass bearing =	W N W $\frac{1}{4}$ W
Or =	N 70° 19′ W
Deviation (for S by E) =	13° 30′ E
Magnetic bearing =	N 56° 49′ W
Variation 18.3° =	18° 18′ W
True bearing =	N 75° 7′ W. Ans.

(21) Consult Arts. **23** to **27,** inclusive.

(22) According to Arts. **36** and **37,** the horizontal danger angle is found as follows: In Fig. 4, the bearing S 51° W is laid off from A and N 68° W from B; the point C where they intersect represents the outermost rock. Then, with C as a center and a radius of 1 mi., according to the scale used, describe a circle. Through the most seaward point x of this circle and the objects A and B draw another circle; then, join x with A and B by straight lines. The angle $A x B$ included between these lines is the horizontal danger angle required, and when measured with a protractor it is found to be very nearly 48°. Ans.

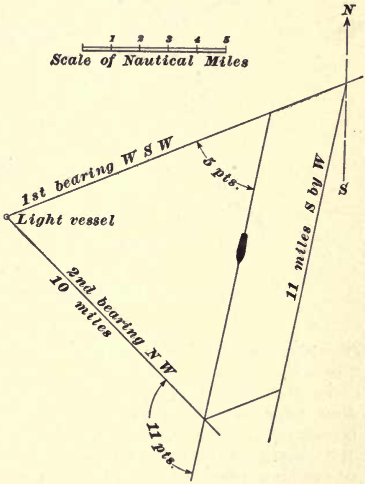

Fig. 5

(23) Proceed as in example 2. Then, entering the proper table with 5 points in the top row and 11 points in the side column, under the former and opposite the latter, the number .90 is found; multiplying this by 11, the number of miles run in the interval between the bearings, will give the distance of the ship from the lightship at the time the second bearing was taken, as shown in Fig. 5. Thus, $.9 \times 11 = 9.9$, or 10 mi., nearly. Ans.

(24) According to Arts. **52** and **56,** the deviation, from Table I, when the ship is heading east, is found to be 7° E, and since the variation given is 7° W (they are equal and of contrary names), it is evident that one counteracts the effect of the other. Therefore, the compass

bearings N by E and N W and the course steered are equivalent to the true bearings and course, respectively, and are plotted accordingly. Proceeding as explained in Arts. **8** to **12**, inclusive, the solution by construction and by the use of tables is as follows: The number of points between the first bearing and the ship's course, Fig. 6, is 7; between the second bearing and ship's course, there are 12 points. Entering the proper table with 7 points in the top row and 12 points in the side column, below the former and opposite the latter, the number 1.18 is found; multiplying this by 15, the number of miles run in the interval between the bearings, will give the distance of the ship

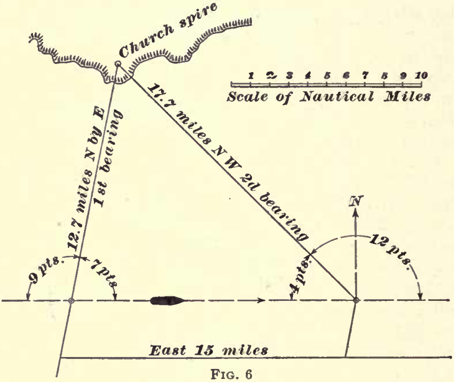

FIG. 6

from the church at the time the second bearing was taken. Thus, $1.18 \times 15 = 17.7$ mi. Ans.

To find the distance when the first bearing was taken, enter the same table with the supplement of the second number of points (supplement of 12 points is 4 points) in the top row and the supplement of the first number of points (supplement of 7 points is 9 points) in the side column, when, under the former and opposite the latter, the number .85 is found; multiplying this by 15, as in the preceding case, the required distance at first bearing is $.85 \times 15 = 12.75$ mi. Ans.

(25) (*a*) Apply the proper formula of Art. **13**. Thus,

$$d = \frac{.56\,h}{v} = \frac{.56 \times 161}{11} = 8.2 \text{ naut. mi. Ans.}$$

(*b*) Apply the proper formula of Art. **13**. Thus,

$$d = \frac{.65\,h}{v} = \frac{.65 \times 161}{11} = 9.5 \text{ stat. mi. Ans.}$$

(26) Consult Art. **28**.

(27) Consult Art. **39.**

(28) Consult Art. **18.**

(29) (*a*) The lead and the sounding machine.
(*b*) Consult Arts. **33** and **34.**

(30) Apply the rules of Arts. **51** and **53.** Thus,

(*a*)

Compass course =	S S E $\frac{1}{2}$ E
Leeway =	$1\frac{1}{4}$ points
	S by E $\frac{1}{4}$ E
Or =	S 14° 4′ E
Deviation $1\frac{1}{2}$ points =	16° 52′ W
Magnetic course =	S 30° 56′ E
Variation $2\frac{1}{4}$ points =	25° 19′ W
True course =	S 56° 15′ E
Or =	S E by E. Ans.

(*b*)

True course =	N W by W $\frac{1}{2}$ W
Or =	N 61° 52′ W
Variation =	19° 0′ E
Magnetic course =	N 80° 52′ W
Deviation $1\frac{1}{2}$ points =	16° 52′ W
	N 64° 0′ W
Leeway $\frac{3}{4}$ point =	8° 26′
Compass course =	N 72° 26′ W. Ans.

(*c*)

True Course =	E S E $\frac{1}{2}$ E
Or =	S 73° 7′ E
Variation =	23° 0′ E
Magnetic course =	S 96° 7′ E
Or =	N 83° 53′ E
Deviation (for N 84° E mag.) =	5° 30′ E
Compass course =	N 78° 23′ E. Ans.

(*d*)

Compass course =	East
Or =	N 90° 0′ E
Leeway 2 points =	22° 30′
	N 67° 30′ E
Deviation (for East) =	7° 0′ E
Magnetic course =	N 74° 30′ E
Variation 15.5° =	15° 30′ E
True course =	N 90° 0′ E, or East. Ans.

DEAD RECKONING

(PART 1)

(1) First find the D. Lat. expressed in miles. Thus,

$$\begin{aligned} \text{Lat. from} &= 3^\circ\,52'\ \text{S} \\ \text{Lat. in.} &= 4^\circ\,30'\ \text{N} \\ \hline \text{D. Lat.} &= 8^\circ\,22' = 502'\ \text{N} \end{aligned}$$

Then, knowing the course and D. Lat., the required Dist. and the Dep. are calculated as follows:

Dist. = D. Lat. × sec C		Dep. = D. Lat. × tan C	
log 502 =	2.70070	log 502 =	2.70070
log sec 61° 52′ =	10.32650	log tan 61° 52′ =	10.27189
log Dist. =	3.02720	log Dep. =	2.97259
Dist. =	1,065 mi. Ans.	Dep. =	938.8 mi. W. Ans.

The required diagram is shown in Fig. 1. To solve the problem by

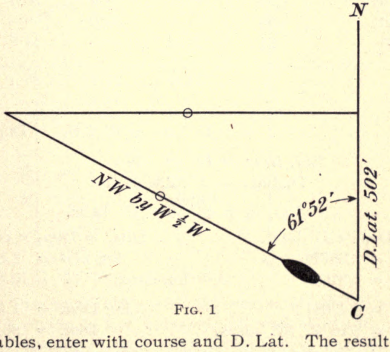

FIG. 1

Traverse Tables, enter with course and D. Lat. The result thus found will agree very nearly with that obtained by calculation. Thus,

for D. Lat. 141.4 is found 300 Dist. and 264.6 Dep.

	×3	×3	×3
	424.2	900	793.8
and for D. Lat.	77.8 is found	165 Dist. and	145.5 Dep.

Whence, for 502 is found 1,065 Dist. and 939.3 Dep. Ans.

(2) Entering the Traverse Tables with E S E ½ E as course and 273 as Dist., the corresponding D. Lat. and Dep. is 79.2 and 261.2, respectively. Hence, Dep. = 261.2 mi. E. Ans.

Lat. from = 54° 10′ N
D. Lat. 79.2′ = 1° 19.2′ S

Lat. in = 52° 50.8′ N. Ans.

(3) (*a*) Consult Arts. **15** and **16**.
(*b*) and (*c*) Consult Art. **6**.

(4) Follow instructions given in Art. **17**. Thus,

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S by W	42	——	41.2	——	8.2
W S W	36	——	13.8	——	33.3
West	18	——	——	——	18.0
E S E	22	——	8.4	20.3	——
South	34	——	34.0	——	——
N E	21	14.8	——	14.8	——
		14.8	97.4	35.1	59.5
			14.8		35.1
		D. Lat. =	82.6′ S	Dep. =	24.4′ W

Lat. from = 49° 58′ N
D. Lat. = 1° 22.6′ S

Lat. in = 48° 35.4′ N. Ans.

tan C = Dep. ÷ D. Lat.
log 24.4 = 1.38739
log 82.6 = 1.91698

log tan C = 9.47041
Course = S 16° 27′ W. Ans.

Dist. = Dep. × cosec C
log 24.4 = 1.38739
log cosec 16° 27′ = 10.54794

log Dist. = 1.93533
Dist. = 86.2 mi. Ans.

Fig. 2 represents the required diagram, from the center C of which are laid out the different courses and distances sailed by the ship.

(5) Let C, Fig. 3, represent the starting point, CW the direction and distance run by ship A, and CE that run by ship B. Connect W with E, and from C draw a line CD perpendicular to WE. Now,

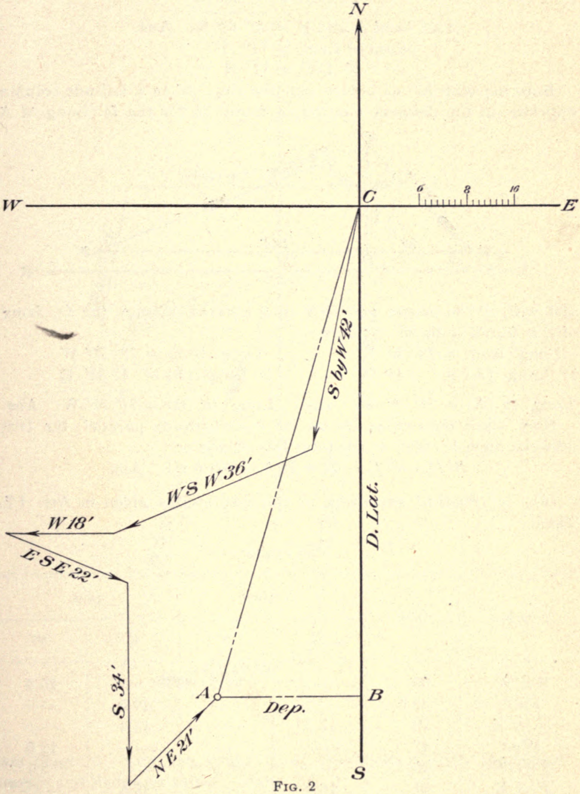

FIG. 2

entering the Traverse Tables with the course 45° and the Dist. 41 mi. gives 29′ for both the D. Lat. and Dep. Hence, WD and CD are each equal to 29′. Entering again with 69° as course and 81 mi. as Dist. gives 29′ for D. Lat. (= CD) and 75.6′ for Dep. (= DE). Then,

since the D. Lat. made by each ship is the same, they must both be on the same parallel W E. Thus, to find the Lat. and Long. in.,

$$\begin{aligned} \text{Lat. from} &= 37^\circ\ 15'\ \text{N} \\ \text{D. Lat.} &= 29'\ \text{S} \\ \hline \text{Lat. in of A and B} &= 36^\circ\ 46'\ \text{N.}\quad \text{Ans.} \\ \text{Sum of Lats.} &= 74^\circ\ \ 1' \\ \text{M. Lat.} &= 37^\circ\ \text{N} \end{aligned}$$

Entering with 37° as course and the Dep. 29′ in a latitude column, opposite, in the distance column, is found 36 for the D. Long of A;

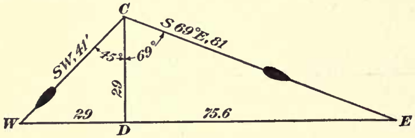

FIG. 3

and with 37° as course and 75.6′ in a latitude column, the D. Long. of B is found to be 95′. Hence,

$$\begin{aligned} \text{Long. from} &= 73^\circ\ 30'\ \text{W} \\ \text{D. Long. (A)} &= 36'\ \text{W} \\ \hline \text{Long. in (A)} &= 74^\circ\ \ 6'\ \text{W.}\quad \text{Ans.} \end{aligned} \qquad \begin{aligned} \text{Long. from} &= 73^\circ\ 30'\ \text{W} \\ \text{D. Long. (B)} &= \ 1^\circ\ 35'\ \text{E} \\ \hline \text{Long. in (B)} &= 71^\circ\ 55'\ \text{W.}\quad \text{Ans.} \end{aligned}$$

Now, since both ships are on the same latitude parallel, the Dist. between them is equal to the total Dep. made or

$$WD + DE = 29 + 75.6 = 104.6 \text{ mi.}\quad \text{Ans.}$$

(6) (*a*) Proceed according to the instructions given in Art. **17.** Thus,

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S S W	69	——	63.7	——	26.4
East	29.5	——	——	29.5	——
N by E	79	77.5	——	15.4	——
West	42	——	——	——	42.0
S by W	26	——	25.5	——	5.1
E N E	31	11.9	——	28.6	——
		89.4	89.2	73.5	73.5
		89.2			73.5
	D. Lat. =	.2′ N		Dep. =	0

Neglecting the two-tenths D. Lat., the ship has evidently returned to the place from which she sailed. This fact is indicated by the result of the traverse, as well as by Fig. 4, which represents a com-

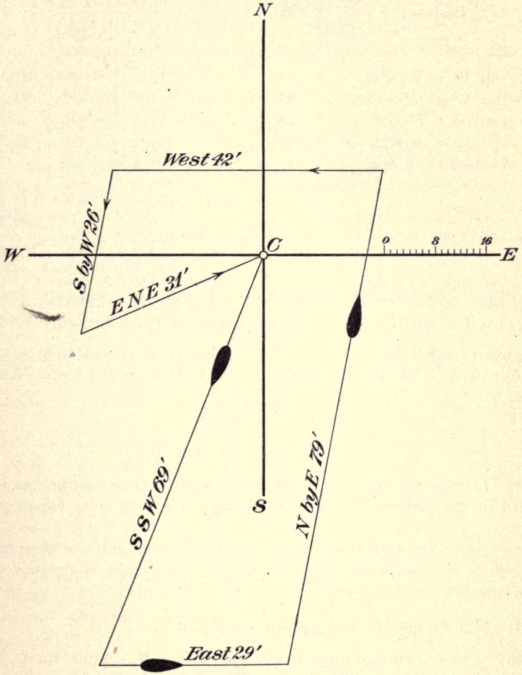

FIG. 4

pass diagram, from the center C of which are plotted the several courses and distances sailed. Ans.

(*b*) None. Ans.

(7) (*a*) Construct a traverse as usual. Thus,

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S 67° 30′ W = W S W	51	——	19.5	——	47.1′
N 78° 45′ W = W by N	35	6.8	——	——	34.3
S 1 point E = S by E	45	——	44.1	8.8	——
S W by W	55	——	30.6	——	45.7
S 2 points E = S S E	41	——	37.9	15.7	——
		6.8	132.1	24.5	127.1
			6.8		24.5
		D. Lat. =	125.3′ S	Dep. =	102.6′ W

tan C = Dep. ÷ D. Lat.
log 102.6 = 2.01115
log 125.3 = 2.09795
log tan C = 9.91320
Course = S 39° 19′ W. Ans.

Dist. = Dep. × cosec C
log 102.6 = 2.01115
log cosec 39° 19′ = 10.19818
log Dist. = 2.20933
Dist. = 161.9 mi. Ans.

(*b*) Lat. from = 50° 13′ N
D. Lat. = 2° 5.3′ S
Lat. in = 48° 7.7′ N. Ans.

(8) (*a*) According to Art. **28**, when steering due east or due west within a few degrees of the equator and the Dist. run is not over 300 mi.

(*b*) When steering in any direction in low latitudes, the Dep. made good may be considered equal to the D. Long. and vice versa, provided the Dist. is limited to a run of 300 or 400 mi.

(9) Middle-latitude and Mercator's sailing.

(10) Proceed as explained in example 2, Art. **8**. Thus, the D. Lat. and the Dep. corresponding to a distance of 54.5 mi. on a course S 65° E are found to be 23′ S and 49.4′ E, respectively.

Lat. from = 38° 26′ N
D. Lat. = 23′ S
Lat. in = 38° 3′ N. Ans.
Sum of Lats. = 76° 29′
M. Lat. = 38° 14′ N

By following illustrations given in Art. **37**, the required D. Long. is found to be 63′, or 1° 3′ E.

Long. from = 74° 50′ W
D. Long. = 1° 3′ E

Long. in = 73° 47′ W. Ans.

(11) Lat. from = 50° 25′ N Long. from = 27° 15′ W
Lat. in = 47° 12′ N Long. in = 30° 20′ W

D. Lat. = 3° 13′ = 193′ S D. Long. = 3° 5′ = 185′ W
Sum of Lats. = 97° 37′
M. Lat. = 48° 48′

tan C = cos M. Lat. × D. Long. ÷ D. Lat.

log cos 48° 48′ = 9.81868
log 185 = 2.26717
a. c. log. 193 = 7.71444

log tan C = 9.80029
Course = S 32° 16′ W. Ans.

Dist. = D. Lat. × sec C
log 193 = 2.28556
log sec 32° 16′ = 10.07285

log Dist. = 2.35841
Dist. = 228.2 mi. Ans.

(12) Consult Art. **38**.

(13) Construct a traverse and find the required data by proper formulas. Thus,

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N E	240	169.7	——	169.7	——
W S W ½ W	120	——	34.8	——	114.8
N by E	250	245.2	——	48.8	——
S E by S	120	——	99.8	66.7	——
W N W	300	114.8	——	——	277.2
S S W ½ W	210	——	185.2	——	99.0
West	120	——	——	——	120.0
		529.7	319.8	285.2	611.0
		319.8			285.2

D. Lat. = 209.9′ N Dep. = 325.8′ W

tan C = Dep. ÷ D Lat.
log 325.8 = 2.51295
log 209.9 = 2.32201

log tan C = 10.19094
Course = N 57° 12′ W. Ans.

Dist. = D. Lat. × sec C
log 209.9 = 2.32201
log sec 57° 12′ = 10.26623

log Dist. = 2.58824
Dist. = 387.5 mi. Ans.

(14) Consult Art. **49**.

(15) Consult Art. **45**.

(16)

$$\text{D. Lat.} = \text{Dist.} \times \cos C$$

$$\begin{aligned} \log 175.2 &= 2.24353 \\ \log \cos 48° 58' &= 9.81723 \\ \hline \log \text{D. Lat.} &= 2.06076 \\ \text{D. Lat.} &= 115' \text{ S} \end{aligned}$$

$$\begin{aligned} \text{Lat. from} &= 47° 10' \text{ N} \\ \text{D. Lat.} &= 1° 55' \text{ S} \\ \hline \text{Lat. in} &= 45° 15' \text{ N. Ans.} \\ \text{Sum of Lats.} &= 92° 25' \\ \text{M. Lat.} &= 46° 12' \end{aligned}$$

$$\text{D. Long.} = \text{D. Lat.} \times \tan C \times \sec \text{M. Lat.}$$

$$\begin{aligned} \log 115 &= 2.06076 \\ \log \tan 48° 58' &= 10.06033 \\ \log \sec 46° 12' &= 10.15980 \\ \hline \log \text{D. Long.} &= 2.28089 \\ \text{D. Long.} &= 191' \text{ W} \end{aligned}$$

$$\begin{aligned} \text{Long. from} &= 32° 15' \text{ W} \\ \text{D. Long.} &= 3° 11' \text{ W} \\ \hline \text{Long. in} &= 35° 26' \text{ W. Ans.} \end{aligned}$$

(17) First find the true course by applying the corrections for deviation and variation. Thus,

$$\begin{aligned} \text{Comp. course E N E} &= \text{N } 67° 30' \text{ E} \\ \text{Deviation} &= 4° 30' \text{ E} \\ \hline \text{Magnetic course} &= \text{N } 72° \ 0' \text{ E} \\ \text{Variation} &= 18° \ 0' \text{ E} \\ \hline \text{True course} &= \text{N } 90° \ 0' \text{ E} \end{aligned}$$

The course being true east, the case is evidently one of parallel sailing. Therefore, use formula **1**, Art. **23**, D. Long. = Dep. × sec Lat. Thus,

$$\begin{aligned} \log 112 &= 2.04922 \\ \log \sec 65° &= 10.37405 \\ \hline \log \text{D. Long.} &= 2.42327 \\ \text{D. Long.} &= 265' \text{ E} \end{aligned}$$

$$\begin{aligned} \text{Long. from} &= 25° 42' \text{ W} \\ \text{D. Long.} &= 4° 25' \text{ E} \\ \hline \text{Long. in} &= 21° 17' \text{ W. Ans.} \end{aligned}$$

(18)

$$\begin{aligned} \text{Lat. from} &= 4° 24' \text{ N} \ldots\ldots\ldots\ldots \text{M. P. } 262.5 \\ \text{Lat. in} &= 8° 48' \text{ S} \ldots\ldots\ldots\ldots \text{M. P. } 526.6 \\ \hline \text{D. Lat.} &= 13° 12' = 792' \text{ S} \qquad \text{M. D. Lat. } 789.1 \end{aligned}$$

$$\begin{aligned} \text{Long from} &= 7° 46' \text{ W} \\ \text{Long. in} &= 13° \ 8' \text{ E} \\ \hline \text{D. Long.} &= 20° 54' = 1{,}254' \text{ E} \end{aligned}$$

$$\tan C = \text{D. Long.} \div \text{M. D. Lat.}$$

$$\begin{aligned} \log 1{,}254 &= 3.09830 \\ \log 789.1 &= 2.89713 \\ \hline \log \tan C &= 10.20117 \\ \text{True course} &= \text{S } 57° 49' \text{ E} \\ \text{Variation} &= 20° \ 0' \text{ W} \\ \hline \text{Magnetic course} &= \text{S } 37° 49' \text{ E. Ans.} \end{aligned}$$

$$\text{Dist.} = \text{D. Lat.} \div \cos C$$

$$\begin{aligned} \log 792 &= 2.89873 \\ \log \cos 57° 49' &= 9.72643 \\ \hline \log \text{Dist.} &= 3.17230 \\ \text{Dist.} &= 1{,}487 \text{ mi. Ans.} \end{aligned}$$

(19)

D. Lat. = Dist. × cos C

log 226 = 2.35411

log cos 48° = 9.82551

log D. Lat. = 2.17962

D. Lat. = 151′ S

Lat. from = 51° 37′ N

D. Lat. = 2° 31′ S

Lat. in = 49° 6′ N. Ans.

Sum of Lats. = 100° 43′

M. Lat. = 50° 21′

D. Long. = D. Lat. × tan C × sec M. Lat.

log 151 = 2.17898

log tan 48° = 10.04556

log sec 50° 21′ = 10.19511

log D. Long. = 2.41965

D. Long. = 262.8′ W

Long. from = 20° 24′ W

D. Long. = 4° 23′ W

Long. in = 24° 47′ W. Ans.

(20) In this case the latitude is sought; therefore, use formula **3**, Art. **23**, substituting the term Dist. for Dep. Thus,

cos Lat. = Dist. ÷ D. Long.

log 28 = 1.44716

log 60 = 1.77815

log cos Lat. = 9.66901

Lat. = 62° 11′ N or S. Ans.

(21) D. Lat. = Dist. × cos C

log 513.5 = 2.71054

log cos 26° 36′ = 9.95141

log D. Lat. = 2.66195

D. Lat. = 459.1′ N

Lat. from = 49° 52′ S . . . . M. P. = 3444.5

D. Lat. = 7° 39′ N

Lat. in = 42° 13′ S. Ans. . . M. P. = 2783.8

M. D. Lat. = 660.7

D. Long. = M. D. Lat. × tan C

log 660.7 = 2.82000

log tan 26° 36′ = 9.69963

log D. Long. = 2.51963

D. Long. = 330.9′ E

Long. from = 17° 22′ W

D. Long. = 5° 31′ E

Long. in. = 11° 51′ W. Ans.

(22) Lat. from = 0° 43′ N

Lat. in = 0° 27.5′ S

D. Lat. = 1° 10.5′

Or = 70.5′ S

Long. from = 105° 17.2′ E

Long. in = 101° 38′ E

D. Long. = 3° 39.2′

Or = 219.2′ W = Dep.

According to Art. **34**, the Dep. may, in this case, be considered equal to the D. Long.

tan C = Dep. ÷ D. Lat.

log 219.2 = 2.34084

log 70.5 = 1.84819

log tan C = 10.49265

Course = S 72° 10′ W. Ans.

Dist. = D. Lat. ÷ cos C

log 70.5 = 1.84819

log cos 72° 10′ = 9.48607

log Dist. = 2.36212

Dist. = 230.2 mi. Ans.

(23) Mercator's sailing should be used in this case.

$$\begin{array}{llll}
\text{1st Lat.} = 33^\circ\ 56'\ S & \ldots\ldots & \text{M. P.} = & 2153.8 \\
\text{2d Lat.} = \underline{35^\circ\ 15'\ N} & \ldots\ldots & \text{M. P.} = & \underline{2249.4} \\
\text{D. Lat.} = 69^\circ\ 11' & & \text{M. D. Lat.} = & 4403.2 \\
\text{Or} = 4{,}151'\ N & & &
\end{array}$$

$$\begin{aligned}
\text{1st Long.} &= 18^\circ\ 29'\ E \\
\text{2d Long.} &= \underline{75^\circ\ 30'\ W} \\
\text{D. Long.} &= 93^\circ\ 59'\ W \\
\text{Or} &= 5{,}639'\ W
\end{aligned}$$

$$\begin{array}{ll}
\tan C = \text{D. Long.} \div \text{M. D. Lat.} & \text{Dist.} = \text{D. Lat.} \div \cos C \\
\log 5{,}639 = 3.75120 & \log 4{,}151 = 3.61815 \\
\log 4{,}403 = \underline{3.64375} & \log \cos 52^\circ\ 1' = \underline{9.78918} \\
\log \tan C = 10.10745 & \log \text{Dist.} = 3.82897 \\
\text{Course} = \text{N } 52^\circ\ 1'\ \text{W. Ans.} & \text{Dist.} = 6{,}745 \text{ mi. Ans.}
\end{array}$$

(24) Reduce the longitude 4° 15′ to minutes, and calculate the Dist. according to formula **2**, Art. **23**. Thus,

$$\begin{aligned}
\text{Dist.} &= \text{D. Long.} \times \text{cos. Lat.} \\
\log 255 &= 2.40654 \\
\log \cos 32^\circ\ 14' &= \underline{9.92731} \\
\log \text{Dist.} &= 2.33385 \\
\text{Dist.} &= 215.7 \text{ mi. Ans.}
\end{aligned}$$

(25)

$$\begin{array}{llll}
\text{1st Lat.} = 46^\circ\ 40'\ N & \ldots\ldots & \text{M. P.} = & 3156.8 \\
\text{2d Lat.} = \underline{32^\circ\ 43'\ N} & \ldots\ldots & \text{M. P.} = & \underline{2066.9} \\
\text{D. Lat.} = 13^\circ\ 57'\ S = 837'\ S & & \text{M. D. Lat.} = & 1089.9
\end{array}$$

$$\begin{aligned}
\text{1st Long.} &= 53^\circ\ \ 7'\ W \\
\text{2d Long.} &= \underline{16^\circ\ 40'\ W} \\
\text{D. Long.} &= 36^\circ\ 27' = 2{,}187'\ E
\end{aligned}$$

$$\begin{array}{ll}
\tan C = \text{D. Long.} \div \text{M. D. Lat.} & \text{Dist.} = \text{D. Lat.} \div \cos C \\
\log 2{,}187 = 3.33985 & \log 837 = 2.92273 \\
\log 1090 = \underline{3.03743} & \log \cos 63^\circ\ 31' = \underline{9.64927} \\
\log \tan C = 10.30242 & \log \text{Dist.} = 3.27346 \\
C = 63^\circ\ 31' & \text{Dist.} = 1{,}877 \text{ mi. Ans.}
\end{array}$$

$$\begin{aligned}
\text{True course} &= \text{S } 63^\circ\ 31'\ \text{E} \\
\text{Variation} &= \underline{29^\circ\ \ 0'\ \text{W}} \\
\text{Magnetic course} &= \text{S } 34^\circ\ 31'\ \text{E} \\
\text{Deviation} &= \underline{12^\circ\ \ 0'\ \text{E}} \\
\text{Comp. course} &= \text{S } 46^\circ\ 31'\ \text{E. Ans.}
\end{aligned}$$

DEAD RECKONING

(PART 2)

(1) (*a*) and (*b*) Consult Arts. **1** and **2**.
(*c*) Consult Arts. **16** and **17**.

(2) (*a*) Consult Art. **24**.
(*b*) Consult Art. **25**.

(3) Consult Art. **27**.

(4) By construction, the solution is effected as follows: Draw a line *N A*, Fig. 1, to represent the meridian. From any point *A* on that line lay off the bearing *A B* in a N W by N direction and make it

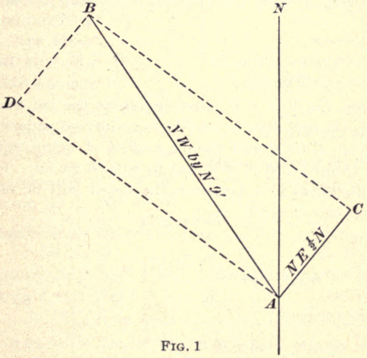

FIG. 1

equal in length to the ship's speed per hr., or 9 mi. Then draw *A C* to represent the set and drift of the current, or N E $\frac{1}{2}$ N 3 mi. Complete the parallelogram with *A B* as a diagonal. The angle *N A D* will then be the proper course to steer. Measuring this angle, it is found to be N 53° W, nearly. The same result will be obtained from the Traverse Tables, by following instructions given in Art. **19**. Thus,

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N W by N	9	7.5	——	——	5.0
S W ½ S	3	——	2.3	——	1.9

D. Lat. = 5.2′ N Dep. = 6.9′ W

Seeking in the Traverse Tables, it will be found that the course is 53°, which corresponds to the D. Lat. and Dep. found. Hence, the course to steer in order to counteract the effect of the current is N 53° W, or N W ¾ W, nearly. Ans.

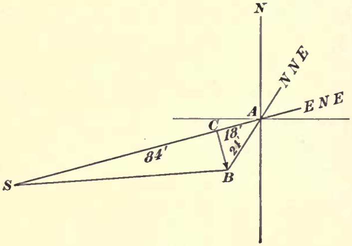
FIG. 2

(5) Consult Art. **29**.

(6) Let *S*, Fig. 2, be the ship's position when the bearing was determined, and *A* the port of destination. Since the ship has sailed E N E for a distance of 10.5 × 8 = 84 mi., her position should be at *C*, but instead she is found to be at *B*; hence, the line *C B* must represent the set and drift of the current. This set and drift is readily found by considering the ship to have sailed from *C* to *A* and thence to *B*, *C A* being equal to E N E (102 − 84 =) 18 mi., and *A B* equal to S S W 24 mi. Entering these courses and distances in a traverse, the result will be as follows:

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
E N E	18	6.9	——	16.6	——
S S W	24	——	22.2	——	9.2

D. Lat. = 15.3′ S Dep. = 7.4′ E

By inspection of the Traverse Tables, it is found that the course and distance corresponding nearest to a D. Lat. of 15.3′ and a Dep. of 7.4 is 26° and 17 mi., respectively. The set of the unknown current is

therefore S 26° E, or S S E $\frac{1}{4}$ E, nearly, and the drift 17 mi. for 8 hr., or very nearly 2 mi. per hr. Ans.

(7) Consult Arts. **32** and **33.**

(8) (*a*) Proceed according to Mercator's sailing, thus:

1st Lat. =	40° 28′ N	M. P. =	2644.5
2d Lat. =	18° 29′ N	M. P. =	1121.5
D. Lat. =	21° 59′ = 1,319′ S	M. D. Lat. =	1523.0

Long. = 74° 0′ W
Long. = 66° 7′ W

D. Long. = 7° 53′ = 473′ E

tan C = D. Long. ÷ M. D. Lat.	Dist. = D. Lat. ÷ cos C
log 473 = 2.67486	log 1,319 = 3.12024
log 1523 = 3.18270	log cos 17° 15′ = 9.98001
log tan C = 9.49216	log Dist. = 3.14023
Course = S 17° 15′ E. Ans.	Dist. = 1,381 mi. Ans.

(*b*) The time required is evidently equal to 1,381 ÷ 12 = $115\frac{1}{12}$ hr., or 4 da. 19 hr., nearly. Ans.

(9) Consult Art. **36.**

(10) Consult Art. **38.**

(11) Proceed according to instructions given in Art. **30.** Correction of courses:

First Course

Compass course =	N N W
Leeway (left) =	$\frac{1}{2}$ point
	N N W $\frac{1}{2}$ W
Or =	N 28° 7′ W
Deviation =	7° 0′ W
Magnetic course =	N 35° 7′ W
Variation =	20° 0′ W
True course =	N 55° 7′ W

Second Course

Compass course =	N 67° 30′ W
Leeway (left) =	8° 26′
	N 75° 56′ W
Deviation =	18° 0′ W
Magnetic course =	N 93° 56′ W
Variation =	20° 0′ W
True course =	N 113° 56′ W
Or =	S 66° W

Third Course

Compass course =	S S W $\frac{1}{2}$ W
Leeway (right) =	$\frac{1}{4}$ point
	S S W $\frac{3}{4}$ W
Or =	S 30° 56′ W
Deviation =	9° 0′ W
Magnetic course =	S 21° 56′ W
Variation =	20° 0′ W
True course =	S 1° 56′ W
Or =	S 2° W

Fourth Course

Compass course =	S by E
Or =	S 11° 15′ E
Deviation =	3° 0′ E
Magnetic course =	S 8° 15′ E
Variation =	20° 0′ W
True course =	S 28° 15′ E

Fifth Course

Compass course	=	S $\frac{1}{2}$ E	
Leeway (left)	=	1 point	
		S by E $\frac{1}{2}$ E	
Or	=	S	16° 52′ E
Deviation	=		2° 0′ E
Magnetic course	=	S	14° 52′ E
Variation	=		20° 0′ W
True course	=	S	34° 52′ E
Or	=	S	35° E

Bearing Reversed

N N E $\frac{1}{2}$ E	=	N	28° 7′ E
Deviation	=		7° 0′ W
Magnetic bearing	=	N	21° 7′ E
Variation	=		20° 0′ W
True bearing	=	N	1° 7′ E

Sixth Course

Compass course	=	N N E	
Leeway (right)	=	$1\frac{1}{4}$ points	
		N E $\frac{3}{4}$ N	
Or	=	N	36° 34′ E
Deviation	=		6° 0′ E
Magnetic course	=	N	42° 34′ E
Variation	=		20° 0′ W
True course	=	N	22° 34′ E
Or	=	N	23° E

Current

Magnetic set	=	S W by W	
Or	=	S	56° 15′ W
Variation	=		20° 0′ W
True set	=	S	36° 15′ W
Or	=	S	36° W

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N 1° E	10	10.0	——	0.2	——
N 55° W	39	22.4	——	——	31.9
S 66° W	36	——	14.6	——	32.9
S 2° W	39	——	39.0	——	1.4
S 28° E	40	——	35.3	18.8	——
S 35° E	35	——	28.7	20.1	——
N 23° E	31	28.5	——	12.1	——
S 36° W	36	——	29.1	——	21.2
		60.9	146.7	51.2	87.4
			60.9		51.2
		D. Lat. =	85.8′ S	Dep. =	36.2′ W

Lat. from = 62° 11′ N
D. Lat. = 1° 26′ S
Lat. in = 60° 45′ N. Ans.
Sum of Lats. = 122° 56′
M. Lat. = 61° 28′

D. Long. = Dep. × sec M. Lat.
log 36.2 = 1.55871
log sec 61° 28′ = 10.32087
log D. Long. = 1.87958
D. Long. = 75.7′ W

Long. from = 166° 10′ W
D. Long. = 1° 16′ W
Long. in = 167° 26′ W. Ans.

(12) Consult Art. **39.**

(13) The course should be shaped according to Mercator's sailing. Since the places are situated nearly on the same meridian (see Art. **38**), there are no advantages worth mentioning in plotting the great-circle track in this case.

(14) Consult Arts. **43** and **44.**

(15) According to Art. **44,** since both places are in the same latitude, the vertex of the great-circle track must be situated midway between them. Hence, the longitude of the vertex equals

$$\frac{52^\circ\ 30' - 12^\circ\ 10'}{2} + 12^\circ\ 10'$$
$$= 32^\circ\ 20'\ \text{W.}\quad \text{Ans.}$$

(16) Consult Arts. **45** and **46.**

(17) See formulas **1** and **2,** Art. **51.**

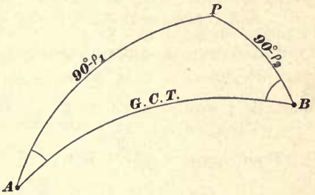

FIG. 3

(18) Referring to Fig. 3, calculate the initial and final course according to formulas **1** and **2,** Art. **48.** Thus,

$$\begin{array}{rclcrcl}
\text{Lat. } A & = & 36^\circ\ \text{N} & \qquad & \text{Long. } A & = & 70^\circ\ \text{W} \\
\text{Lat. } B & = & 50^\circ\ \text{N} & & \text{Long. } B & = & 10^\circ\ \text{W} \\
\hline
\rho_1 - \rho_2 & = & 14^\circ & & \text{D. Long.} & = & 60^\circ \\
\rho_1 + \rho_2 & = & 86^\circ & & \tfrac{1}{2}\ \text{D. Long.} & = & 30^\circ \\
\frac{\rho_1 - \rho_2}{2} & = & 7^\circ & & & & \\
\frac{\rho_1 + \rho_2}{2} & = & 43^\circ & & & &
\end{array}$$

$$\begin{array}{rclcrcl}
\log \cos 7^\circ & = & 9.99675 & \qquad & \log \sin 7^\circ & = & 9.08589 \\
\log \cot 30^\circ & = & 10.23856 & & \log \cot 30^\circ & = & 10.23856 \\
\log \operatorname{cosec} 43^\circ & = & 10.16622 & & \log \sec 43^\circ & = & 10.13587 \\
\hline
\log \tan \tfrac{1}{2}(A + B) & = & 10.40153 & & \log \tan \tfrac{1}{2}(A - B) & = & 9.46032 \\
\tfrac{1}{2}(A + B) & = & 68^\circ\ 22' & & \tfrac{1}{2}(A - B) & = & 16^\circ\ 6'
\end{array}$$

$$A = 68^\circ\ 22' - 16^\circ\ 6' = 52^\circ\ 16'$$
$$B = 68^\circ\ 22' + 16^\circ\ 6' = 84^\circ\ 28'$$

Hence, the initial course = N 52° 16′ E, and the final course = S 84° 28′ E. Ans.

(19) By a straight line connecting the places.

(20) (*a*) Consult Art. **59.**
(*b*) Consult Arts. **63** and **64.**

(21) Correction of courses:

First Course

Compasss course = N 28° 0′ W
Deviation = 14° 50′ W

Magnetic course = N 42° 50′ W
Variation = 23° 0′ W

True course = N 65° 50′ W
Or = N 66° W

Second Course

Compass course = E S E $\frac{1}{4}$ E
Or = S 70° 19′ E
Deviation = 18° 0′ E

Magnetic course = S 52° 19′ E
Variation = 23° 0′ W

True course = S 75° 19′ E

Third Course

Compass course = E S E
Or = S 67° 30′ E
Deviation = 18° 30′ E

Magnetic course = S 49° 0′ E
Variation = 23° 0′ W

True course = S 72° 0′ E

Fourth Course

Compass course = N N E $\frac{1}{4}$ E
Or = N 25° 19′ E
Deviation = 11° 0′ E

Magnetic course = N 36° 19′ E
Variation = 23° 0′ W

True course = N 13° 19′ E

Fifth Course

Compass course = W S W
Or = S 67° 30′ W
Deviation = 16° 30′ W

Magnetic course = S 51° 0′ W
Variation = 23° 0′ W

True course = S 28° 0′ W

Then arrange a traverse and find the latitude and longitude in as usual. Thus,

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
N 66° W	12.3	5.0	——	——	11.2
S 75° E	14.8	——	3.8	14.3	——
S 72° E	12.7	——	3.9	12.1	——
N 13° E	34.1	33.2	——	7.7	——
S 28° W	42.7	——	37.7	——	20.0
		38.2	45.4	34.1	31.2
			38.2	31.2	
		D. Lat. =	7.2′ S	2.9′ E	= Dep.

Lat. from = 48° 10′ N
D. Lat. = 7.2′ S

Lat. in = 48° 2.8′ N. Ans.
Sum of Lats. = 96° 12.8′
M. Lat. = 48° 6′

D. Long. = Dep. × sec M. Lat.
log 2.9 = 0.46240
log sec 48° 6′ = 0.17533

log D. Long. = 0.63773
D. Long. = 4.3′ E

Long. from = 14° 6′ W
D. Long. = 4.3′ E

Long. in = 14° 1.7′ W. Ans.

By inspection of the Traverse Tables it is found that the course made good from noon to noon is 22°, or S 22° E., nearly. Ans.

(22) (*a*) Referring to Fig. 4, proceed according to instructions given in Art. **53**. Thus, for the courses:

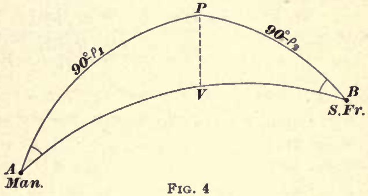

FIG. 4

Manila Lat. = 14° 29′ N
San Francisco Lat. = 37° 49′ N

$\rho_1 - \rho_2 = 23° 20'$
$\rho_1 + \rho_2 = 52° 18'$
$\frac{\rho_1 - \rho_2}{2} = 11° 40'$
$\frac{\rho_1 + \rho_2}{2} = 26° 9'$

Long. = 120° 55′ E
Long. = 122° 30′ W

D. Long. = 116° 35′
$\frac{1}{2}$ D. Long. = 58° 17′

log cos 11° 40′ = 9.99093
log cot 58° 17′ = 9.79100
log cosec 26° 9′ = 10.35583

log tan $\frac{1}{2}(A + B)$ = 10.13776
$\frac{1}{2}(A + B)$ = 53° 56′

log sin 11° 40′ = 9.30582
log cot 58° 17′ = 9.79100
log sec 26° 9′ = 10.04690

log tan $\frac{1}{2}(A - B)$ ∓ 9.14372
$\frac{1}{2}(A - B)$ = 7° 56′

$A = 53° 56' - 7° 56' = 46°$
$B = 53° 56' + 7° 56' = 61° 52'$

Hence, initial course = N 46° E; final course = S 61° 52′ E. Ans.

(*b*) For the latitude of vertex use formula in Art. **50**.

$$\cos \rho_v = \sin A \times \cos \rho_1$$

$$\begin{aligned} \log \sin 46° &= 9.85693 \\ \log \cos 14° 29' &= 9.98597 \\ \hline \log \cos \rho_v &= 9.84290 \\ \rho_v &= 45° 51' \end{aligned}$$

Hence, highest latitude reached = 45° 51′ N. Ans.

(23) It is evident that the difference between the great-circle track and the Mercatorial, or rhumb, track will show the saving in distance of A. To find the distance on the great-circle track,

$$\sin D = \frac{\cos \rho_2 \times \sin \text{D. Long.}}{\sin A},$$

in which, $\rho_2 = 37° 49'$; D. Long. = 116° 35′; and $A = 46°$, according to the preceding example. Hence,

$$\begin{aligned} \log \cos 37° 49' &= 9.89761 \\ \log \sin (116° 35' =)\ 63° 25' &= 9.95148 \\ \log \operatorname{cosec} 46° &= 10.14307 \\ \hline \log \sin D &= 9.99216 \\ D &= 79° 9' \end{aligned}$$

Whence, distance = 180° − 79° 9′ = 100° 51′, or 6,051 mi.

To find the Mercatorial course and distance, proceed as follows:

1st Lat. = 14° 29′ N	M. P. = 872.7
2d Lat. = 37° 49′ N	M. P. = 2440.2
D. Lat. = 23° 20′ = 1,400′	M. D. Lat. = 1567.5

Long. = 120° 55′ E
Long. = 122° 30′ W

D. Long. = 116° 35′ = 6,995′

tan C = D. Long. ÷ M. D. Lat.	Dist. = D. Lat. ÷ cos C
log 6,995 = 3.84479	log 1,400 = 3.14613
log 1567 = 3.19507	log cos 77° 22′ = 9.33987
log tan C = 10.64972	log Dist. = 3.80626
Course = 77° 22′	Dist. = 6,401 mi.

Hence, course of B = N 77° 22′ E. Ans.

Distance on great-circle track = 6,051 mi.
Distance on rhumb track = 6,401 mi.

Difference, or saving in distance by A = 350 mi. Ans.

(24) Consult Arts. **60** to **62**.

(25) Consult Art. **63**.

(26) (*a*) Consult Arts. **54** and **55**.
(*b*) Consult Art. **56**.

(27) Proceed according to instructions given in Art. **53**. Thus,

Cape Hatteras Lat. = 35° 15′ N Long. = 75° 30′ W

Lizard Lat. = 49° 58′ N Long. = 5° 12′ W

$\rho_1 - \rho_2 = 14° 43'$ D. Long. = 70° 18′

$\rho_1 + \rho_2 = 85° 13'$ ½ D. Long. = 35° 9′

$\frac{\rho_1 - \rho_2}{2} = 7° 21'$

$\frac{\rho_1 + \rho_2}{2} = 42° 36'$

log cos 7° 21′ = 9.99642
log cot 35° 9′ = 10.15236
log cosec 42° 36′ = 10.16949
log tan ½ $(A + B)$ = 10.31827
½ $(A + B)$ = 64° 20′

log sin 7° 21′ = 9.10697
log cot 35° 9′ = 10.15236
log sec 42° 36′ = 10.13306
log tan ½ $(A - B)$ = 9.39239
½ $(A - B)$ = 13° 52′

A = 64° 20′ − 13° 52′ = 50° 28′

B = 64° 20′ + 13° 52′ = 78° 12′

Hence, initial course = N 50° 28′ E; final course = S 78° 12′ E. Ans.

For the distance use formula in Art. **49**.

$$\sin D = \frac{\cos \rho_2 \times \sin \text{D. Long.}}{\sin A}$$

log cos 49° 58′ = 9.80837
log sin 70° 18′ = 9.97381
log cosec 50° 28′ = 10.11280
log sin D = 9.89498

Dist. = 51° 44′ = 3,104 mi. Ans.

(28) Consult Art. **39**.

(29) Let A, Fig. 5, be the place referred to, mn the 45th parallel, and x the point of tangency of the track Ax with that parallel. The angle PAx is then the initial course, Ax the required distance from x from A, and ro ($= APx$) the D. Long. To find PAx, the initial course, in the right spherical triangle AxP, where Px = 90° − 45° = 45°, and AP = 90° − 34° 29′ = 55° 31′, Napier's rules are applied. Thus,

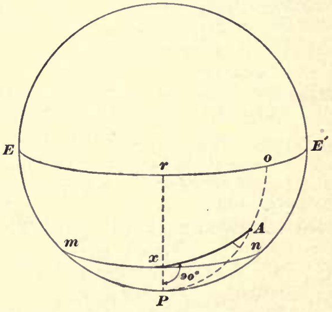

FIG. 5

$$\cos Px = \sin PAx \times \sin AP$$

and
$$\sin Px = \sin PAx \times \sin AP$$

Whence, $\sin PAx = \sin Px \times \operatorname{cosec} AP$

$$\begin{aligned} \log\sin 45°\ 0' &= 9.84949 \\ \log\operatorname{cosec} 55°\ 31' &= \underline{10.08392} \\ \log\sin PAx &= 9.93341 \\ PAx &= 59°\ 5' \end{aligned}$$

Hence, initial course = S 59° 5′ W. Ans.

For the distance Ax in the same right triangle,

$$\begin{aligned} \cos Ax &= \cos AP \times \sec Px \\ \log\cos 55°\ 31' &= 9.75294 \\ \log\sec 45° &= \underline{10.15051} \\ \log\cos Ax &= 9.90345 \\ Ax &= 36°\ 48' \end{aligned}$$

Hence, distance = 2,208 mi. Ans.

For the D. Long.

$$\begin{aligned} \sin APx &= \sin Ax \times \operatorname{cosec} AP \\ \log\sin 36°\ 48' &= 9.77744 \\ \log\operatorname{cosec} 55°\ 31' &= \underline{10.08392} \\ \log\sin APx &= 9.86136 \\ \text{D. Long., or } APx &= 46°\ 37'\ \text{W} \\ \text{Long. of } A &= \underline{18°\ 23'\ \text{E}} \\ \text{Long. of the point } x &= 28°\ 14'\ \text{W. Ans.} \end{aligned}$$

(30) Correction of courses:

First Course

Compass course =	S E by S
Leeway (right) =	1 point
	S S E
Or =	S 22° 30′ E
Deviation =	7° 50′ E
Magnetic course =	S 14° 40′ E
Variation =	2° 49′ E
True course =	S 11° 51′ E
Or =	S 12° E

Second Course

Compass course =	E by S
Leeway (right) =	1 point
	E S E
Or =	S 67° 30′ E
Deviation =	7° 15′ E
Magnetic course =	S 60° 15′ E
Variation =	2° 49′ E
True course =	S 57° 26′ E
Or =	S 57° E

Third Course

Compass course =	E $\frac{3}{4}$ S
Leeway (right) =	$\frac{3}{4}$ point
	E S E $\frac{1}{2}$ E
Or =	S 73° 7′ E
Deviation =	6° 10′ E
Magnetic course =	S 66° 57′ E
Variation =	2° 49′ E
True course =	S 64° 8′ E
Or =	S 64° E

Fourth Course

Compass course =	N W by N
Leeway (left) =	$\frac{1}{2}$ point
	N W $\frac{1}{2}$ N
Or =	N 39° 22′ W
Deviation =	9° 15′ W
Magnetic course =	N 48° 37′ W
Variation =	2° 49′ E
True course =	N 45° 48′ W
Or =	N 46° W

Fifth Course

Compass course = N by W
Or = N 11° 15′ W
Deviation = 2° 57′ E
Magnetic course = N 8° 18′ W
Variation = 2° 49′ E
True course = N 5° 29′ W
Or = N 5° W

Bearing Reversed

Magnetic bearing = S 49° 0′ E
Variation = 2° 49′ E
True bearing = S 46° 11′ E
Or = S 46° E

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S 46° E	12	——	8.3	8.6	——
S 12° E	39	——	38.1	8.1	——
S 57° E	39	——	21.2	32.7	——
S 64° E	21	——	9.2	18.9	——
N 46° W	28	19.5	——	——	20.1
N 5° W	20	19.9	——	——	1.7
		39.4	76.8	68.3	21.8
		76.8			68.3

D. Lat. = 37.4′ S   Dep. = 46.5′ E

Lat. from = 13° 4′ N
D. Lat. = 37.4′ S
Lat. in = 12° 26.6′ N. Ans.
Sum of Lats. = 25° 30.6′
M. Lat. = 12° 45′

D. Long. = Dep. × sec M. Lat.
log 46.5 = 1.66745
log sec 12° 45′ = 0.01084
log D. Long. = 1.67829
D. Long. = 47.7′ E

Long. from = 59° 32′ W
D. Long. = 47.7′ E
Long. in = 58° 44.3′ W. Ans.

(31) The latitude and longitude in by dead reckoning is determined, first; then, the course and distance from that position to the one obtained by astronomical observations will give the set and drift of the current. Thus,

D. Lat. = Dist. × cos C
log 39 = 1.59106
log cos 81° = 9.19433
log D. Lat. = 0.78539
D. Lat. = 0° 6.1′ N
Lat. from = 35° 3′ S
Lat. in = 34° 57′ S
Sum of Lats. = 70° 0′
M. Lat. = 35° 0′

D. Long. = D. Lat. × tan C × sec M. Lat.
log 6.1 = 0.78539
log tan 81° = 10.80029
log sec 35° = 10.08664
log D. Long. = 1.67232
D. Long. = 47′W
Long. from = 26° 52′ E
Long in. = 26° 5′ E

Position by D. R. . . . Lat. 34° 57′ S. . . . M. P. =	2227.5
Position by Obs. . . . Lat. 35° 13′ S. . . . M. P. =	2246.9
D. Lat. = 16′ S M. D. Lat. =	19.4

Long. = 26° 5′ E
Long. = 25° 5′ E
D. Long. = 1° 0′ = 60′ W

tan C = D. Long. ÷ M. D. Lat.	Dist. = D. Lat. ÷ cos C
log 60 = 1.77815	log 16 = 1.20412
log 19.4 = 1.28780	log cos 72° 5′ = 9.48803
log tan C = 10.49035	log Dist. = 1.71609
Course, or set = 72° 5′	Dist. = 52 mi.

Hence, set of current = S 72° W, and drift of current = $\frac{52}{24} = 2\frac{1}{6}$ mi. per hr. Ans.

(32) Correction of courses:

First Course		*Second Course*	
Compass course =	S W by W	Compass course =	S E $\frac{1}{4}$ S
Or =	S 56° 15′ W	Or =	S 42° 11′ E
Deviation =	15° 0′ W	Deviation =	10° 0′ W
Magnetic course =	S 41° 15′ W	Magnetic course =	S 52° 11′ E
Variation =	30° 30′ W	Variation =	30° 30′ W
True course =	S 10° 45′ W	True course =	S 82° 41′ E
Or =	S 11° W	Or =	S 83° E

Third Course		*Fourth Course*	
Compass course =	W N W $\frac{3}{4}$ W	Compass course =	E S E $\frac{1}{2}$ E
Leeway (right) =	$1\frac{1}{4}$ point	Leeway (left) =	$\frac{3}{4}$ point
	N W by W $\frac{1}{2}$ W		E $\frac{3}{4}$ S
Or =	N 61° 52′ W	Or =	S 81° 34′ E
Deviation =	7° 30′ W	Deviation =	3° 30′ E
Magnetic course =	N 69° 22′ W	Magnetic course =	S 78° 4′ E
Variation =	30° 30′ W	Variation =	30° 30′ W
True course =	N 99° 52′ W	True course =	S 108° 34′ E
Or =	S 80° W	Or =	N 71° E

Fifth Course		*Sixth Course*	
Compass course =	N N W $\frac{3}{4}$ W	Compass course =	W S W $\frac{3}{4}$ W
Leeway (right) =	$\frac{1}{2}$ point	Leeway (right) =	$\frac{1}{4}$ point
	N N W $\frac{1}{4}$ W		W by S
Or =	N 25° 19′ W	Or =	S 78° 45′ W
Deviation =	5° 30′ E	Deviation =	12° 0′ W
Magnetic course =	N 19° 49′ W	Magnetic course =	S 66° 45′ W
Variation =	30° 30′ W	Variation =	30° 30′ W
True course =	N 50° 19′ W	True course =	S 36° 15′ W
Or =	N 50° W	Or =	S 36° W

Bearing Reversed

Compass bearing = S ½ W
Or = S 5° 37′ W
Deviation = 15° 0′ W

Magnetic bearing = S 9° 23′ E
Variation = 30° 30′ W

True bearing = S 39° 53′ E
Or = S 40° E

Current

Magnetic set = N E ½ E
Or = N 50° 37′ E
Variation = 30° 30′ W

True set = N 20° 7′ E
Or = N 20° E

TRAVERSE

Courses	Dist.	D. Lat.		Dep.	
		N	S	E	W
S 40° E	14	——	10.7	9.0	——
S 11° W	39	——	38.3	——	7.4
S 83° E	37	——	4.5	36.7	——
S 80° W	38	——	6.6	——	37.4
N 71° E	35	11.4	——	33.1	——
N 50° W	36	23.1	——	——	27.6
S 36° W	32	——	25.9	——	18.8
N 20° E	36	33.8	——	12.3	——
		68.3	86.0	91.1	91.2
			68.3		91.1

D. Lat. = 17.7′ S Dep. = 0.1′ W
Lat. from = 46° 40′ N
Lat. in = 46° 22.3′ N. Ans.

Since the course made good is practically south, there is no D. Long. worth considering; hence, Long. in = 53° 4′ W. Ans.

For the bearing and distance to Cardiff:

Ship's position, Lat. = 46° 22′ N . . . M. P. = 3130.7
Cardiff, Lat. = 51° 28′ N . . . M. P. = 3595.6
D. Lat. = 5° 6′ M. D. Lat. = 464.9
Or = 306′ N
Long. = 53° 4′ W
Long. = 3° 10′ W
D. Long. = 49° 54′ E
Or = 2,994′ E

tan C = D. Long. ÷ M. D. Lat.
log 2,994 = 3.47625
log 464.9 = 2.66736
log tan C = 10.80889
Bearing = N 81° 10′ E. Ans.

Dist. = D. Lat. ÷ cos C
log 306 = 2.48572
log cos 81° 10′ = 9.18628
log Dist. = 3.29944
Dist. = 1,993 mi. Ans.

TABLE

OF

COMMON LOGARITHMS

OF NUMBERS

From 1 to 10,000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
0		**20**	30 103	**40**	60 206	**60**	77 815	**80**	90 309
1	00 000	21	32 222	41	61 278	61	78 533	81	90 849
2	30 103	22	34 242	42	62 325	62	79 239	82	91 381
3	47 712	23	36 173	43	63 347	63	79 934	83	91 908
4	60 206	24	38 021	44	64 345	64	80 618	84	92 428
5	69 897	25	39 794	45	65 321	65	81 291	85	92 942
6	77 815	26	41 497	46	66 276	66	81 954	86	93 450
7	84 510	27	43 136	47	67 210	67	82 607	87	93 952
8	90 309	28	44 716	48	68 124	68	83 251	88	94 448
9	95 424	29	46 240	49	69 020	69	83 885	89	94 939
10	00 000	**30**	47 712	**50**	69 897	**70**	84 510	**90**	95 424
11	04 139	31	49 136	51	70 757	71	85 126	91	95 904
12	07 918	32	50 515	52	71 600	72	85 733	92	96 379
13	11 394	33	51 851	53	72 428	73	86 332	93	96 848
14	14 613	34	53 148	54	73 239	74	86 923	94	97 313
15	17 609	35	54 407	55	74 036	75	87 506	95	97 772
16	20 412	36	55 630	56	74 819	76	88 081	96	98 227
17	23 045	37	56 820	57	75 587	77	88 649	97	98 677
18	25 527	38	57 978	58	76 343	78	89 209	98	99 123
19	27 875	39	59 106	59	77 085	79	89 763	99	99 564
20	30 103	**40**	60 206	**60**	77 815	**80**	90 309	**100**	00 000

N.	L. 0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
141	922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
151	898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	304	329	353	378	403	428	452	477	502	527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	717	741	764	788	811	834	858	881	905	928
186	951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	29	**28**
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	**26**
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	**23**
1	2.4	2.3
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	**21**
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	22	21
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9	19.8	18.9

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7	13.3
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9	17.1

	18
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3	5.4
4	7.2
5	9.0
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7	12.6
8	14.4
9	16.2

	17
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6	10.2
7	11.9
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P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
250	39 794	811	829	846	863	881	898	915	933	950	
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123	**18**
252	40 140	157	175	192	209	226	243	261	278	295	1 1.8
253	312	329	346	364	381	398	415	432	449	466	2 3.6
254	483	500	518	535	552	569	586	603	620	637	3 5.4
255	654	671	688	705	722	739	756	773	790	807	4 7.2
256	824	841	858	875	892	909	926	943	960	976	5 9.0
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	6 10.8
258	41 162	179	196	212	229	246	263	280	296	313	7 12.6
259	330	347	363	380	397	414	430	447	464	481	8 14.4
260	497	514	531	547	564	581	597	614	631	647	9 16.2
261	664	681	697	714	731	747	764	780	797	814	**17**
262	830	847	863	880	896	913	929	946	963	979	1 1.7
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	2 3.4
264	42 160	177	193	210	226	243	259	275	292	308	3 5.1
265	325	341	357	374	390	406	423	439	455	472	4 6.8
266	488	504	521	537	553	570	586	602	619	635	5 8.5
267	651	667	684	700	716	732	749	765	781	797	6 10.2
268	813	830	846	862	878	894	911	927	943	959	7 11.9
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120	8 13.6
270	43 136	152	169	185	201	217	233	249	265	281	9 15.3
271	297	313	329	345	361	377	393	409	425	441	**16**
272	457	473	489	505	521	537	553	569	584	600	1 1.6
273	616	632	648	664	680	696	712	727	743	759	2 3.2
274	775	791	807	823	838	854	870	886	902	917	3 4.8
275	933	949	965	981	996	*012	*028	*044	*059	*075	4 6.4
276	44 091	107	122	138	154	170	185	201	217	232	5 8.0
277	248	264	279	295	311	326	342	358	373	389	6 9.6
278	404	420	436	451	467	483	498	514	529	545	7 11.2
279	560	576	592	607	623	638	654	669	685	700	8 12.8
280	716	731	747	762	778	793	809	824	840	855	9 14.4
281	871	886	902	917	932	948	963	979	994	*010	**15**
282	45 025	040	056	071	086	102	117	133	148	163	1 1.5
283	179	194	209	225	240	255	271	286	301	317	2 3.0
284	332	347	362	378	393	408	423	439	454	469	3 4.5
285	484	500	515	530	545	561	576	591	606	621	4 6.0
286	637	652	667	682	697	712	728	743	758	773	5 7.5
287	788	803	818	834	849	864	879	894	909	924	6 9.0
288	939	954	969	984	*000	*015	*030	*045	*060	*075	7 10.5
289	46 090	105	120	135	150	165	180	195	210	225	8 12.0
290	240	255	270	285	300	315	330	345	359	374	9 13.5
291	389	404	419	434	449	464	479	494	509	523	**14**
292	538	553	568	583	598	613	627	642	657	672	1 1.4
293	687	702	716	731	746	761	776	790	805	820	2 2.8
294	835	850	864	879	894	909	923	938	953	967	3 4.2
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114	4 5.6
296	47 129	144	159	173	188	202	217	232	246	261	5 7.0
297	276	290	305	319	334	349	363	378	392	407	6 8.4
298	422	436	451	465	480	494	509	524	538	553	7 9.8
299	567	582	596	611	625	640	654	669	683	698	8 11.2
300	712	727	741	756	770	784	799	813	828	842	9 12.6
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
301	857	871	885	900	914	929	943	958	972	986
302	48 001	015	029	044	058	073	087	101	116	130
303	144	159	173	187	202	216	230	244	259	273
304	287	302	316	330	344	359	373	387	401	416
305	430	444	458	473	487	501	515	530	544	558
306	572	586	601	615	629	643	657	671	686	700
307	714	728	742	756	770	785	799	813	827	841
308	855	869	883	897	911	926	940	954	968	982
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
311	276	290	304	318	332	346	360	374	388	402
312	415	429	443	457	471	485	499	513	527	541
313	554	568	582	596	610	624	638	651	665	679
314	693	707	721	734	748	762	776	790	803	817
315	831	845	859	872	886	900	914	927	941	955
316	969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50 106	120	133	147	161	174	188	202	215	229
318	243	256	270	284	297	311	325	338	352	365
319	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
321	651	664	678	691	705	718	732	745	759	772
322	786	799	813	826	840	853	866	880	893	907
323	920	934	947	961	974	987	*001	*014	*028	*041
324	51 055	068	081	095	108	121	135	148	162	175
325	188	202	215	228	242	255	268	282	295	308
326	322	335	348	362	375	388	402	415	428	441
327	455	468	481	495	508	521	534	548	561	574
328	587	601	614	627	640	654	667	680	693	706
329	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101
332	52 114	127	140	153	166	179	192	205	218	231
333	244	257	270	284	297	310	323	336	349	362
334	375	388	401	414	427	440	453	466	479	492
335	504	517	530	543	556	569	582	595	608	621
336	634	647	660	673	686	699	711	724	737	750
337	763	776	789	802	815	827	840	853	866	879
338	892	905	917	930	943	956	969	982	994	*007
339	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	250	263
341	275	288	301	314	326	339	352	364	377	390
342	403	415	428	441	453	466	479	491	504	517
343	529	542	555	567	580	593	605	618	631	643
344	656	668	681	694	706	719	732	744	757	769
345	782	794	807	820	832	845	857	870	882	895
346	908	920	933	945	958	970	983	995	*008	*020
347	54 033	045	058	070	083	095	108	120	133	145
348	158	170	183	195	208	220	233	245	258	270
349	283	295	307	320	332	345	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	15	**14**	**13**	**12**
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4	6.0	5.6	5.2	4.8
5	7.5	7.0	6.5	6.0
6	9.0	8.4	7.8	7.2
7	10.5	9.8	9.1	8.4
8	12.0	11.2	10.4	9.6
9	13.5	12.6	11.7	10.8

N.	L. 0	1	2	3	4	5	6	7	8	9
350	54 407	419	432	444	456	469	481	494	506	518
351	531	543	555	568	580	593	605	617	630	642
352	654	667	679	691	704	716	728	741	753	765
353	777	790	802	814	827	839	851	864	876	888
354	900	913	925	937	949	962	974	986	998	*011
355	55 023	035	047	060	072	084	096	108	121	133
356	145	157	169	182	194	206	218	230	242	255
357	267	279	291	303	315	328	340	352	364	376
358	388	400	413	425	437	449	461	473	485	497
359	509	522	534	546	558	570	582	594	606	618
360	630	642	654	666	678	691	703	715	727	739
361	751	763	775	787	799	811	823	835	847	859
362	871	883	895	907	919	931	943	955	967	979
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56 110	122	134	146	158	170	182	194	205	217
365	229	241	253	265	277	289	301	312	324	336
366	348	360	372	384	396	407	419	431	443	455
367	467	478	490	502	514	526	538	549	561	573
368	585	597	608	620	632	644	656	667	679	691
369	703	714	726	738	750	761	773	785	797	808
370	820	832	844	855	867	879	891	902	914	926
371	937	949	961	972	984	996	*008	*019	*031	*043
372	57 054	066	078	089	101	113	124	136	148	159
373	171	183	194	206	217	229	241	252	264	276
374	287	299	310	322	334	345	357	368	380	392
375	403	415	426	438	449	461	473	484	496	507
376	519	530	542	553	565	576	588	600	611	623
377	634	646	657	669	680	692	703	715	726	738
378	749	761	772	784	795	807	818	830	841	852
379	864	875	887	898	910	921	933	944	955	967
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58 092	104	115	127	138	149	161	172	184	195
382	206	218	229	240	252	263	274	286	297	309
383	320	331	343	354	365	377	388	399	410	422
384	433	444	456	467	478	490	501	512	524	535
385	546	557	569	580	591	602	614	625	636	647
386	659	670	681	692	704	715	726	737	749	760
387	771	782	794	805	816	827	838	850	861	872
388	883	894	906	917	928	939	950	961	973	984
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
391	218	229	240	251	262	273	284	295	306	318
392	329	340	351	362	373	384	395	406	417	428
393	439	450	461	472	483	494	506	517	528	539
394	550	561	572	583	594	605	616	627	638	649
395	660	671	682	693	704	715	726	737	748	759
396	770	780	791	802	813	824	835	846	857	868
397	879	890	901	912	923	934	945	956	966	977
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60 097	108	119	130	141	152	163	173	184	195
400	206	217	228	239	249	260	271	282	293	304
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	13
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	12
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7	8.4
8	9.6
9	10.8

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9	9.0

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
400	60 206	217	228	239	249	260	271	282	293	304
401	314	325	336	347	358	369	379	390	401	412
402	423	433	444	455	466	477	487	498	509	520
403	531	541	552	563	574	584	595	606	617	627
404	638	649	660	670	681	692	703	713	724	735
405	746	756	767	778	788	799	810	821	831	842
406	853	863	874	885	895	906	917	927	938	949
407	959	970	981	991	*002	*013	*023	*034	*045	*055
408	61 066	077	087	098	109	119	130	140	151	162
409	172	183	194	204	215	225	236	247	257	268
410	278	289	300	310	321	331	342	352	363	374
411	384	395	405	416	426	437	448	458	469	479
412	490	500	511	521	532	542	553	563	574	584
413	595	606	616	627	637	648	658	669	679	690
414	700	711	721	731	742	752	763	773	784	794
415	805	815	826	836	847	857	868	878	888	899
416	909	920	930	941	951	962	972	982	993	*003
417	62 014	024	034	045	055	066	076	086	097	107
418	118	128	138	149	159	170	180	190	201	211
419	221	232	242	252	263	273	284	294	304	315
420	325	335	346	356	366	377	387	397	408	418
421	428	439	449	459	469	480	490	500	511	521
422	531	542	552	562	572	583	593	603	613	624
423	634	644	655	665	675	685	696	706	716	726
424	737	747	757	767	778	788	798	808	818	829
425	839	849	859	870	880	890	900	910	921	931
426	941	951	961	972	982	992	*002	*012	*022	*033
427	63 043	053	063	073	083	094	104	114	124	134
428	144	155	165	175	185	195	205	215	225	236
429	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
431	448	458	468	478	488	498	508	518	528	538
432	548	558	568	579	589	599	609	619	629	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	849	859	869	879	889	899	909	919	929	939
436	949	959	969	979	988	*998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	108	118	128	137
438	147	157	167	177	187	197	207	217	227	237
439	246	256	266	276	286	296	306	316	326	335
440	345	355	365	375	385	395	404	414	424	434
441	444	454	464	473	483	493	503	513	523	532
442	542	552	562	572	582	591	601	611	621	631
443	640	650	660	670	680	689	699	709	719	729
444	738	748	758	768	777	787	797	807	816	826
445	836	846	856	865	875	885	895	904	914	924
446	933	943	953	963	972	982	992	*002	*011	*021
447	65 031	040	050	060	070	079	089	099	108	118
448	128	137	147	157	167	176	186	196	205	215
449	225	234	244	254	263	273	283	292	302	312
450	321	331	341	350	360	369	379	389	398	408
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	11
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9	8.1

P. P.

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450	65 321	331	341	350	360	369	379	389	398	408
451	418	427	437	447	456	466	475	485	495	504
452	514	523	533	543	552	562	571	581	591	600
453	610	619	629	639	648	658	667	677	686	696
454	706	715	725	734	744	753	763	772	782	792
455	801	811	820	830	839	849	858	868	877	887
456	896	906	916	925	935	944	954	963	973	982
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077
458	66 087	096	106	115	124	134	143	153	162	172
459	181	191	200	210	219	229	238	247	257	266
460	276	285	295	304	314	323	332	342	351	361
461	370	380	389	398	408	417	427	436	445	455
462	464	474	483	492	502	511	521	530	539	549
463	558	567	577	586	596	605	614	624	633	642
464	652	661	671	680	689	699	708	717	727	736
465	745	755	764	773	783	792	801	811	820	829
466	839	848	857	867	876	885	894	904	913	922
467	932	941	950	960	969	978	987	997	*006	*015
468	67 025	034	043	052	062	071	080	089	099	108
469	117	127	136	145	154	164	173	182	191	201
470	210	219	228	237	247	256	265	274	284	293
471	302	311	321	330	339	348	357	367	376	385
472	394	403	413	422	431	440	449	459	468	477
473	486	495	504	514	523	532	541	550	560	569
474	578	587	596	605	614	624	633	642	651	660
475	669	679	688	697	706	715	724	733	742	752
476	761	770	779	788	797	806	815	825	834	843
477	852	861	870	879	888	897	906	916	925	934
478	943	952	961	970	979	988	997	*006	*015	*024
479	68 034	043	052	061	070	079	088	097	106	115
480	124	133	142	151	160	169	178	187	196	205
481	215	224	233	242	251	260	269	278	287	296
482	305	314	323	332	341	350	359	368	377	386
483	395	404	413	422	431	440	449	458	467	476
484	485	494	502	511	520	529	538	547	556	565
485	574	583	592	601	610	619	628	637	646	655
486	664	673	681	690	699	708	717	726	735	744
487	753	762	771	780	789	797	806	815	824	833
488	842	851	860	869	878	886	895	904	913	922
489	931	940	949	958	966	975	984	993	*002	*011
490	69 020	028	037	046	055	064	073	082	090	099
491	108	117	126	135	144	152	161	170	179	188
492	197	205	214	223	232	241	249	258	267	276
493	285	294	302	311	320	329	338	346	355	364
494	373	381	390	399	408	417	425	434	443	452
495	461	469	478	487	496	504	513	522	531	539
496	548	557	566	574	583	592	601	609	618	627
497	636	644	653	662	671	679	688	697	705	714
498	723	732	740	749	758	767	775	784	793	801
499	810	819	827	836	845	854	862	871	880	888
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501	984	992	*001	*010	*018	*027	*036	*044	*053	*062
502	70 070	079	088	096	105	114	122	131	140	148
503	157	165	174	183	191	200	209	217	226	234
504	243	252	260	269	278	286	295	303	312	321
505	329	338	346	355	364	372	381	389	398	406
506	415	424	432	441	449	458	467	475	484	492
507	501	509	518	526	535	544	552	561	569	578
508	586	595	603	612	621	629	638	646	655	663
509	672	680	689	697	706	714	723	731	740	749
510	757	766	774	783	791	800	808	817	825	834
511	842	851	859	868	876	885	893	902	910	919
512	927	935	944	952	961	969	978	986	995	*003
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514	096	105	113	122	130	139	147	155	164	172
515	181	189	198	206	214	223	231	240	248	257
516	265	273	282	290	299	307	315	324	332	341
517	349	357	366	374	383	391	399	408	416	425
518	433	441	450	458	466	475	483	492	500	508
519	517	525	533	542	550	559	567	575	584	592
520	600	609	617	625	634	642	650	659	667	675
521	684	692	700	709	717	725	734	742	750	759
522	767	775	784	792	800	809	817	825	834	842
523	850	858	867	875	883	892	900	908	917	925
524	933	941	950	958	966	975	983	991	999	*008
525	72 016	024	032	041	049	057	066	074	082	090
526	099	107	115	123	132	140	148	156	165	173
527	181	189	198	206	214	222	230	239	247	255
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532	591	599	607	616	624	632	640	648	656	665
533	673	681	689	697	705	713	722	730	738	746
534	754	762	770	779	787	795	803	811	819	827
535	835	843	852	860	868	876	884	892	900	908
536	916	925	933	941	949	957	965	973	981	989
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070
538	73 078	086	094	102	111	119	127	135	143	151
539	159	167	175	183	191	199	207	215	223	231
540	239	247	255	263	272	280	288	296	304	312
541	320	328	336	344	352	360	368	376	384	392
542	400	408	416	424	432	440	448	456	464	472
543	480	488	496	504	512	520	528	536	544	552
544	560	568	576	584	592	600	608	616	624	632
545	640	648	656	664	672	679	687	695	703	711
546	719	727	735	743	751	759	767	775	783	791
547	799	807	815	823	830	838	846	854	862	870
548	878	886	894	902	910	918	926	933	941	949
549	957	965	973	981	989	997	*005	*013	*020	*028
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P. P.

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552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
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561	896	904	912	920	927	935	943	950	958	966	1 0.8
562	974	981	989	997	*005	*012	*020	*028	*035	*043	2 1.6
563	75 051	059	066	074	082	089	097	105	113	120	3 2.4
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566	282	289	297	305	312	320	328	335	343	351	6 4.8
567	358	366	374	381	389	397	404	412	420	427	7 5.6
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572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
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575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
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589	77 012	019	026	034	041	048	056	063	070	078	9 6.3
590	085	093	100	107	115	122	129	137	144	151	
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592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
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602	960	967	974	981	988	996	*003	*010	*017	*025
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606	247	254	262	269	276	283	290	297	305	312
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608	390	398	405	412	419	426	433	440	447	455
609	462	469	476	483	490	497	504	512	519	526
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611	604	611	618	625	633	640	647	654	661	668
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615	888	895	902	909	916	923	930	937	944	951
616	958	965	972	979	986	993	*000	*007	*014	*021
617	79 029	036	043	050	057	064	071	078	085	092
618	099	106	113	120	127	134	141	148	155	162
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620	239	246	253	260	267	274	281	288	295	302
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622	379	386	393	400	407	414	421	428	435	442
623	449	456	463	470	477	484	491	498	505	511
624	518	525	532	539	546	553	560	567	574	581
625	588	595	602	609	616	623	630	637	644	650
626	657	664	671	678	685	692	699	706	713	720
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628	796	803	810	817	824	831	837	844	851	858
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631	80 003	010	017	024	030	037	044	051	058	065
632	072	079	085	092	099	106	113	120	127	134
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635	277	284	291	298	305	312	318	325	332	339
636	346	353	359	366	373	380	387	393	400	407
637	414	421	428	434	441	448	455	462	468	475
638	482	489	496	502	509	516	523	530	536	543
639	550	557	564	570	577	584	591	598	604	611
640	618	625	632	638	645	652	659	665	672	679
641	686	693	699	706	713	720	726	733	740	747
642	754	760	767	774	781	787	794	801	808	814
643	821	828	835	841	848	855	862	868	875	882
644	889	895	902	909	916	922	929	936	943	949
645	956	963	969	976	983	990	996	*003	*010	*017
646	81 023	030	037	043	050	057	•064	070	077	084
647	090	097	104	111	117	124	131	137	144	151
648	158	164	171	178	184	191	198	204	211	218
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654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
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659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	**7**
661	82 020	027	033	040	046	053	060	066	073	079	1 0.7
662	086	092	099	105	112	119	125	132	138	145	2 1.4
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669	543	549	556	562	569	575	582	588	595	601	9 6.3
670	607	614	620	627	633	640	646	653	659	666	
671	672	679	685	692	698	705	711	718	724	730	
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673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
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681	315	321	327	334	340	347	353	359	366	372	**6**
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688	759	765	771	778	784	790	797	803	809	816	7 4.2
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697	323	330	336	342	348	354	361	367	373	379	
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701	572	578	584	590	597	603	609	615	621	628
702	634	640	646	652	658	665	671	677	683	689
703	696	702	708	714	720	726	733	739	745	751
704	757	763	770	776	782	788	794	800	807	813
705	819	825	831	837	844	850	856	862	868	874
706	880	887	893	899	905	911	917	924	930	936
707	942	948	954	960	967	973	979	985	991	997
708	85 003	009	016	022	028	034	040	046	052	058
709	065	071	077	083	089	095	101	107	114	120
710	126	132	138	144	150	156	163	169	175	181
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715	431	437	443	449	455	461	467	473	479	485
716	491	497	503	509	516	522	528	534	540	546
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718	612	618	625	631	637	643	649	655	661	667
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722	854	860	866	872	878	884	890	896	902	908
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724	974	980	986	992	998	*004	*010	*016	*022	*028
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726	094	100	106	112	118	124	130	136	141	147
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731	392	398	404	410	415	421	427	433	439	445
732	451	457	463	469	475	481	487	493	499	504
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737	747	753	759	764	770	776	782	788	794	800
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739	864	870	876	882	888	894	900	906	911	917
740	923	929	935	941	947	953	958	964	970	976
741	982	988	994	999	*005	*011	*017	*023	*029	*035
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744	157	163	169	175	181	186	192	198	204	210
745	216	221	227	233	239	245	251	256	262	268
746	274	280	286	291	297	303	309	315	320	326
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P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
750	87 506	512	518	523	529	535	541	547	552	558	
751	564	570	576	581	587	593	599	604	610	616	
752	622	628	633	639	645	651	656	662	668	674	
753	679	685	691	697	703	708	714	720	726	731	
754	737	743	749	754	760	766	772	777	783	789	
755	795	800	806	812	818	823	829	835	841	846	
756	852	858	864	869	875	881	887	892	898	904	
757	910	915	921	927	933	938	944	950	955	961	
758	967	973	978	984	990	996	*001	*007	*013	*018	
759	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	**6**
761	138	144	150	156	161	167	173	178	184	190	1 0.6
762	195	201	207	213	218	224	230	235	241	247	2 1.2
763	252	258	264	270	275	281	287	292	298	304	3 1.8
764	309	315	321	326	332	338	343	349	355	360	4 2.4
765	366	372	377	383	389	395	400	406	412	417	5 3.0
766	423	429	434	440	446	451	457	463	468	474	6 3.6
767	480	485	491	497	502	508	513	519	525	530	7 4.2
768	536	542	547	553	559	564	570	576	581	587	8 4.8
769	593	598	604	610	615	621	627	632	638	643	9 5.4
770	649	655	660	666	672	677	683	589	694	700	
771	705	711	717	722	728	734	739	745	750	756	
772	762	767	773	779	784	790	795	801	807	812	
773	818	824	829	835	840	846	852	857	863	868	
774	874	880	885	891	897	902	908	913	919	925	
775	930	936	941	947	953	958	964	969	975	981	
776	986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89 042	048	053	059	064	070	076	081	087	092	
778	098	104	109	115	120	126	131	137	143	148	
779	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	**5**
781	265	271	276	282	287	293	298	304	310	315	1 0.5
782	321	326	332	337	343	348	354	360	365	371	2 1.0
783	376	382	387	393	398	404	409	415	421	426	3 1.5
784	432	437	443	448	454	459	465	470	476	481	4 2.0
785	487	492	498	504	509	515	520	526	531	537	5 2.5
786	542	548	553	459	564	570	575	581	586	592	6 3.0
787	597	603	609	614	620	625	631	636	642	647	7 3.5
788	653	658	664	669	675	680	686	691	697	702	8 4.0
789	708	713	719	724	730	735	741	746	752	757	9 4.5
790	763	768	774	779	785	790	796	801	807	812	
791	818	823	829	834	840	845	851	856	862	867	
792	873	878	883	889	894	900	905	911	916	922	
793	927	933	938	944	949	955	960	966	971	977	
794	982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90 037	042	048	053	059	064	069	075	080	086	
796	091	097	102	108	113	119	124	129	135	140	
797	146	151	157	162	168	173	179	184	189	195	
798	200	206	211	217	222	227	233	238	244	249	
799	255	260	266	271	276	282	287	293	298	304	
800	309	314	320	325	331	336	342	347	352	358	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
800	90 309	314	320	325	331	336	342	347	352	358
801	363	369	374	380	385	390	396	401	407	412
802	417	423	428	434	439	445	450	455	461	466
803	472	477	482	488	493	499	504	509	515	520
804	526	531	536	542	547	553	558	563	569	574
805	580	585	590	596	601	607	612	617	623	628
806	634	639	644	650	655	660	666	671	677	682
807	687	693	698	703	709	714	720	725	730	736
808	741	747	752	757	763	768	773	779	784	789
809	795	800	806	811	816	822	827	832	838	843
810	849	854	859	865	870	875	881	886	891	897
811	902	907	913	918	924	929	934	940	945	950
812	956	961	966	972	977	982	988	993	998	*004
813	91 009	014	020	025	030	036	041	046	052	057
814	062	068	073	078	084	089	094	100	105	110
815	116	121	126	132	137	142	148	153	158	164
816	169	174	180	185	190	196	201	206	212	217
817	222	228	233	238	243	249	254	259	265	270
818	275	281	286	291	297	302	307	312	318	323
819	328	334	339	344	350	355	360	365	371	376
820	381	387	392	397	403	408	413	418	424	429
821	434	440	445	450	455	461	466	471	477	482
822	487	492	498	503	508	514	519	524	529	535
823	540	545	551	556	561	566	572	577	582	587
824	593	598	603	609	614	619	624	630	535	640
825	645	651	656	661	666	672	677	682	687	693
826	698	703	709	714	719	724	730	735	740	745
827	751	756	761	766	772	777	782	787	793	798
828	803	808	814	819	824	829	834	840	845	850
829	855	861	866	871	876	882	887	892	897	903
830	908	913	918	924	929	934	939	944	950	955
831	960	965	971	976	981	986	991	997	*002	*007
832	92 012	018	023	028	033	038	044	049	054	059
833	065	070	075	080	085	091	096	101	106	111
834	117	122	127	132	137	143	148	153	158	163
835	169	174	179	184	189	195	200	205	210	215
836	221	226	231	236	241	247	252	257	262	267
837	273	278	283	288	293	298	304	309	314	319
838	324	330	335	340	345	350	355	361	366	371
839	376	381	387	392	397	402	407	412	418	423
840	428	433	438	443	449	454	459	464	469	474
841	480	485	490	495	500	505	511	516	521	526
842	531	536	542	547	552	557	562	567	572	578
843	583	588	593	598	603	609	614	619	624	629
844	634	639	645	650	655	660	665	670	675	681
845	686	691	696	701	706	711	716	722	727	732
846	737	742	747	752	758	763	768	773	778	783
847	788	793	799	804	809	814	819	824	829	834
848	840	845	850	855	860	865	870	875	881	886
849	891	896	901	906	911	916	921	927	932	937
850	942	947	952	957	962	967	973	978	983	988
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

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9	4.5

P. P.

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851	993	998	*003	*008	*013	*018	*024	*029	*034	*039
852	93 044	049	054	059	064	069	075	080	085	090
853	095	100	105	110	115	120	125	131	136	141
854	146	151	156	161	166	171	176	181	186	192
855	197	202	207	212	217	222	227	232	237	242
856	247	252	258	263	268	273	278	283	288	293
857	298	303	308	313	318	323	328	334	339	344
858	349	354	359	364	369	374	379	384	389	394
859	399	404	409	414	420	425	430	435	440	445
860	450	455	460	465	470	475	480	485	490	495
861	500	505	510	515	520	526	531	536	541	546
862	551	556	561	566	571	576	581	586	591	596
863	601	606	611	616	621	626	631	636	641	646
864	651	656	661	666	671	676	682	687	692	697
865	702	707	712	717	722	727	732	737	742	747
866	752	757	762	767	772	777	782	787	792	797
867	802	807	812	817	822	827	832	837	842	847
868	852	857	862	867	872	877	882	887	892	897
869	902	907	912	917	922	927	932	937	942	947
870	952	957	962	967	972	977	982	987	992	997
871	94 002	007	012	017	022	027	032	037	042	047
872	052	057	062	067	072	077	082	086	091	096
873	101	106	111	116	121	126	131	136	141	146
874	151	156	161	166	171	176	181	186	191	196
875	201	206	211	216	221	226	231	236	240	245
876	250	255	260	265	270	275	280	285	290	295
877	300	305	310	315	320	325	330	335	340	345
878	349	354	359	364	369	374	379	384	389	394
879	399	404	409	414	419	424	429	433	438	443
880	448	453	458	463	468	473	478	483	488	493
881	498	503	507	512	517	522	527	532	537	542
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883	596	601	606	611	616	621	626	630	635	640
884	645	650	655	660	665	670	675	680	685	689
885	694	699	704	709	714	719	724	729	734	738
886	743	748	753	758	763	768	773	778	783	787
887	792	797	802	807	812	817	822	827	832	836
888	841	846	851	856	861	866	871	876	880	885
889	890	895	900	905	910	915	919	924	929	934
890	939	944	949	954	959	963	968	973	978	983
891	988	993	998	*002	*007	*012	*017	*022	*027	*032
892	95 036	041	046	051	056	061	066	071	075	080
893	085	090	095	100	105	109	114	119	124	129
894	134	139	143	148	153	158	163	168	173	177
895	182	187	192	197	202	207	211	216	221	226
896	231	236	240	245	250	255	260	265	270	274
897	279	284	289	294	299	303	308	313	318	323
898	328	332	337	342	347	352	357	361	366	371
899	376	381	386	390	395	400	405	410	415	419
900	424	429	434	439	444	448	453	458	463	468
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

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9	3.6

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
900	95 424	429	434	439	444	448	453	458	463	468	
901	472	477	482	487	492	497	501	506	511	516	
902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
909	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	**5**
911	952	957	961	966	971	976	980	985	990	995	1 0.5
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	2 1.0
913	96 047	052	057	061	066	071	076	080	085	090	3 1.5
914	095	099	104	109	114	118	123	128	133	137	4 2.0
915	142	147	152	156	161	166	171	175	180	185	5 2.5
916	190	194	199	204	209	213	218	223	227	232	6 3.0
917	237	242	246	251	256	261	265	270	275	280	7 3.5
918	284	289	294	298	303	308	313	317	322	327	8 4.0
919	332	336	341	346	350	355	360	365	369	374	9 4.5
920	379	384	388	393	398	402	407	412	417	421	
921	426	431	435	440	445	450	454	459	464	468	
922	473	478	483	487	492	497	501	506	511	515	
923	520	525	530	534	539	544	548	553	558	562	
924	567	572	577	581	586	591	595	600	605	609	
925	614	619	624	628	633	638	642	647	652	656	
926	661	666	670	675	680	685	689	694	699	703	
927	708	713	717	722	727	731	736	741	745	750	
928	755	759	764	769	774	778	783	788	792	797	
929	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	**4**
931	895	900	904	909	914	918	923	928	932	937	1 0.4
932	942	946	951	956	960	965	970	974	979	984	2 0.8
933	988	993	997	*002	*007	*011	*016	*021	*025	*030	3 1.2
934	97 035	039	044	049	053	058	063	067	072	077	4 1.6
935	081	086	090	095	100	104	109	114	118	123	5 2.0
936	128	132	137	142	146	151	155	160	165	169	6 2.4
937	174	179	183	188	192	197	202	206	211	216	7 2.8
938	220	225	230	234	239	243	248	253	257	262	8 3.2
939	267	271	276	280	285	290	294	299	304	308	9 3.6
940	313	317	322	327	331	336	340	345	350	354	
941	359	364	368	373	377	382	387	391	396	400	
942	405	410	414	419	424	428	433	437	442	447	
943	451	456	460	465	470	474	479	483	488	493	
944	497	502	506	511	516	520	525	529	534	539	
945	543	548	552	557	562	566	571	575	580	585	
946	589	594	598	603	607	612	617	621	626	630	
947	635	640	644	649	653	658	663	667	672	676	
948	681	685	690	695	699	704	708	713	717	722	
949	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
950	97 772	777	782	786	791	795	800	804	809	813	
951	818	823	827	832	836	841	845	850	855	859	
952	864	868	873	877	882	886	891	896	900	905	
953	909	914	918	923	928	932	937	941	946	950	
954	955	959	964	968	973	978	982	987	991	996	
955	98 000	005	009	014	019	023	028	032	037	041	
956	046	050	055	059	064	068	073	078	082	087	
957	091	096	100	105	109	114	118	123	127	132	
958	137	141	146	150	155	159	164	168	173	177	
959	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	**5**
961	272	277	281	186	290	295	299	304	308	313	1 0.5
962	318	322	327	331	336	340	345	349	354	358	2 1.0
963	363	367	372	376	381	385	390	394	399	403	3 1.5
964	408	412	417	421	426	430	435	439	444	448	4 2.0
965	453	457	462	466	471	475	480	484	489	493	5 2.5
966	498	502	507	511	516	520	525	529	534	538	6 3.0
967	543	547	552	556	561	565	570	574	579	583	7 3.5
968	588	592	597	601	605	610	614	619	623	628	8 4.0
969	632	637	641	646	650	655	659	664	668	673	9 4.5
970	677	682	686	691	695	700	704	709	713	717	
971	722	726	731	735	740	744	749	753	758	762	
972	767	771	776	780	784	789	793	798	802	807	
973	811	816	820	825	829	834	838	843	847	851	
974	856	860	865	869	874	878	883	887	892	896	
975	900	905	909	914	918	923	927	932	936	941	
976	945	949	954	958	963	967	972	976	981	985	
977	989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99 034	038	043	047	052	056	061	065	069	074	
979	078	083	087	092	096	100	195	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	
981	167	171	176	180	185	189	193	198	202	207	**4**
982	211	216	220	224	229	233	238	242	247	251	1 0.4
983	255	260	264	269	273	277	282	286	291	295	2 0.8
984	300	304	308	313	317	322	326	330	335	339	3 1.2
985	344	348	352	357	361	366	370	374	379	383	4 1.6
986	388	392	396	401	405	410	414	419	423	427	5 2.0
987	432	436	441	445	449	454	458	463	467	471	6 2.4
988	476	480	484	489	493	498	502	506	511	515	7 2.8
989	520	524	528	533	537	542	546	550	555	559	8 3.2
990	564	568	572	577	581	585	590	594	599	603	9 3.6
991	607	612	616	621	625	629	634	638	642	647	
992	651	656	660	664	669	673	677	682	686	691	
993	695	699	704	708	712	717	721	726	730	734	
994	739	743	747	752	756	760	765	769	774	778	
995	782	787	791	795	800	804	808	813	817	822	
996	826	830	835	839	843	848	852	856	861	865	
997	870	874	878	883	887	891	896	900	904	909	
998	913	917	922	926	930	935	939	944	948	952	
999	957	961	965	970	974	978	983	987	991	996	
1000	00 000	004	009	013	017	022	026	030	035	039	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

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8	.00233	1.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
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12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.90949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	89°		88°		87°		86°		85°		′

′	5°		6°		7°		8°		9°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	84°		83°		82°		81°		80°		′

′	10°		11°		12°		13°		14°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.07987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	64°		63°		62°		61°		60°		′

′	30° Sine.	30° Cosine.	31° Sine.	31° Cosine.	32° Sine.	32° Cosine.	33° Sine.	33° Cosine.	34° Sine.	34° Cosine.	′
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	′
	59°		58°		57°		56°		55°		

′	35° Sine.	35° Cosine.	36° Sine.	36° Cosine.	37° Sine.	37° Cosine.	38° Sine.	38° Cosine.	39° Sine.	39° Cosine.	′
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69362	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	Cosine.	Sine.	
′	49°		48°		47°		46°		45°		′

′	0°		1°		2°		3°		4°		′
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	60
1	.00029	3,437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	.07022	14.2411	59
2	.00058	1,718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	.07051	14.1821	58
3	.00087	1,145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	.07080	14.1235	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	.07110	14.0655	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	.07139	14.0079	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	.07168	13.9507	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	.07197	13.8940	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	.07227	13.8378	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	.07256	13.7821	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	.07285	13.7267	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	.07314	13.6719	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	.07344	13.6174	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	.07373	13.5634	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	.07402	13.5098	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	.07431	13.4566	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	.07461	13.4039	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	.07490	13.3515	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	.07519	13.2996	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	.07548	13.2480	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	.07578	13.1969	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	.07607	13.1461	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	.07636	13.0958	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	.07665	13.0458	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	.07695	12.9962	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	.07724	12.9469	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	.07753	12.8981	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	.07782	12.8496	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	.07812	12.8014	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	.07841	12.7536	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	.07870	12.7062	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	.07899	12.6591	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	.07929	12.6124	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	.07958	12.5660	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	.07987	12.5199	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	.08017	12.4742	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	.08046	12.4288	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	.08075	12.3838	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	.08104	12.3390	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	.08134	12.2946	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	.08163	12.2505	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	.08192	12.2067	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	.08221	12.1632	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	.08251	12.1201	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	.08280	12.0772	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	.08309	12.0346	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	.08339	11.9923	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	.08368	11.9504	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	.08397	11.9087	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	.08427	11.8673	11
50	.01455	68 7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	.08456	11.8262	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	.08485	11.7853	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	.08514	11.7448	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	.08544	11.7045	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	.08573	11.6645	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	.08602	11.6248	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	.08632	11.5853	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	.08661	11.5461	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	.08690	11.5072	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	.08720	11.4685	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	.08749	11.4301	0
	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	
′	89°		88°		87°		86°		85°		′

′	5°		6°		7°		8°		9°		′
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	60
1	.08778	11.3919	.10540	9.48781	.12308	8.12481	.14084	7.10038	.15868	6.30189	59
2	.08807	11.3540	.10569	9.46141	.12338	8.10536	.14113	7.08546	.15898	6.29007	58
3	.08837	11.3163	.10599	9.43515	.12367	8.08600	.14143	7.07059	.15928	6.27829	57
4	.08866	11.2789	.10628	9.40904	.12397	8.06674	.14173	7.05579	.15958	6.26655	56
5	.08895	11.2417	.10657	9.38307	.12426	8.04756	.14202	7.04105	.15988	6.25486	55
6	.08925	11.2048	.10687	9.35724	.12456	8.02848	.14232	7.02637	.16017	6.24321	54
7	.08954	11.1681	.10716	9.33155	.12485	8.00948	.14262	7.01174	.16047	6.23160	53
8	.08983	11.1316	.10746	9.30599	.12515	7.99058	.14291	6.99718	.16077	6.22003	52
9	.09013	11.0954	.10775	9.28058	.12544	7.97176	.14321	6.98268	.16107	6.20851	51
10	.09042	11.0594	.10805	9.25530	.12574	7.95302	.14351	6.96823	.16137	6.19703	50
11	.09071	11.0237	.10834	9.23016	.12603	7.93438	.14381	6.95385	.16167	6.18559	49
12	.09101	10.9882	.10863	9.20516	.12633	7.91582	.14410	6.93952	.16196	6.17419	48
13	.09130	10.9529	.10893	9.18028	.12662	7.89734	.14440	6.92525	.16226	6.16283	47
14	.09159	10.9178	.10922	9.15554	.12692	7.87895	.14470	6.91104	.16256	6.15151	46
15	.09189	10.8829	.10952	9.13093	.12722	7.86064	.14499	6.89688	.16286	6.14023	45
16	.09218	10.8483	.10981	9.10646	.12751	7.84242	.14529	6.88278	.16316	6.12899	44
17	.09247	10.8139	.11011	9.08211	.12781	7.82428	.14559	6.86874	.16346	6.11779	43
18	.09277	10.7797	.11040	9.05789	.12810	7.80622	.14588	6.85475	.16376	6.10664	42
19	.09306	10.7457	.11070	9.03379	.12840	7.78825	.14618	6.84082	.16405	6.09552	41
20	.09335	10.7119	.11099	9.00983	.12869	7.77035	.14648	6.82694	.16435	6.08444	40
21	.09365	10.6783	.11128	8.98598	.12899	7.75254	.14678	6.81312	.16465	6.07340	39
22	.09394	10.6450	.11158	8.96227	.12929	7.73480	.14707	6.79936	.16495	6.06240	38
23	.09423	10.6118	.11187	8.93867	.12958	7.71715	.14737	6.78564	.16525	6.05143	37
24	.09453	10.5789	.11217	8.91520	.12988	7.69957	.14767	6.77199	.16555	6.04051	36
25	.09482	10.5462	.11246	8.89185	.13017	7.68208	.14796	6.75838	.16585	6.02962	35
26	.09511	10.5136	.11276	8.86862	.13047	7.66466	.14826	6.74483	.16615	6.01878	34
27	.09541	10.4813	.11305	8.84551	.13076	7.64732	.14856	6.73133	.16645	6.00797	33
28	.09570	10.4491	.11335	8.82252	.13106	7.63005	.14886	6.71789	.16674	5.99720	32
29	.09600	10.4172	.11364	8.79964	.13136	7.61287	.14915	6.70450	.16704	5.98646	31
30	.09629	10.3854	.11394	8.77689	.13165	7.59575	.14945	6.69116	.16734	5.97576	30
31	.09658	10.3538	.11423	8.75425	.13195	7.57872	.14975	6.67787	.16764	5.96510	29
32	.09688	10.3224	.11452	8.73172	.13224	7.56176	.15005	6.66463	.16794	5.95448	28
33	.09717	10.2913	.11482	8.70931	.13254	7.54487	.15034	6.65144	.16824	5.94390	27
34	.09746	10.2602	.11511	8.68701	.13284	7.52806	.15064	6.63831	.16854	5.93335	26
35	.09776	10.2294	.11541	8.66482	.13313	7.51132	.15094	6.62523	.16884	5.92283	25
36	.09805	10.1988	.11570	8.64275	.13343	7.49465	.15124	6.61219	.16914	5.91236	24
37	.09834	10.1683	.11600	8.62078	.13372	7.47806	.15153	6.59921	.16944	5.90191	23
38	.09864	10.1381	.11629	8.59893	.13402	7.46154	.15183	6.58627	.16974	5.89151	22
39	.09893	10.1080	.11659	8.57718	.13432	7.44509	.15213	6.57339	.17004	5.88114	21
40	.09923	10.0780	.11688	8.55555	.13461	7.42871	.15243	6.56055	.17033	5.87080	20
41	.09952	10.0483	.11718	8.53402	.13491	7.41240	.15272	6.54777	.17063	5.86051	19
42	.09981	10.0187	.11747	8.51259	.13521	7.39616	.15302	6.53503	.17093	5.85024	18
43	.10011	9.98931	.11777	8.49128	.13550	7.37999	.15332	6.52234	.17123	5.84001	17
44	.10040	9.96007	.11806	8.47007	.13580	7.36389	.15362	6.50970	.17153	5.82982	16
45	.10069	9.93101	.11836	8.44896	.13609	7.34786	.15391	6.49710	.17183	5.81966	15
46	.10099	9.90211	.11865	8.42795	.13639	7.33190	.15421	6.48456	.17213	5.80953	14
47	.10128	9.87338	.11895	8.40705	.13669	7.31600	.15451	6.47206	.17243	5.79944	13
48	.10158	9.84482	.11924	8.38625	.13698	7.30018	.15481	6.45961	.17273	5.78938	12
49	.10187	9.81641	.11954	8.36555	.13728	7.28442	.15511	6.44720	.17303	5.77936	11
50	.10216	9.78817	.11983	8.34496	.13758	7.26873	.15540	9.43484	.17333	5.76937	10
51	.10246	9.76009	.12013	8.32446	.13787	7.25310	.15570	6.42253	.17363	5.75941	9
52	.10275	9.73217	.12042	8.30406	.13817	7.23754	.15600	6.41026	.17393	5.74949	8
53	.10305	9.70441	.12072	8.28376	.13846	7.22204	.15630	6.39804	.17423	5.73960	7
54	.10334	9.67680	.12101	8.26355	.13876	7.20661	.15660	6.38587	.17453	5.72974	6
55	.10363	9.64935	.12131	8.24345	.13906	7.19125	.15689	6.37374	.17483	5.71992	5
56	.10393	9.62205	.12160	8.22344	.13935	7.17594	.15719	6.36165	.17513	5.71013	4
57	.10422	9.59490	.12190	8.20352	.13965	7.16071	.15749	6.34961	.17543	5.70037	3
58	.10452	9.56791	.12219	8.18370	.13995	7.14553	.15779	6.33761	.17573	5.69064	2
59	.10481	9.54106	.12249	8.16398	.14024	7.13042	.15809	6.32566	.17603	5.68094	1
60	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	.17633	5.67128	0
	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	
′	84°		83°		82°		81°		80°		′

′	10°		11°		12°		13°		14°		′
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.17633	5.67128	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	60
1	.17663	5.66165	.19468	5.13658	.21286	4.69791	.23117	4.32573	.24964	4.00582	50
2	.17693	5.65205	.19498	5.12862	.21316	4.69121	.23148	4.32001	.24995	4.00086	58
3	.17723	5.64248	.19529	5.12069	.21347	4.68452	.23179	4.31430	.25026	3.99592	57
4	.17753	5.63295	.19559	5.11279	.21377	4.67786	.23209	4.30860	.25056	3.99099	56
5	.17783	5.62344	.19589	5.10490	.21408	4.67121	.23240	4.30291	.25087	3.98607	55
6	.17813	5.61397	.19619	5.09704	.21438	4.66458	.23271	4.29724	.25118	3.98117	54
7	.17843	5.60452	.19649	5.08921	.21469	4.65797	23301	4.29159	.25149	3.97627	53
8	.17873	5.59511	.19680	5.08139	.21499	4.65138	.23332	4.28595	.25180	3.97139	52
9	.17903	5.58573	.19710	5.07360	.21529	4.64480	.23363	4.28032	.25211	3.96651	51
10	.17933	5.57638	.19740	5.06584	.21560	4.63825	.23393	4.27471	.25242	3.96165	50
11	.17963	5.56706	.19770	5.05809	.21590	4.63171	.23424	4.26911	.25273	3.95680	49
12	.17993	5.55777	.19801	5.05037	.21621	4.62518	.23455	4.26352	.25304	3.95196	48
13	.18023	5.54851	.19831	5.04267	.21651	4.61868	.23485	4.25795	.25335	3.94713	47
14	.18053	5.53927	.19861	5.03499	.21682	4.61219	.23516	4.25239	.25366	3.94232	46
15	.18083	5.53007	.19891	5.02734	.21712	4.60572	.23547	4.24685	.25397	3.93751	45
16	.18113	5.52090	.19921	5.01971	.21743	4.59927	.23578	4.24132	.25428	3.93271	44
17	.18143	5.51176	.19952	5.01210	.21773	4.59283	.23608	4.23580	.25459	3.92793	43
18	.18173	5.50264	.19982	5.00451	.21804	4.58641	.23639	4.23030	.25490	3.92316	42
19	.18203	5.49356	.20012	4.99695	.21834	4.58001	.23670	4.22481	.25521	3.91839	41
20	.18233	5.48451	.20042	4.98940	.21864	4.57363	.23700	4.21933	.25552	3.91364	40
21	.18263	5.47548	.20073	4.98188	.21895	4.56726	.23731	4.21387	.25583	3.90890	39
22	.18293	5.46648	.20103	4.97438	.21925	4.56091	.23762	4.20842	.25614	3.90417	38
23	.18323	5.45751	.20133	4.96690	.21956	4.55458	.23793	4.20298	.25645	3.89945	37
24	.18353	5.44857	.20164	4.95945	.21986	4.54826	.23823	4.19756	.25676	3.89474	36
25	.18384	5.43966	.20194	4.95201	.22017	4.54196	.23854	4.19215	.25707	3.89004	35
26	.18414	5.43077	.20224	4.94460	.22047	4.53568	.23885	4.18675	.25738	3.88536	34
27	.18444	5.42192	.20254	4.93721	.22078	4.52941	.23916	4.18137	.25769	3.88068	33
28	.18474	5.41309	.20285	4.92984	.22108	4.52316	.23946	4.17600	.25800	3.87601	32
29	.18504	5.40429	.20315	4.92249	.22139	4.51693	.23977	4.17064	.25831	3.87136	31
30	.18534	5.39552	.20345	4.91516	.22169	4.51071	.24008	4.16530	.25862	3.86671	30
31	.18564	5.38677	.20376	4.90785	.22200	4.50451	.24039	4.15997	.25893	3.86208	29
32	.18594	5.37805	.20406	4.90056	.22231	4.49832	.24069	4.15465	.25924	3.85745	28
33	.18624	5.36936	.20436	4.89330	.22261	4.49215	.24100	4.14934	.25955	3.85284	27
34	.18654	5.36070	.20466	4.88605	.22292	4.48600	.24131	4.14405	.25986	3.84824	26
35	.18684	5.35206	.20497	4.87882	.22322	4.47986	.24162	4.13877	.26017	3.84364	25
36	.18714	5.34345	.20527	4.87162	.22353	4.47374	.24193	4.13350	.26048	3.83906	24
37	.18745	5.33487	.20557	4.86444	.22383	4.46764	.24223	4.12825	.26079	3.83449	23
38	.18775	5.32631	.20588	4.85727	.22414	4.46155	.24254	4.12301	.26110	3.82992	22
39	.18805	5.31778	.20618	4.85013	.22444	4.45548	.24285	4.11778	.26141	3.82537	21
40	.18835	5.30928	.20648	4.84300	.22475	4.44942	.24316	4.11256	.26172	3.82083	20
41	.18865	5.30080	.20679	4.83590	.22505	4.44338	.24347	4.10736	.26203	3.81630	19
42	.18895	5.29235	.20709	4.82882	.22536	4.43735	.24377	4.10216	.26235	3.81177	18
43	.18925	5.28393	.20739	4.82175	.22567	4.43134	.24408	4.09699	.26266	3.80726	17
44	.18955	5.27553	.20770	4.81471	.22597	4.42534	.24439	4.09182	.26297	3.80276	16
45	.18986	5.26715	.20800	4.80769	.22628	4.41936	.24470	4.08666	.26328	3.79827	15
46	.19016	5.25880	.20830	4.80068	.22658	4.41340	.24501	4.08152	.26359	3.79378	14
47	.19046	5.25048	.20861	4.79370	.22689	4.40745	.24532	4.07639	.26390	3.78931	13
48	.19076	5.24218	.20891	4.78673	.22719	4.40152	.24562	4.07127	.26421	3.78485	12
49	.19106	5.23391	.20921	4.77978	.22750	4.39560	.24593	4.06616	.26452	3.78040	11
50	.19136	5.22566	.20952	4.77286	.22781	4.38969	.24624	4.06107	.26483	3.77595	10
51	.19166	5.21744	.20982	4.76595	.22811	4.38381	.24655	4.05599	.26515	3.77152	9
52	.19197	5.20925	.21013	4.75906	.22842	4.37793	.24686	4.05092	.26546	3.76709	8
53	.19227	5.20107	.21043	4.75219	.22872	4.37207	.24717	4.04586	.26577	3.76268	7
54	.19257	5.19293	.21073	4.74534	.22903	4.36623	.24747	4.04081	.26608	3.75828	6
55	.19287	5.18480	.21104	4.73851	.22934	4.36040	.24778	4.03578	.26639	3.75388	5
56	.19317	5.17671	.21134	4.73170	.22964	4.35459	.24809	4.03076	.26670	3.74950	4
57	.19347	5.16863	.21164	4.72490	.22995	4.34879	.24840	4.02574	.26701	3.74512	3
58	.19378	5.16058	.21195	4.71813	.23026	4.34300	.24871	4.02074	.26733	3.74075	2
59	.19408	5.15256	.21225	4.71137	.23056	4.33723	.24902	4.01576	.26764	3.73640	1
60	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	0
	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.26795	3.73205	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.26826	3.72771	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.26857	3.72338	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.26888	3.71907	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.26920	3.71476	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.26951	3.71046	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.26982	3.70616	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.27013	3.70188	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.27044	3.69761	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.27076	3.69335	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.27107	3.68909	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.27138	3.68485	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.27169	3.68061	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.27201	3.67638	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.27232	3.67217	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.27263	3.66796	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.27294	3.66376	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.27326	3.65957	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.27357	3.65538	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.27388	3.65121	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.27419	3.64705	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.27451	3.64289	.29337	5.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.27482	3.63874	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.27513	3.63461	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.27545	3.63048	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.27576	3.62636	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.27607	3.62224	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.27638	3.61814	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.27670	3.61405	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.27701	3.60996	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.27732	3.60588	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.27764	3.60181	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.27795	3.59775	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.27826	3.59370	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.27858	3.58966	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.27889	3.58562	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.27921	3.58160	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.27952	3.57758	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.27983	3.57357	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.28015	3.56957	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.28046	3.56557	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.28077	3.56159	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.28109	3.55761	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.28140	3.55364	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.28172	3.54968	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.28203	3.54573	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.28234	3.54179	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.28266	3.53785	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.28297	3.53393	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.28329	3.53001	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.28360	3.52609	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.28391	3.52219	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.28423	3.51829	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.28454	3.51441	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.28486	3.51053	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.28517	3.50666	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.28549	3.50279	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.28580	3.49894	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.28612	3.49509	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.28643	3.49125	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	.44558	2.24428	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	.44593	2.24252	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	.44627	2.24077	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	.44662	2.23902	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	.44697	2.23727	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	.44732	2.23553	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	.44767	2.23378	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	.44802	2.23204	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	.44837	2.23030	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	.44872	2.22857	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	.44907	2.22683	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	.44942	2.22510	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	.44977	2.22337	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	.45012	2.27164	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	.45047	2.21992	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	.45082	2.21819	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	.45117	2.21647	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	.45152	2.21475	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	.45187	2.21304	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	43136	2.31826	.45222	2.21132	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	.45257	2.20961	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	.45292	2.20790	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43230	2.31271	.45327	2.20619	37
24	.37190	2.68892	.39190	2.35170	.41217	2.42618	.43274	2.31086	.45362	2.20449	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	.45397	2.20278	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	.45432	2.20108	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	.45467	2.19938	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	.45502	2.19769	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	.45538	2.19599	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	.45573	2.19430	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	.45608	2.19261	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	.45643	2.19092	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	.45678	2.18923	27
34	.37521	2 66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	.45713	2.18755	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	.45748	2.18587	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	.45784	2.18419	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	.45819	2.18251	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	.45854	2.18084	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	.45889	2.17916	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	.45924	2.17749	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	.45960	2.17582	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	.45995	2.17416	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	.46030	2.17249	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	.46065	2.17083	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	.46101	2.16917	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	.46136	2.16751	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	.46171	2.16585	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	.46206	2.16420	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	.46242	2.16255	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	.46277	2.16090	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	.46312	2.15925	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	.46348	2.15760	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	.46383	2.15596	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	.46418	2.15432	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	.46454	2.15268	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	.46489	2.15104	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	.46525	2.14940	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	.46560	2.14777	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	.46595	2.14614	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	.46631	2.14451	0
	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Co4ang.	Tang.	Cotang.	Tang.	
′	69°		68°		67°		66°		65°		′

′	25°		26°		27°		28°		29°		′
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	60
1	.46666	2.14288	.48809	2.04879	.50989	1.96120	.53208	1.87941	.55469	1.80281	59
2	.46702	2.14125	.48845	2.04728	.51026	1.95979	.53246	1.87809	.55507	1.80158	58
3	.46737	2.13963	.48881	2.04577	.51063	1.95838	.53283	1.87677	.55545	1.80034	57
4	.46772	2.13801	.48917	2.04426	.51099	1.95698	.53320	1.87546	.55583	1.79911	56
5	.46808	2.13639	.48953	2.04276	.51136	1.95557	.53358	1.87415	.55621	1.79788	55
6	.46843	2.13477	.48989	2.04125	.51173	1.95417	.53395	1.87283	.55659	1.79665	54
7	.46879	2.13316	.49026	2.03975	.51209	1.95277	.53432	1.87152	.55697	1.79542	53
8	.46914	2.13154	.49062	2.03825	.51246	1.95137	.53470	1.87021	.55736	1.79419	52
9	.46950	2.12993	.49098	2.03675	.51283	1.94997	.53507	1.86891	.55774	1.79296	51
10	.46985	2.12832	.49134	2.03526	.51319	1.94858	.53545	1.86760	.55812	1.79174	50
11	.47021	2.12671	.49170	2.03376	.51356	1.94718	.53582	1.86630	.55850	1.79051	49
12	.47056	2.12511	.49206	2.03227	.51393	1.94579	.53620	1.86499	.55888	1.78929	48
13	.47092	2.12350	.49242	2.03078	.51430	1.94440	.53657	1.86369	.55926	1.78807	47
14	.47128	2.12190	.49278	2.02929	.51467	1.94301	.53694	1.86239	.55964	1.78685	46
15	.47163	2.12030	.49315	2.02780	.51503	1.94162	.53732	1.86109	.56003	1.78563	45
16	.47199	2.11871	.49351	2.02631	.51540	1.94023	.53769	1.85979	.56041	1.78441	44
17	.47234	2.11711	.49387	2.02483	.51577	1.93885	.53807	1.85850	.56079	1.78319	43
18	.47270	2.11552	.49423	2.02335	.51614	1.93746	.53844	1.85720	.56117	1.78198	42
19	.47305	2.11392	.49459	2.02187	.51651	1.93608	.53882	1.85591	.56156	1.78077	41
20	.47341	2.11233	.49495	2.02039	.51688	1.93470	.53920	1.85462	.56194	1.77955	40
21	.47377	2.11075	.49532	2.01891	.51724	1.93332	.53957	1.85333	.56232	1.77834	39
22	.47412	2.10916	.49568	2.01743	.51761	1.93195	.53995	1.85204	.56270	1.77713	38
23	.47448	2.10758	.49604	2.01596	.51798	1.93057	.54032	1.85075	.56309	1.77592	37
24	.47483	2.10600	.49640	2.01449	.51835	1.92920	.54070	1.84946	.56347	1.77471	36
25	.47519	2.10442	.49677	2.01302	.51872	1.92782	.54107	1.84818	.56385	1.77351	35
26	.47555	2.10284	.49713	2.01155	.51909	1.92645	.54145	1.84689	.56424	1.77230	34
27	.47590	2.10126	.49749	2.01008	.51946	1.92508	.54183	1.84561	.56462	1.77110	33
28	.47626	2.09969	.49786	2.00862	.51983	1.92371	.54220	1.84433	.56501	1.76990	32
29	.47662	2.09811	.49822	2.00715	.52020	1.92235	.54258	1.84305	.56539	1.76869	31
30	.47698	2.09654	.49858	2.00569	.52057	1.92098	.54296	1.84177	.56577	1.76749	30
31	.47733	2.09498	.49894	2.00423	.52094	1.91962	.54333	1.84049	.56616	1.76629	29
32	.47769	2.09341	.49931	2.00277	.52131	1.91826	.54371	1.83922	.56654	1.76510	28
33	.47805	2.09184	.49967	2.00131	.52168	1.91690	.54409	1.83794	.56693	1.76390	27
34	.47840	2.09028	.50004	1.99986	.52205	1.91554	.54446	1.83667	.56731	1.76271	26
35	.47876	2.08872	.50040	1.99841	.52242	1.91418	.54484	1.83540	.56769	1.76151	25
36	.47912	2.08716	.50076	1.99695	.52279	1.91282	.54522	1.83413	.56808	1.76032	24
37	.47948	2.08560	.50113	1.99550	.52316	1.91147	.54560	1.83286	.56846	1.75913	23
38	.47984	2.08405	.50149	1.99406	.52353	1.91012	.54597	1.83159	.56885	1.75794	22
39	.48019	2.08250	.50185	1.99261	.52390	1.90876	.54635	1.83033	.56923	1.75675	21
40	.48055	2.08094	.50222	1.99116	.52427	1.90741	.54673	1.82906	.56962	1.75556	20
41	.48091	2.07939	.50258	1.98972	.52464	1.90607	.54711	1.82780	.57000	1.75437	19
42	.48127	2.07785	.50295	1.98828	.52501	1.90472	.54748	1.82654	.57039	1.75319	18
43	.48163	2.07630	.50331	1.98684	.52538	1.90337	.54786	1.82528	.57078	1.75200	17
44	.48198	2.07476	.50368	1.98540	.52575	1.90203	.54824	1.82402	.57116	1.75082	16
45	.48234	2.07321	.50404	1.98396	.52613	1.90069	.54862	1.82276	.57155	1.74964	15
46	.48270	2.07167	.50441	1.98253	.52650	1.89935	.54900	1.82150	.57193	1.74846	14
47	.48306	2.07014	.50477	1.98110	.52687	1.89801	.54938	1.82025	.57232	1.74728	13
48	.48342	2.06860	.50514	1.97966	.52724	1.89667	.54975	1.81899	.57271	1.74610	12
49	.48378	2.06706	.50550	1.97823	.52761	1.89533	.55013	1.81774	.57309	1.74492	11
50	.48414	2.06553	.50587	1.97681	.52798	1.89400	.55051	1.81649	.57348	1.74375	10
51	.48450	2.06400	.50623	1.97538	.52836	1.89266	.55089	1.81524	.57386	1.74257	9
52	.48486	2.06247	.50660	1.97395	.52873	1.89133	.55127	1.81399	.57425	1.74140	8
53	.48521	2.06094	.50696	1.97253	.52910	1.89000	.55165	1.81274	.57464	1.74022	7
54	.48557	2.05942	.50733	1.97111	.52947	1.88867	.55203	1.81150	.57503	1.73905	6
55	.48593	2.05790	.50769	1.96969	.52985	1.88734	.55241	1.81025	.57541	1.73788	5
56	.48629	2.05637	.50806	1.96827	.53022	1.88602	.55279	1.80901	.57580	1.73671	4
57	.48665	2.05485	.50843	1.96685	.53059	1.88469	.55317	1.80777	.57619	1.73555	3
58	.48701	2.05333	.50879	1.96544	.53096	1.88337	.55355	1.80653	.57657	1.73438	2
59	.48737	2.05182	.50916	1.96402	.53134	1.88205	.55393	1.80529	.57696	1.73321	1
60	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	.57735	1.73205	0
	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	
′	64°		63°		62°		61°		60°		′

′	30°		31°		32°		33°		34°		′
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.57735	1.73205	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	60
1	.57774	1.73089	.60126	1.66318	.62527	1.59930	.64982	1.53888	.67493	1.48163	59
2	.57813	1.72973	.60165	1.66209	.62568	1.59826	.65024	1.53791	.67536	1.48070	58
3	.57851	1.72857	.60205	1.66099	.62608	1.59723	.65065	1.53693	.67578	1.47977	57
4	.57890	1.72741	.60245	1.65990	.62649	1.59620	.65106	1.53595	.67620	1.47885	56
5	.57929	1.72625	.60284	1.65881	.62689	1.59517	.65148	1.53497	.67663	1.47792	55
6	.57968	1.72509	.60324	1.65772	.62730	1.59414	.65189	1.53400	.67705	1.47699	54
7	.58007	1.72393	.60364	1.65663	.62770	1.59311	.65231	1.53302	.67748	1.47607	53
8	.58046	1.72278	.60403	1.63554	.62811	1.59208	.65272	1.53205	.67790	1.47514	52
9	.58085	1.72163	.60443	1.65445	.62852	1.59105	.65314	1.53107	.67832	1.47422	51
10	.58124	1.72047	.60483	1.65337	.62892	1.59002	.65355	1.53010	.67875	1.47330	50
11	.58162	1.71932	.60522	1.65228	.62933	1.58900	.65397	1.52913	.67917	1.47238	49
12	.58201	1.71817	.60562	1.65120	.62973	1.58797	.65438	1.52816	.67960	1.47146	48
13	.58240	1.71702	.60602	1.65011	.63014	1.58695	.65480	1.52719	.68002	1.47053	47
14	.58279	1.71588	.60642	1.64903	.63055	1.58593	.65521	1.52622	.68045	1.46962	46
15	.58318	1.71473	.60681	1.64795	.63095	1.58490	.65563	1.52525	.68088	1.46870	45
16	.58357	1.71358	.60721	1.64687	.63136	1.58388	.65604	1.52429	.68130	1.46778	44
17	.58396	1.71244	.60761	1.64579	.63177	1.58286	.65646	1.52332	.68173	1.46686	43
18	.58435	1.71129	.60801	1.64471	.63217	1.58184	.65688	1.52235	.68215	1.46595	42
19	.58474	1.71015	.60841	1.64363	.63258	1.58083	.65729	1.52139	.68258	1.46503	41
20	.58513	1.70901	.60881	1.64256	.63299	1.57981	.65771	1.52043	.68301	1.46411	40
21	.58552	1.70787	.60921	1.64148	.63340	1.57879	.65813	1.51946	.68343	1.46320	39
22	.58591	1.70673	.60960	1.64041	.63380	1.57778	.65854	1.51850	.68386	1.46229	38
23	.5863[illegible]	1.70560	.61000	1.63934	.63421	1.57676	.65896	1.51754	.68429	1.46137	37
24	.58670	1.70446	.61040	1.63826	.63462	1.57575	.65938	1.51658	.68471	1.46046	36
25	.58709	1.70332	.61080	1.63719	.63503	1.57474	.65980	1.51562	.68514	1.45955	35
26	.58748	1.70219	.61120	1.63612	.63544	1.57372	.66021	1.51466	.68557	1.45864	34
27	.58787	1.70106	.61160	1.63505	.63584	1.57271	.66063	1.51370	.68600	1.45773	33
28	.58826	1.69992	.61200	1.63398	.63625	1.57170	.66105	1.51275	.68642	1.45682	32
29	.58865	1.69879	.61240	1.63292	.63666	1.57069	.66147	1.51179	.68685	1.45592	31
30	.58905	1.69766	.61280	1.63185	.63707	1.56969	.66189	1.51084	.68728	1.45501	30
31	.58944	1.69653	.61320	1.63079	.63748	1.56868	.66230	1.50988	.68771	1.45410	29
32	.58983	1.69541	.61360	1.62972	.63789	1.56767	.66272	1.50893	.68814	1.45320	28
33	.59022	1.69428	.61400	1.62866	.63830	1.56667	.66314	1.50797	.68857	1.45229	27
34	.59061	1.69316	.61440	1.62760	.63871	1.56566	.66356	1.50702	.68900	1.45139	26
35	.59101	1.69203	.61480	1.62654	.63912	1.56466	.66398	1.50607	.68942	1.45049	25
36	.59140	1.69091	.61520	1.62548	.63953	1.56366	.66440	1.50512	.68985	1.44958	24
37	.59179	1.68979	.61561	1.62442	.63994	1.56265	.66482	1.50417	.69028	1.44868	23
38	.59218	1.68866	.61601	1.62336	.64035	1.56165	.66524	1.50322	.69071	1.44778	22
39	.59258	1.68754	.61641	1.62230	.64076	1.56065	.66566	1.50228	.69114	1.44688	21
40	.59297	1.68643	.61681	1.62125	.64117	1.55966	.66608	1.50133	.69157	1.44598	20
41	.59336	1.68531	.61721	1.62019	.64158	1.55866	.66650	1.50038	.69200	1.44508	19
42	.59376	1.68419	.61761	1.61914	.64199	1.55766	.66692	1.49944	.69243	1.44418	18
43	.59415	1.68308	.61801	1.61808	.64240	1.55666	.66734	1.49849	.69286	1.44329	17
44	.59454	1.68196	.61842	1.61703	.64281	1.55567	.66776	1.49755	.69329	1.44239	16
45	.59494	1.68085	.61882	1.61598	.64322	1.55467	.66818	1.49661	.69372	1.44149	15
46	.59533	1.67974	.61922	1.61493	.64363	1.55368	.66860	1.49566	.69416	1.44060	14
47	.59573	1.67863	.61962	1.61388	.64404	1.55269	.66902	1.49472	.69459	1.43970	13
48	.59612	1.67752	.62003	1.61283	.64446	1.55170	.66944	1.49378	.69502	1.43881	12
49	.59651	1.67641	.62043	1.61179	.64487	1.55071	.66986	1.49284	.69545	1.43792	11
50	.59691	1.67530	.62083	1.61074	.64528	1.54972	.67028	1.49190	.69588	1.43703	10
51	.59730	1.67419	.62124	1.60970	.64569	1.54873	.67071	1.49097	.69631	1.43614	9
52	.59770	1.67309	.62164	1.60865	.64610	1.54774	.67113	1.49003	.69675	1.43525	8
53	.59809	1.67198	.62204	1.60761	.64652	1.54675	.67155	1.48909	.69718	1.43436	7
54	.59849	1.67088	.62245	1.60657	.64693	1.54576	.67197	1.48816	.69761	1.43347	6
55	.59888	1.66978	.62285	1.60553	.64734	1.54478	.67239	1.48722	.69804	1.43258	5
56	.59928	1.66867	.62325	1.60449	.64775	1.54379	.67282	1.48629	.69847	1.43169	4
57	.59967	1.66757	.62366	1.60345	.64817	1.54281	.67324	1.48536	.69891	1.43080	3
58	.60007	1.66647	.62406	1.60241	.64858	1.54183	.67366	1.48442	.69934	1.42992	2
59	.60046	1.66538	.62446	1.60137	.64899	1.54085	.67409	1.48349	.69977	1.42903	1
60	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	0
	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	
′	59°		58°		57°		56°		55°		′

′	35°		36°		37°		38°		39°		′
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.70021	1.42815	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.70064	1.42726	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.70107	1.42638	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.70151	1.42550	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.70194	1.42462	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.70238	1.42374	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.70281	1.42286	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.70325	1.42198	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.70368	1.42110	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.70412	1.42022	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.70455	1.41934	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.70499	1.41847	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.70542	1.41759	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.70586	1.41672	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.70629	1.41584	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.70673	1.41497	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.70717	1.41409	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.70760	1.41322	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.70804	1.41235	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.70848	1.41148	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.70891	1.41061	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.70935	1.40974	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.70979	1.40887	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.71023	1.40800	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.71066	1.40714	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.71110	1.40627	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.71154	1.40540	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.71198	1.40454	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.71242	1.40367	.73906	1.35307	.76640	1.30480	.79499	1.25867	.82336	1.21454	32
29	.71285	1.40281	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.71329	1.40195	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.71373	1.40109	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.71417	1.40022	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.71461	1.39936	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.71505	1.39850	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.71549	1.39764	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.71593	1.39679	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.71637	1.39593	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.71681	1.39507	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.71725	1.39421	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.71769	1.39336	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.71813	1.39250	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.71857	1.39165	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.71901	1.39079	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.71946	1.38994	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.71990	1.38909	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.72034	1.38824	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.72078	1.38738	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.72122	1.38653	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.72167	1.38568	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.72211	1.38484	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.72255	1.38399	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.72299	1.38314	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.72344	1.38229	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.72388	1.38145	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.72432	1.38060	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.72477	1.37976	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.72521	1.37891	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.72565	1.37807	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.72610	1.37722	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	
0	.83910	1.19175	.86929	1.15037	.90049	1.11061	.93252	1.07237	.96569	1.03553	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	.96625	1.03493	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	.96681	1.03433	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	.96738	1.03372	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	.96794	1.03312	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	.96850	1.03252	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	.96907	1.03192	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	.96963	1.03132	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	.97020	1.03072	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	.97076	1.03012	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	.97133	1.02952	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	.97189	1.02892	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	.97246	1.02832	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	.97302	1.02772	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	.97359	1.02713	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	.97416	1.02653	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	.97472	1.02593	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	.97529	1.02533	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	.97586	1.02474	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	.97643	1.02414	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	.97700	1.02355	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	.97756	1.02295	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	.97813	1.02236	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	.97870	1.02176	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	.97927	1.02117	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	.97984	1.02057	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	.98041	1.01998	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	.98098	1.01939	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	.98155	1.01879	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	.98213	1.01820	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	.98270	1.01761	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	.98327	1.01702	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	.98384	1.01642	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	.98441	1.01583	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	.98499	1.01524	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	.98556	1.01465	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	.98613	1.01406	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	.98671	1.01347	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	.98728	1.01288	22
39	.85862	1,16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	.98786	1.01229	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	.98843	1.01170	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	.98901	1.01112	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	.98958	1.01053	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	.99016	1.00994	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	.99073	1.00935	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	.99131	1.00876	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	.99189	1.00818	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	.99247	1.00759	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	.99304	1.00701	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	.99362	1.00642	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	.99420	1.00583	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	.99478	1.00525	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	.99536	1.00467	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	.99594	1.00408	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	.99652	1.00350	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	.99710	1.00291	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	.99768	1.00233	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	.99826	1.00175	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	.99884	1.00116	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	.99942	1.00058	1
60	.86929	1.15037	.80040	1.11061	.93252	1.07237	.96569	1.03553	1.00000	1.00000	0
	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	Cotang.	Tang.	
′	49°		48°		47°		46°		45°		′

0° | 179°

M.	Hr. A.M.	Hr. P.M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	12 0 0	0 0 0	Inf. neg.	Infinite.	Inf. neg.	Infinite.	10.00000	10.00000	60
1	11 59 52	0 8	6.46373	13.53627	6.46373	13.53627	00000	00000	59
2	59 44	0 16	76476	23524	76476	23524	00000	00000	58
3	59 36	0 24	94085	05915	94085	05915	00000	00000	57
4	59 28	0 32	7.06579	12.93421	7.06579	12.93421	00000	00000	56
5	11 59 20	0 0 40	7.16270	12.83730	7.16270	12.83730	10.00000	10.00000	55
6	59 12	0 48	24188	75812	24188	75812	00000	00000	54
7	59 4	0 56	30882	69118	30882	69118	00000	00000	53
8	58 56	1 4	36682	63318	36682	63318	00000	00000	52
9	58 48	1 12	41797	58203	41797	58203	00000	00000	51
10	11 58 40	0 1 20	7.46373	12.53627	7.46373	12.53627	10.00000	10.00000	50
11	58 32	1 28	50512	49488	50512	49488	00000	00000	49
12	58 24	1 36	54291	45709	54291	45709	00000	00000	48
13	58 16	1 44	57767	42233	57767	42233	00000	00000	47
14	58 8	1 52	60985	39015	60986	39014	00000	00000	46
15	11 58 0	0 2 0	7.63982	12.36018	7.63982	12.36018	10.00000	10.00000	45
16	57 52	2 8	66784	33216	66785	33215	00000	00000	44
17	57 44	2 16	69417	30583	69418	30582	00001	9.99999	43
18	57 36	2 24	71900	28100	71900	28100	00001	99999	42
19	57 28	2 32	74248	25752	74248	25752	00001	99999	41
20	11 57 20	0 2 40	7.76475	12.23525	7.76476	12.23524	10.00001	9.99999	40
21	57 12	2 48	78594	21406	78595	21405	00001	99999	39
22	57 4	2 56	80615	19385	80615	19385	00001	99999	38
23	56 56	3 4	82545	17455	82546	17454	00001	99999	37
24	56 48	3 12	84393	15607	84394	15606	00001	99999	36
25	11 56 40	0 3 20	7.86166	12.13834	7.86167	12.13833	10.00001	9.99999	35
26	56 32	3 28	87870	12130	87871	12129	00001	99999	34
27	56 24	3 36	89509	10491	89510	10490	00001	99999	33
28	56 16	3 44	91088	08912	91089	08911	00001	99999	32
29	56 8	3 52	92612	07388	92613	07387	00002	99998	31
30	11 56 0	0 4 0	7.94084	12.05916	7.94086	12.05914	10.00002	9.99998	30
31	55 52	4 8	95508	04492	95510	04490	00002	99998	29
32	55 44	4 16	96887	03113	96889	03111	00002	99998	28
33	55 36	4 24	98223	01777	98225	01775	00002	99998	27
34	55 28	4 32	99520	00480	99522	00478	00002	99998	26
35	11 55 20	0 4 40	8.00779	11.99221	8.00781	11.99219	10.00002	9.99998	25
36	55 12	4 48	02002	97998	02004	97996	00002	99998	24
37	55 4	4 56	03192	96808	03194	96806	00003	99997	23
38	54 56	5 4	04350	95650	04353	95647	00003	99997	22
39	54 48	5 12	05478	94522	05481	94519	00003	99997	21
40	11 54 40	0 5 20	8.06578	11.93422	8.06581	11.93419	10.00003	9.99997	20
41	54 32	5 28	07650	92350	07653	92347	00003	99997	19
42	54 24	5 36	08696	91304	08700	91300	00003	99997	18
43	54 16	5 44	09718	90282	09722	90278	00003	99997	17
44	54 8	5 52	10717	89283	10720	89280	00004	99996	16
45	11 54 0	0 6 0	8.11693	11.88307	8.11696	11.88304	10.00004	9.99996	15
46	53 52	6 8	12647	87353	12651	87349	00004	99996	14
47	53 44	6 16	13581	86419	13585	86415	00004	99996	13
48	53 36	6 24	14495	85505	14500	85500	00004	99996	12
49	53 28	6 32	15391	84609	15395	84605	00004	99996	11
50	11 53 20	0 6 40	8.16268	11.83732	8.16273	11.83727	10.00005	99995	10
51	53 12	6 48	17128	82872	17133	82867	00005	99995	9
52	53 4	6 56	17971	82029	17976	82024	00005	99995	8
53	52 56	7 4	18798	81202	18804	81196	00005	99995	7
54	52 48	7 12	19610	80390	19616	80384	00005	99995	6
55	11 52 40	0 7 20	8.20407	11.79593	8.20413	11.79587	10.00006	9.99994	5
56	52 32	7 28	21189	78811	21195	78805	00006	99994	4
57	52 24	7 36	21958	78042	21964	78036	00006	99994	3
58	52 16	7 44	22713	77287	22720	77280	00006	99994	2
59	52 8	7 52	23456	76544	23462	76538	00006	99994	1
60	52 0	8 0	24186	75814	24192	75808	00007	99993	0
M.	Hr. P.M.	Hr. A.M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

90° | 89°

1°									178°
M.	Hr. A.M.	Hr. P.M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	11 52 0	0 8 0	8.24186	11.75814	8.24192	11.75808	10.00007	9.99993	60
1	51 52	8 8	24903	75097	24910	75090	00007	99993	59
2	51 44	8 16	25609	74391	25616	74384	00007	99993	58
3	51 36	8 24	26304	73696	26312	73688	00007	99993	57
4	51 28	8 32	26988	73012	26996	73004	00008	99992	56
5	11 51 20	0 8 40	8.27661	11.72339	8.27669	11.72331	10.00008	9.99992	55
6	51 12	8 48	28324	71676	28332	71668	00008	99992	54
7	51 4	8 56	28977	71023	28986	71014	00008	99992	53
8	50 56	9 4	29621	70379	29629	70371	00008	99992	52
9	50 48	9 12	30255	69745	30263	69737	00009	99991	51
10	11 50 40	0 9 20	8.30879	11.69121	8.30888	11.69112	10.00009	9.99991	50
11	50 32	9 28	31495	68505	31505	68495	00009	99991	49
12	50 24	9 36	32103	67897	32112	67888	00010	99990	48
13	50 16	9 44	32702	67298	32711	67289	00010	99990	47
14	50 8	9 52	33292	66708	33302	66698	00010	99990	46
15	11 50 0	0 10 0	8.33875	11.66125	8.33886	11.66114	10.00010	9.99990	45
16	49 52	10 8	34450	65550	34461	65539	00011	99989	44
17	49 44	10 16	35018	64982	35029	64971	00011	99989	43
18	49 36	10 24	35578	64422	35590	64410	00011	99989	42
19	49 28	10 32	36131	63869	36143	63857	00011	99989	41
20	11 49 20	0 10 40	8.36678	11.63322	8.36689	11.63311	10.00012	9.99988	40
21	49 12	10 48	37217	62783	37229	62771	00012	99988	39
22	49 4	10 56	37750	62250	37762	62238	00012	99988	38
23	48 56	11 4	38276	61724	38289	61711	00013	99987	37
24	48 48	11 12	38796	61204	38809	61191	00013	99987	36
25	11 48 40	0 11 20	8.39310	11.60690	8.39323	11.60677	10.00013	9.99987	35
26	48 32	11 28	39818	60182	39832	60168	00014	99986	34
27	48 24	11 36	40320	59680	40334	59666	00014	99986	33
28	48 16	11 44	40816	59184	40830	59170	00014	99986	32
29	48 8	11 52	41307	58693	41321	58679	00015	99985	31
30	11 48 0	0 12 0	8.41792	11.58208	8.41807	11.58193	10.00015	9.99985	30
31	47 52	12 8	42272	57728	42287	57713	00015	99985	29
32	47 44	12 16	42746	57254	42762	57238	00016	99984	28
33	47 36	12 24	43216	56784	43232	56768	00016	99984	27
34	47 28	12 32	43680	56320	43696	56304	00016	99984	26
35	11 47 20	0 12 40	8.44139	11.55861	8.44156	11.55844	10.00017	9.99983	25
36	47 12	12 48	44594	55406	44611	55389	00017	99983	24
37	47 4	12 56	45044	54956	45061	54939	00017	99983	23
38	46 56	13 4	45489	54511	45507	54493	00018	99982	22
39	46 48	13 12	45930	54070	45948	54052	00018	99982	21
40	11 46 40	0 13 20	8.46366	11.53634	8.46385	11.53615	10.00018	9.99982	20
41	46 32	13 28	46799	53201	46817	53183	00019	99981	19
42	46 24	13 36	47226	52774	47245	52755	00019	99981	18
43	46 16	13 44	47650	52350	47669	52331	00019	99981	17
44	46 8	13 52	48069	51931	48089	51911	00020	99980	16
45	11 46 0	0 14 0	8.48485	11.51515	8.48505	11.51495	10.00020	9.99980	15
46	45 52	14 8	48896	51104	48917	51083	00021	99979	14
47	45 44	14 16	49304	50696	49325	50675	00021	99979	13
48	45 36	14 24	49708	50292	49729	50271	00021	99979	12
49	45 28	14 32	50108	49892	50130	49870	00022	99978	11
50	11 45 20	0 14 40	8.50504	11.49496	8.50527	11.49473	10.00022	9.99978	10
51	45 12	14 48	50897	49103	50920	49080	00023	99977	9
52	45 4	14 56	51287	48713	51310	48690	00023	99977	8
53	44 56	15 4	51673	48327	51696	48304	00023	99977	7
54	44 48	15 12	52055	47945	52079	47921	00024	99976	6
55	11 44 40	0 15 20	8.52434	11.47566	8.52459	11.47541	10.00024	9.99976	5
56	44 32	15 28	52810	47190	52835	47165	00025	99975	4
57	44 24	15 36	53183	46817	53208	46792	00025	99975	3
58	44 16	15 44	53552	46448	53578	46422	00026	99974	2
59	44 8	15 52	53919	46081	53945	46055	00026	99974	1
60	44 0	16 0	54282	45718	54308	45692	00026	99974	0
M.	Hr. P.M.	Hr. A.M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
91°									88°

2°									177°
M.	Hr. A.M.	Hr. P.M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	11 44 0	0 16 0	8.54282	11.45718	8.54308	11.45692	10.00026	9.99974	60
1	43 52	16 8	54642	45358	54669	45331	00027	99973	59
2	43 44	16 16	54999	45001	55027	44973	00027	99973	58
3	43 36	16 24	55354	44646	55382	44618	00028	99972	57
4	43 28	16 32	55705	44295	55734	44266	00028	99972	56
5	11 43 20	0 16 40	8.56054	11.43946	8.56083	11.43917	10.00029	9.99971	55
6	43 12	16 48	56400	43600	56429	43571	00029	99971	54
7	43 4	16 56	56743	43257	56773	43227	00030	99970	53
8	42 56	17 4	57084	42916	57114	42886	00030	99970	52
9	42 48	17 12	57421	42579	57452	42548	00031	99969	51
10	11 42 40	0 17 20	8.57757	11.42243	8.57788	11.42212	10.00031	9.99969	50
11	42 32	17 28	58089	41911	58121	41879	00032	99968	49
12	42 24	17 36	58419	41581	58451	41549	00032	99968	48
13	42 16	17 44	58747	41253	58779	41221	00033	99967	47
14	42 8	17 52	59072	40928	59105	40895	00033	99967	46
15	11 42 0	0 18 0	8.59395	11.40605	8.59428	11.40572	10.00033	9.99967	45
16	41 52	18 8	59715	40285	59749	40251	00034	99966	44
17	41 44	18 16	60033	39967	60068	39932	00034	99966	43
18	41 36	18 24	60349	39651	60384	39616	00035	99965	42
19	41 28	18 32	60662	39338	60698	39302	00036	99964	41
20	11 41 20	0 18 40	8.60973	11.39027	8.61009	11.38991	10.00036	9.99964	40
21	41 12	18 48	61282	38718	61319	38681	00037	99963	39
22	41 4	18 56	61589	38411	61626	38374	00037	99963	38
23	40 56	19 4	61894	38106	61931	38069	00038	99962	37
24	40 48	19 12	62196	37804	62234	37766	00038	99962	36
25	11 40 40	0 19 20	8.62497	11.37503	8.62535	11.37465	10.00039	9.99961	35
26	40 32	19 28	62795	37205	62834	37166	00039	99961	34
27	40 24	19 36	63091	36909	63131	36869	00040	99960	33
28	40 16	19 44	63385	36615	63426	36574	00040	99960	32
29	40 8	19 52	63678	36322	63718	36282	00041	99959	31
30	11 40 0	0 20 0	8.63968	11.36032	8.64009	11.35991	10.00041	9.99959	30
31	39 52	20 8	64256	35744	64298	35702	00042	99958	29
32	39 44	20 16	64543	35457	64585	35415	00042	99958	28
33	39 36	20 24	64827	35173	64870	35130	00043	99957	27
34	39 28	20 32	65110	34890	65154	34846	00044	99956	26
35	11 39 20	0 20 40	8.65391	11.34609	8.65435	11.34565	10.00044	9.99956	25
36	39 12	20 48	65670	34330	65715	34285	00045	99955	24
37	39 4	20 56	65947	34053	65993	34007	00045	99955	23
38	38 56	21 4	66223	33777	66269	33731	00046	99954	22
39	38 48	21 12	66497	33503	66543	33457	00046	99954	21
40	11 38 40	0 21 20	8.66769	11.33231	8.66816	11.33184	10.00047	9.99953	20
41	38 32	21 28	67039	32961	67087	32913	00048	99952	19
42	38 24	21 36	67308	32692	67356	32644	00048	99952	18
43	38 16	21 44	67575	32425	67624	32376	00049	99951	17
44	38 8	21 52	67841	32159	67890	32110	00049	99951	16
45	11 38 0	0 22 0	8.68104	11.31896	8.68154	11.31846	10.00050	9.99950	15
46	37 52	22 8	68367	31633	68417	31583	00051	99949	14
47	37 44	22 16	68627	31373	68678	31322	00051	99949	13
48	37 36	22 24	68886	31114	68938	31062	00052	99948	12
49	37 28	22 32	69144	30856	69196	30804	00052	99948	11
50	11 37 20	0 22 40	8.69400	11.30600	8.69453	11.30547	10.00053	9.99947	10
51	37 12	22 48	69654	30346	69708	30292	00054	99946	9
52	37 4	22 56	69907	30093	69962	30038	00054	99946	8
53	36 56	23 4	70159	29841	70214	29786	00055	99945	7
54	36 48	23 12	70409	29591	70465	29535	00056	99944	6
55	11 36 40	0 23 20	8.70658	11.29342	8.70714	11.29286	10.00056	9.99944	5
56	36 32	23 28	70905	29095	70962	29038	00057	99943	4
57	36 24	23 36	71151	28849	71208	28792	00058	99942	3
58	36 16	23 44	71395	28605	71453	28547	00058	99942	2
59	36 8	23 52	71638	28362	71697	28303	00059	99941	1
60	36 0	24 0	71880	28120	71940	28060	00060	99940	0
M.	Hr. P.M.	Hr. A.M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
92°									87°

3° 176°

M.	Hr. A.M.	Hr. P.M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	11 36 0	0 24 0	8.71880	11.28120	8.71940	11.28060	10.00060	9.99940	60
1	35 52	24 8	72120	27880	72181	27819	00060	99940	59
2	35 44	24 16	72359	27641	72420	27580	00061	99939	58
3	35 36	24 24	72597	27403	72659	27341	00062	99938	57
4	35 28	24 32	72834	27166	72896	27104	00062	99938	56
5	11 35 20	0 24 40	8.73069	11.26931	8.73132	11.26868	10.00063	9.99937	55
6	35 12	24 48	73303	26697	73366	26634	00064	99936	54
7	35 4	24 56	73535	26465	73600	26400	00064	99936	53
8	34 56	25 4	73767	26233	73832	26168	00065	99935	52
9	34 48	25 12	73997	26003	74063	25937	00066	99934	51
10	11 34 40	0 25 20	8.74226	11.25774	8.74292	11.25708	10.00066	9.99934	50
11	34 32	25 28	74454	25546	74521	25479	00067	99933	49
12	34 24	25 36	74680	25320	74748	25252	00068	99932	48
13	34 16	25 44	74906	25094	74974	25026	00068	99932	47
14	34 8	25 52	75130	24870	75199	24801	00069	99931	46
15	11 34 0	0 26 0	8.75353	11.24647	8.75423	11.24577	10.00070	9.99930	45
16	33 52	26 8	75575	24425	75645	24355	00071	99929	44
17	33 44	26 16	75795	24205	75867	24133	00071	99929	43
18	33 36	26 24	76015	23985	76087	23913	00072	99928	42
19	33 28	26 32	76234	23766	76306	23694	00073	99927	41
20	11 33 20	0 26 40	8.76451	11.23549	8.76525	11.23475	10.00074	9.99926	40
21	33 12	26 48	76667	23333	76742	23258	00074	99926	39
22	33 4	26 56	76883	23117	76958	23042	00075	99925	38
23	32 56	27 4	77097	22903	77173	22827	00076	99924	37
24	32 48	27 12	77310	22690	77387	22613	00077	99923	36
25	11 32 40	0 27 20	8.77522	11.22478	8.77600	11.22400	10.00077	9.99923	35
26	32 32	27 28	77733	22267	77811	22189	00078	99922	34
27	32 24	27 36	77943	22057	78022	21978	00079	99921	33
28	32 16	27 44	78152	21848	78232	21768	00080	99920	32
29	32 8	27 52	78360	21640	78441	21559	00080	99920	31
30	11 32 0	0 28 0	8.78568	11.21432	8.78649	11.21351	10.00081	9.99919	30
31	31 52	28 8	78774	21226	78855	21145	00082	99918	29
32	31 44	28 16	78979	21021	79061	20939	00083	99917	28
33	31 36	28 24	79183	20817	79266	20734	00083	99917	27
34	31 28	28 32	79386	20614	79470	20530	00084	99916	26
35	11 31 20	0 28 40	8.79588	11.20412	8.79673	11.20327	10.00085	9.99915	25
36	31 12	28 48	79789	20211	79875	20125	00086	99914	24
37	31 4	28 56	79990	20010	80076	19924	00087	99913	23
38	30 56	29 4	80189	19811	80277	19723	00087	99913	22
39	30 48	29 12	80388	19612	80476	19524	00088	99912	21
40	11 30 40	0 29 20	8.80585	11.19415	8.80674	11.19326	10.00089	9.99911	20
41	30 32	29 28	80782	19218	80872	19128	00090	99910	19
42	30 24	29 36	80978	19022	81068	18932	00091	99909	18
43	30 16	29 44	81173	18827	81264	18736	00091	99909	17
44	30 8	29 52	81367	18633	81459	18541	00092	99908	16
45	11 30 0	0 30 0	8.81560	11.18440	8.81653	11.18347	10.00093	9.99907	15
46	29 52	30 8	81752	18248	81846	18154	00094	99906	14
47	29 44	30 16	81944	18056	82038	17962	00095	99905	13
48	29 36	30 24	82134	17866	82230	17770	00096	99904	12
49	29 28	30 32	82324	17676	82420	17580	00096	99904	11
50	11 29 20	0 30 40	8.82513	11.17487	8.82610	11.17390	10.00097	9.99903	10
51	29 12	30 48	82701	17299	82799	17201	00098	99902	9
52	29 4	30 56	82888	17112	82987	17013	00099	99901	8
53	28 56	31 4	83075	16925	83175	16825	00100	99900	7
54	28 48	31 12	83261	16739	83361	16639	00101	99899	6
55	11 28 40	0 31 20	8.83446	11.16554	8.83547	11.16453	10.00102	9.99898	5
56	28 32	31 28	83630	16370	83732	16268	00102	99898	4
57	28 24	31 36	83813	16187	83916	16084	00103	99897	3
58	28 16	31 44	83996	16004	84100	15900	00104	99896	2
59	28 8	31 52	84177	15823	84282	15718	00105	99895	1
60	28 0	32 0	84358	15642	84464	15536	00106	99894	0
M.	Hr. P.M.	Hr. A.M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

93° 86°